2. Temperature, Pressure, Wind, and Minor Constituents.

2.1 Distributions of temperature, pressure and wind.

Close examination of Figs. 1.7-1.10 of MS reveals the following features that cry out for explanation:

• In the zonal mean cross sections (Figs. 1.7 and 1.8), zonal mean wind (positive from the west, or westerly) increases with height where temperature decreases toward the pole.

• Seasonal mean and instantaneous wind direction at mid-tropospheric and stratospheric heights follows contours of constant height on isobaric surfaces (Figs. 1.9-1.12).

• Seasonal mean and instantaneous wind speed at mid-tropospheric and stratospheric heights tends to be inversely proportional to spacing of contours (Figs. 1.9-1.12).

• As a consequence of the preceding two bullets, the relationship between the wind and height contours on an isobaric surface tends to resemble flow between walls of a channel; however, since the contours evolve over time, this flow is more aptly described as flow channeled by "flexible membranes".

Several other features are noteworthy in Fig. 1.7, e.g., decrease of tropopause height toward the poles, increase of lower stratospheric temperature toward the poles, decrease of mid-stratospheric temperature from summer to winter pole at solstice seasons, and increase of mesopause level temperature from summer to winter pole at solstice seasons. Because of the first bullet above, these thermal features are associated with the following noteworthy features in Fig. 1.9 and 1.10: jets (closed contour wind speed maxima in the latitude height plane) at the mid-latitude tropopause, and stratosphere-mesosphere summer easterlies and winter westerlies with jets in the upper mesosphere. Figs. 8 and 9 reveal equatorward and poleward meanders of the prevailing westerly winds with sharper features of smaller scale on the daily map. On the monthly mean map, ridges (poleward meanders) tend to occur near north-south mountain ranges or continental west coasts, and troughs (equatorward meanders) tending to occur over continental east coasts and western oceans. The patterns depicted in Fig. 8 are typical of autumn, winter, and spring. Weaker patterns, displaced toward higher latitude, occur in summer. Figs. 1.11-1.12 show similar features, but the horizontal scale of the meanders tends to be larger in the stratospheric maps.

In the next two sections we will investigate the mechanism responsible for the features noted in the four bullets above.

2.2 Geostrophic balance.

If the planetary rotation rate is fast enough, pressure and horizontal wind fields are in approximate geostrophic balance as a result of the coriolis force. To analyze this balance, we will use a cartesian coordinate system that is locally tangent to the planet's surface with coordinate directions, unit vectors, and wind velocity components as follows:

Coordinates	Unit vectors	Velocity components	
x	$\hat{\mathbf{i}}$	u	eastward
y	$\hat{\mathbf{j}}$	v	northward
z	$\hat{\mathbf{k}}$	w	upward

The horizontal wind is

$$\mathbf{v_h} = \hat{\mathbf{i}}u + \hat{\mathbf{j}}v \; . \tag{2.1}$$

On the large scales of the trough ridge systems in MS Figs 1.9-1.12, the vertical wind component is at least two orders of magnitude smaller than the horizontal wind components, so we consider here only the horizontal components. These are driven by the horizontal pressure gradient, which is a force per unit volume

$$\nabla_{\mathbf{h}} p = \hat{\mathbf{i}}\frac{\partial p}{\partial x} + \hat{\mathbf{j}}\frac{\partial p}{\partial y} \; . \tag{2.2}$$

If $\Omega = \dfrac{2\pi}{\text{rotation} - \text{period}} = 0.729\text{x}10^{-4}\,\text{s}^{-1}$ for Earth and φ= latitude, geostrophic balance is given in terms of the coriolis parameter, $f \equiv 2\Omega\sin\varphi$:

$$2\Omega\sin\varphi \cdot \hat{\mathbf{k}}\mathbf{x}\mathbf{v_h} = -\frac{1}{\rho}\nabla_{\mathbf{h}} p \tag{2.3}$$

geostrophic balance

(Note that rotation period is the period of the sidereal day, not the solar day).

