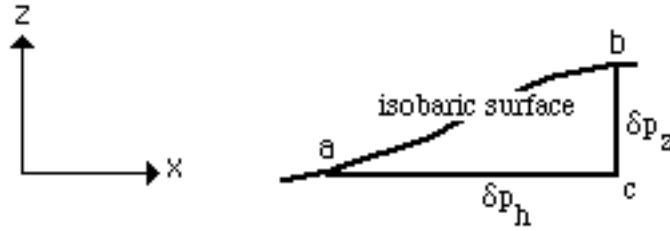


(2.4) Geostrophic wind expressed in terms of height contours on isobaric surfaces.

It is more convenient to deal with geopotential heights of isobaric surfaces than with pressure on constant geopotential height surfaces. To do this, we make the transformation

$$\frac{1}{\rho} \nabla_h p \rightarrow g \nabla_p z \quad (2.9)$$

where $\nabla_p = \mathbf{i} \frac{\partial}{\partial x_p} + \mathbf{j} \frac{\partial}{\partial y_p}$, the subscript h indicates evaluation at constant geopotential height and the subscript p indicates evaluation at constant pressure, i.e., on isobaric surfaces. The transformation (2.9) can be understood with the help of the following figure illustrating a sloping isobaric surface in the x-z plane.



The line ab corresponds to the isobaric surface. Because ab is an isobaric surface,

$\delta p_h + \delta p_z = 0$ so that $\delta p_h = -\delta p_z = -\frac{\partial p}{\partial z} \delta z_p = \rho g \delta z_p$. Dividing by δx ,

$\left(\frac{\delta p}{\delta x} \right)_h = \rho g \left(\frac{\delta z}{\delta x} \right)_p$. Generalizing to two dimensions and taking the limit,

$$\frac{1}{\rho} \nabla_h p = g \nabla_p z.$$

The equation of geostrophic balance and its components transforms to

$$\mathbf{v}_g = \frac{g}{f} \mathbf{k} \times \nabla_p z, \quad u_g = -\frac{g}{f} \left(\frac{\partial z}{\partial y} \right)_p, \quad v_g = \frac{g}{f} \left(\frac{\partial z}{\partial x} \right)_p. \quad (2.10)$$

According to (2.10), geostrophic wind follows isobaric height contours as if the contours were flexible membranes.

2.5 Thermal wind.

Taking the derivative of (2.10) with respect to $\ln p$ and applying the hydrostatic equation,

$$\frac{\partial \mathbf{v}_g}{\partial \ln p} = \frac{g}{f} \mathbf{k} \times \nabla_p \left(\frac{\partial z}{\partial \ln p} \right) = -\frac{R}{f} \mathbf{k} \times \nabla_p T \quad (2.11a)$$

with components

$$\boxed{\frac{\partial u_g}{\partial \ln p} = \frac{R}{f} \frac{\partial T}{\partial y_p}, \quad \frac{\partial v_g}{\partial \ln p} = -\frac{R}{f} \frac{\partial T}{\partial x_p}}. \quad (2.11b)$$

Since $\frac{\partial}{\partial \ln p} \approx \frac{\partial z}{\partial \ln p} \frac{\partial}{\partial z} = -\frac{RT}{g} \frac{\partial}{\partial z}$, eq. 2.11 can also be written in vector and component forms:

$$\boxed{\frac{\partial \mathbf{v}_g}{\partial z} = \frac{g}{fT} \mathbf{k} \times \nabla_p T, \quad \frac{\partial u_g}{\partial z} = -\frac{g}{fT} \frac{\partial T}{\partial y_p}, \quad \frac{\partial v_g}{\partial z} = \frac{g}{fT} \frac{\partial T}{\partial x_p}}. \quad (2.12)$$

Equation 2.11 states that vertical shear of the geostrophic wind follows isobaric contours of temperature like flexible membranes. We can also integrate (2.11) over a layer of finite thickness Δz to obtain the equation for the components of the thermal wind $(\Delta u_g, \Delta v_g)$ for the layer,

