Exponentials

Anything that increases at a rate proportional to how much is already there is growing *exponentially*. This is also sometimes called "geometrical" growth. It is distinguished from linear growth, in which the actual amount of increase is the same per unit time (adding $5 per year to your bank account is linear growth, adding 5% per year is exponential).

The rate of growth can be converted to a doubling time. For example, 4% per year is an 18 year doubling time. This means if we start with 100 units, there will be 200 after 18 years, 400 after 36 years, 800 after 54 years, etc.

Examples of exponentially growing quantities: germ population in your body following an initial infection by just a few, microbe population in a petri dish (until they approach the limits of the dish), world population (of humans, that is), world energy consumption

Exponential growth cannot be maintained in the natural world. Rather, it represents a transition to a new level or state. It frequently represents the rapid exploitation of some new potential before the limits of the system have been approached. At some point, constraints in the system set in.

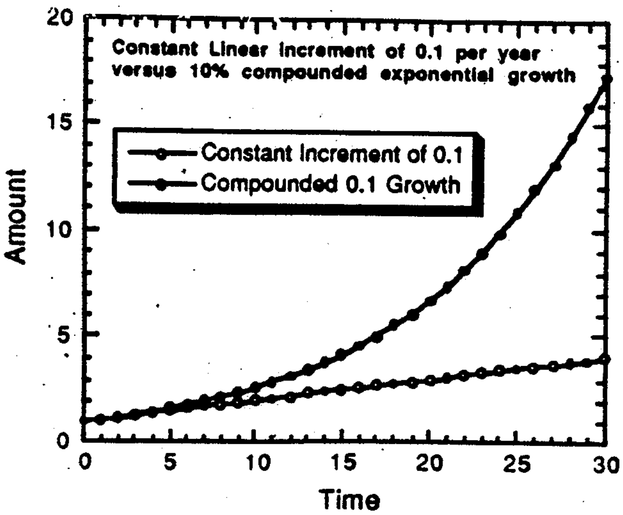
Biological reproduction is inherently capable of exponential growth, since each member in each new generation is capable of producing multiple offspring.

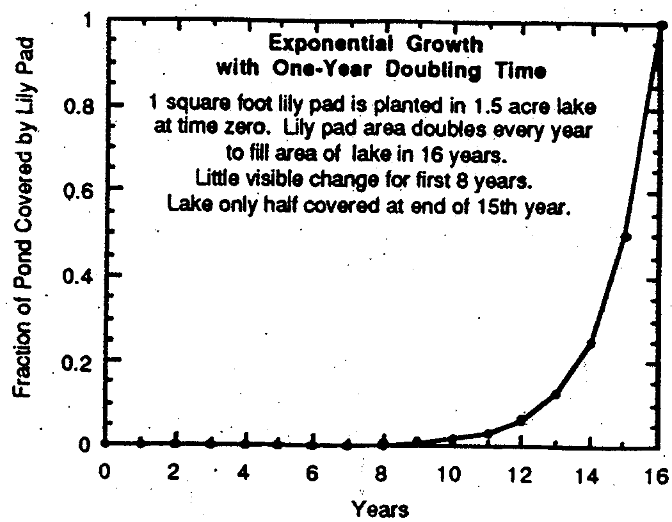
Would you rather have $100 a day for a month or get a penny on the first day and double that amount on each of the next 29 days?

William Nieremberg, former Director of Scripps Institute of Oceanography, said that when he and his colleagues first saw the *curve* in the CO_2 record from Mauno Loa (visible after less than 10 years of measurements so this would have been in the mid-1960's), they knew they had a terribly serious finding. [And Nierenberg is a "global warming skeptic", by the way]. A curve in a plot of concentration over time is indicative of exponential growth.

Exponential Growth

1) Compound Versus Simple Interest





Linear: $y = y_0 + y_0 R t$

Expon.: $y = y_0 (1+R)^t$

y = amount at time t
y_0 = initial amount
R = rate of increase (%/yr)
t = time (years)

Suppose you put a dollar in an investment that pays you 10% every year and then you put the ten cents in your mattress. How much money do you have in 30 years? About $4. What if, instead, you put the 10% back into the investment every year so that your gains compound? At the end of 30 years you have about $17.45, a difference of about 548% in your gain.

2) The Pond and the Lily Pad Problem

Suppose you have a 1.5 acre (65,536 sq.ft.) fish pond on your farm. You want to improve the looks of your pond and the habitat for your trout, so you plant a 1 square foot patch of pond lily. A year later you notice that the area of the pond lily has doubled. At this rate, how long before the pond is covered with lily pads, and how much of the pond is covered the year before total coverage is achieved? It takes 15 years to cover half the pond with lily pads, but one year later at the end of the 16th year all the pond is covered. For the first 8 years, a negligibly small fraction of the pond is covered. It takes over 12 years to cover 10% of the pond.

Lessons:

- Compound interest pays big dividends. The first dollar you save is the biggest.
- Even though the growth is a steady rate, like 10% or 100% per year as in these examples, the magnitude of the year-to-year change increases with time.
- If you are approaching an upper limit exponentially, that limit is approached very quickly after a relatively long wait. For a long time it seems like nothing is happening, then suddenly change is rapid and it's over.