Rearranging (2.3),

$$\begin{aligned} \mathbf{v_h} &= \frac{1}{\rho f}\hat{\mathbf{k}}\mathbf{x}\nabla_{\mathbf{h}} p \\ u &= -\frac{1}{\rho f}\frac{\partial p}{\partial y} \\ v &= \frac{1}{\rho f}\frac{\partial p}{\partial x} \; . \end{aligned} \tag{2.4}$$

This geostrophic wind has the properties that: (a) horizontal wind is parallel to pressure contours on a geopotential surface, (b) wind speed is approximately inversely proportional to contour spacing (proportional to pressure gradient), and as a consequence of (a) and (b), (c) as pressure contours evolve over time, the flow evolves with it (approximately) as if the pressure contours act like "flexible membranes", channeling the flow.

Why is geostrophic balance a good approximation? To answer this, we proceed in two steps, first deriving the coriolis effect in the equation of motion using a heuristic approach (see MS Chap. 10 for a more formal derivation), and second performing a simple scaling analysis to identify the dominant terms in the equation of motion.

For our heuristic derivation, consider separately the coriolis force acting on east-west and on north-south flow relative to a coordinate frame rotating with the planet. The effects on an arbitrary horizontal flow can be obtained by superposing these two effects.

(a) On eastward (westward) flow, the coriolis force appears as excess (deficit) centrifugal force relative to the centrifugal force acting on a resting object due to planetary rotation. The magnitude of this force is obtained by expanding the total centrifugal acceleration, recognizing that atmospheric wind speeds are small compared with the rotational speed of the planet. Thus total centrifugal acceleration is

$$\Omega_{tot}^2 R = \left(\Omega + \frac{u}{R}\right)^2 R = \Omega^2 R + 2\Omega u + \mathrm{HOT}$$

where Ω_{tot}^2 is total rotation rate, due to both planet and zonal wind, $R = a\cos\varphi$ is radial distance outward from the planetary rotation axis, u is relative eastward velocity, and HOT represents a higher order term in the expansion that is much smaller than the explicitly expressed terms. Term $\Omega^2 R$ is the planetary centrifugal acceleration, acting radially outward from the rotation axis. This term has already been incorporated in apparent gravity and in our use of geopotential surfaces as vertical coordinate surfaces. Term $2\Omega u$ is the coriolis acceleration acting on the eastward relative motion speed u. Since it acts radially outward, it has two components in the tangent plane (x,y,z) coordinate system: $2\Omega u\cos\varphi$ upward and $-2\Omega u\sin\varphi$ poleward (note that since $\sin\varphi$ changes sign between hemispheres, this acceleration is in the -y direction in the northern hemisphere, and in the +y direction in the southern hemisphere). The HOT are real but small effects due to curvature of horizontal motion on the (nearly) spherical planet. They need not concern us here. Since we are only concerned with horizontal motions and accelerations in the horizontal, the vertical component of the coriolis force (which is typically 4 orders of magnitude smaller than gravitational acceleration) also need not concern us here. We can write the partial y-component equation of horizontal motion including inertial and coriolis accelerations as

$$\frac{dv}{dt} = -2\Omega u\sin\varphi \equiv -fu\,. \qquad (2.5)$$

(b) On equatorward (poleward) motion, coriolis force appears as a westward (eastward) acceleration arising as an effect of angular momentum conservation. Imagine looking down on Earth's North pole from space. The projection on the equatorial plane of the displacement vector for a southward moving object would be seen to be radially outward from the rotation axis. The tendency for such an object to conserve angular momentum as it increases its radial distance from the axis would produce a westward velocity increase (decrease in angular velocity) in proportion to the outward displacement of the object δR. The corresponding change in velocity over a small time interval would appear as westward acceleration in proportion to the rate of radial displacement, in other words, in proportion to radial velocity. This can be analyzed beginning with the angular momentum conservation principle that implies that $\Omega_{tot}R^2$ is conserved following the motion of the object. Expanding this expression for a small radial displacement δR from initial values $(\Omega_{init}, R_{init})$,

$$\Omega_{init}R_{init}^2 = \left(\Omega_{init} + \frac{\delta u}{R}\right)\left(R_{init} + \delta R\right)^2 = \Omega_{init}R_{init}^2 + R_{init}\delta u + 2\Omega_{init}R_{init}\delta R + \text{HOT}.$$