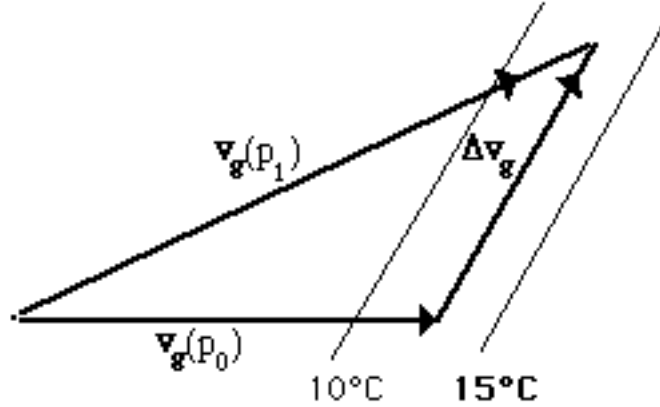
$$\boxed{\Delta u_g = \frac{R}{f} \frac{\partial}{\partial y} \left(\int_{\Delta p} T d \ln p \right), \quad \Delta v_g = -\frac{R}{f} \frac{\partial}{\partial x} \left(\int_{\Delta p} T d \ln p \right)}. \quad (2.13)$$

Equation (2.13) has several important implications.

(1) Geostrophic wind change between two pressure levels, the thermal wind, is related to contours of layer mean temperature in the same way that geostrophic wind relates to isobaric height contours. In particular, thermal wind follows the "flexible membrane rules" with layer mean temperature contours acting as the "flexible membranes". In the Northern Hemisphere, low temperatures are to the left of the thermal wind flow direction, high temperatures are to the right, with the reverse relationship in the Southern Hemisphere because of the reversal in sign of f .

(2) To an accuracy of order Ro , wind fields can be constructed from knowledge of the three dimensional distribution of temperature together with the two dimensional distribution of heights on a single isobaric surface. Since climatological mean surface geostrophic winds tend to be weak, upper level winds can be crudely approximated from the temperature distribution alone, neglecting surface geostrophic wind entirely. The fact that most of the information about the wind field resides in the temperature field is observationally important. It is much easier to sense the three dimensional distribution of temperature remotely, e.g., by satellite measurements of radiation emitted by the planet, than it is to remotely sense three dimensional distributions of either wind or pressure.

(3) Geostrophic wind on any isobaric surface can be constructed by vector addition of the geostrophic wind on any other surface and the thermal wind for the intervening layer, as in the figure below. A further implication is that, to an accuracy of R_0 , the horizontal advection of temperature averaged over a layer can be estimated from knowledge of the wind vectors at the bottom and top of the layer (see example 2 in the next section).



(4) The zonal mean temperature and zonal wind relationships shown in MS Figs. 1.7 and 1.8 are explained by the thermal wind relationship. In the troposphere, temperature decreases toward the poles, especially in mid-latitudes, and westerly winds increase with height. In the lower stratosphere, temperature increases toward the poles in mid-latitudes, and westerly winds decrease with height, so there are westerly wind maxima (jets) at the mid-latitude tropopause. In the upper stratosphere and lower mesosphere during solstice seasons, temperature increases toward the summer pole and increases toward the winter pole. Consequently westerly winds increase with height in the winter hemisphere and easterly winds increase with height in the summer hemisphere. In the upper mesosphere, the temperature gradient reverses in both hemispheres so thermal wind shear reverses and easterly and westerly jets occur near 70km.

2.6 Applications.

Example 1: Zonal mean eastward wind on the 1000hPa isobaric surface at latitude 55°N is about 10ms^{-1} . Estimate the slope of the isobaric surface at 55°N .

$$\frac{\partial z}{\partial y_p} \approx -\frac{f}{g} u_g(1000\text{hPa}) \approx \frac{-1.2 \times 10^{-5} \text{s}^{-1} \times 10 \text{ms}^{-1}}{10 \text{ms}^{-2}} \approx 1.2 \times 10^{-4} = 120 \text{m} / 1000 \text{km} .$$

The average temperature of the 1000-500hPa layer is 280K at 40°N and 260K at 50°N . Estimate the geostrophic zonal wind at 500hPa and 55°N .

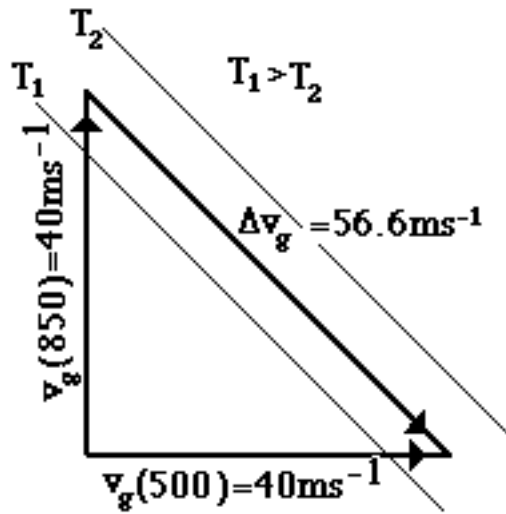
$$\Delta u_g \approx -\frac{R}{f} \frac{\partial \bar{T}}{\partial y_p} \Delta \ln p = \frac{287 \text{m}^2 \text{s}^{-2} \times 20 \text{ms}^{-1}}{1.19 \text{s}^{-1} \times 0.349 \times 6.37 \times 10^6 \text{m}} \times 0.693 = 15.0 \text{ms}^{-1},$$

$$u_g(500\text{hPa}) = u_g(1000\text{hPa}) + \Delta u_g \approx +25\text{ms}^{-1}.$$

The positive sign means that the wind is from the west (westerly).

Example 2: At latitude 45°N , a rawinsonde measures 850hPa wind from due south (180°) at 40ms^{-1} . At 500hPa, it measures wind from due west (270°) at 40ms^{-1} . Estimate the direction and magnitude of the average temperature gradient in the 850-500hPa layer.

The solution is illustrated below:



From the vector addition, the thermal wind is from 315° (NW) at 56.6ms^{-1} . Then the magnitude of $\nabla_p \bar{T}$ is $|\nabla_p \bar{T}| \approx \frac{f}{R} |\Delta \mathbf{v}_g| \div \ln\left(\frac{850}{500}\right) \approx 38\text{K}/1000\text{km}$, and its direction is toward 225° (SW).

What is the local rate of temperature change due to horizontal temperature advection?

This contribution to local temperature change follows from the expansion of the substantial derivative using the chain rule, $\frac{d\bar{T}}{dt} = \frac{\partial \bar{T}}{\partial t} + \overline{\mathbf{v}_h \cdot \nabla_h \bar{T}} + w \frac{\partial \bar{T}}{\partial z}$ where $\frac{\partial \bar{T}}{\partial t}$ is local rate of change of layer mean temperature, $-\overline{\mathbf{v}_h \cdot \nabla_h \bar{T}}$ is rate of change due to horizontal advection through the layer, and $-w \frac{\partial \bar{T}}{\partial z}$ is the layer average rate of change due to vertical advection. Then our answer is given by

$$\frac{\partial \bar{T}}{\partial t} \approx -\overline{\mathbf{v}_h \cdot \nabla_h \bar{T}} \approx -\mathbf{v}_g \cdot \nabla_p \bar{T}$$

where the geostrophic wind in the last expression can be evaluated at any level in the layer, including the bottom and top levels, under the assumption that thermal wind is constant through the layer (if thermal wind is constant through the layer, the tip of all wind vectors in the layer lies along the thermal wind vector, so the component of all wind vectors in the direction of the temperature gradient is the same). The answer is $+93\text{K/dy}$. Because the winds in the layer are blowing from warmer toward colder, horizontal advection will warm the layer. This is a very large rate of temperature change because the assumed winds are strong and change substantially with height so that the temperature gradient must be large. Moreover, the large change in direction of wind with height implies that the component of the wind in the direction of the temperature gradient is large. Such a large horizontal advection would usually be countered to a significant degree by vertical advective change of opposite sign. Moreover, such a large rate of horizontal advection through such a deep layer would rarely persist for as long as a day.

Example 3: Review MS figs. 1.9-1.12 again to verify the real-world applicability of the "flexible membrane rule", the general low isobaric contour heights (at 500 and 10hPa) at high latitudes compared with low latitudes arising from the generally cooler temperatures at high latitudes, the association of westerly winds with this pattern, the pattern of large-scale troughs and ridges in the flow, which will be associated with large-scale troughs and ridges in both the underlying 100hPa isobaric heights and the underlying temperatures, the climatological occurrence of ridges near west coasts, and the overall larger scales at 10hPa than at 500hPa.