Dividing by δt and taking the limit $\delta t \to 0$,

$$\frac{du}{dt} = -2\Omega\frac{dR}{dt} = 2\Omega v\sin\varphi - 2\Omega w\cos\varphi \approx fv \qquad (2.6)$$

since

$$\frac{dR}{dt} = -\sin\varphi\frac{dy}{dt} + \cos\varphi\frac{dz}{dt} = -v\sin\varphi + w\cos\varphi$$

and the contribution from the very small vertical motion can be neglected. [Note that the time derivatives in these equations are derivatives following the fluid flow or "substantial derivatives"]. Adding the pressure gradient contributions to (2.5) and (2.6) and dividing by density to convert from force per unit volume to force per unit mass (acceleration), we obtain the inviscid horizontal equations of motion (except for small terms due to Earth's curvature as mentioned above; inviscid implies neglect of any terms due to either molecular viscosity or the effects of small scale turbulence).

$$\boxed{\begin{aligned} &\frac{du}{dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} \\ &\frac{dv}{dt} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} \\ &\frac{d\mathbf{v_h}}{\mathbf{d}t} + f\hat{\mathbf{k}}\mathbf{x}\mathbf{v_h} = -\frac{1}{\rho}\nabla_{\mathbf{h}}p \end{aligned}} \qquad (2.7)$$

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We can now do a simple scale analysis on the vector form of these equations (last equation in the box of eq. 2.7). If any of the three terms in the equation (inertial acceleration, coriolis acceleration, pressure gradient acceleration) can be shown to be much smaller than either of the other two terms, then the two larger terms must be in approximate balance. To see which is the smaller term under the conditions shown in the large scale maps in MS Figs. 9-12, we need only compare terms I and II above by assuming velocity, length, and time scales V, L, and T. For large-scale atmospheric motions, the time-scale characterizing changes in the system is T = L/V. This is the time required to change the flow by advection (transport of fluid properties by the flow). Then the scale of term I in (2.7) is V^2/L, and the scale of term II is fV. The ratio of these terms is

$$\mathrm{Ro} = \frac{V}{fL}\ .$$

This is an important parameter characterizing the flow called the Rossby Number. For large-scale motions in Earth's atmosphere (except at very low latitudes), $V \approx 10\,\mathrm{ms}^{-1}$, $L \approx 10^6\,\mathrm{m}$, $f \approx 10^{-4}\,\mathrm{s}^{-1}$ so that

$$\boxed{\mathrm{Ro} = \frac{V}{fL} << 1}. \qquad (2.8)$$

That is, coriolis acceleration is much larger than inertial acceleration and there must be an approximate balance between terms II and III in eq. 2.7. In other words, eq. 2.4 holds with an error of order Ro, and the coriolis acceleration on the rapidly rotating Earth is strong enough to ensure that Ro<<1 for large-scale motions except in very low latitudes. The resulting approximate balance between pressure and coriolis accelerations defines the geostrophic wind of eqs. 2.3 and 2.4. We will use $\mathbf{v}_g$ to denote the geostrophic wind.

Note that, to the extent that spatial variations in ρ and f can be neglected, the geostrophic wind is approximately non-divergent,

$$\nabla_{\mathbf{h}} \bullet \mathbf{v}_{\mathbf{g}} \approx \frac{1}{\rho f}\nabla_{\mathbf{h}} \bullet \left(\hat{\mathbf{k}}\,\mathbf{x}\,\nabla_{\mathbf{h}} p\right) = 0\,.$$

This has two implications. First, the geostrophic wind tends to follow contours on horizontal surfaces of a scalar field (in this case pressure), which acts as a stream-function for the flow, i.e., the contours of stream-function act like "flexible membranes". Second, since vertical velocity is connected with divergence of the horizontal wind, large-scale vertical velocity is constrained to be small (even smaller than the geometric constraint of a large ratio of horizontal to vertical scale would imply).