from
Horel & Geisler, Global Environmental Change, 1997

also includes "Fractions" review sheet at end

Appendix

A.1 About Numbers

Some numbers are too large or too small to express conveniently by writing them out in full. Any number can more compactly be written in terms of powers of ten. This is called exponential notation. For example, a million (1,000,000) can be written as 1×10^6 and three billion (3,000,000,000) can be written as 3×10^9. Fractions can also be written in terms of powers of ten. A thousandth (.001) can be written as 1×10^{-3} (that is, $1/10^3$) while a billionth (.000000001) is 1×10^{-9}.

In this book, we often state numbers approximately. The term *roughly* means that the number lies within a range of 10 percent about that number. When we say that there are roughly 10 items, then there could be 9 or 11. The term *negligible* also has a specific meaning. It indicates that one number is several powers of ten larger than another. For example, carbon dioxide in the atmosphere constitutes only a tiny fraction of all of the gas in the atmosphere (roughly 3×10^{-4} of the total). So, we can say that carbon dioxide constitutes a negligible fraction of the atmosphere as a whole. However, carbon dioxide gas has a profound effect on the behavior of the earth-atmosphere system, even if its concentration is negligible compared to other gases.

A.2 About Units

The convention in all scientific disciplines is to use the SI (Système Internationale) units. Units are necessary in order to quantify distance, mass, time, and so on, as shown in the following table.

Quantity	Unit	Symbol
distance	meter	m
mass	kilogram	kg
time	second	s
temperature	kelvin	K
power	watt	W

Because of the large range of numbers that we will use, it is convenient to use prefixes such as kilo and milli. Kilometer (a thousand meters) and milligram (a thousandth of a gram) are common measures of distance and mass respectively. The following table lists prefixes that we will use to express the size of things:

Prefix	Size	Power of Ten
micro	millionth	10^{-6}
milli	thousandth	10^{-3}
kilo	thousand	10^{3}
mega	million	10^{6}

We will have an occasional need to convert quantities in SI units to the more familiar English system of units (miles, pounds, degrees, etc.).

Temperature in Kelvin can be obtained from temperature in degrees Celsius by adding 273. For reference, the melting temperature of ice is 273 K (0°C) and the boiling temperature of water (at sea level) is 373 K (100°C). To convert from degrees Celsius to degrees Fahrenheit, the following equation is used:

$$°F = 1.8 \times °C + 32.$$

Ice melts at 32°F and water boils at 212°F.

A.3 About Graphs

We show a number of figures in this book that convey how one quantity depends on another quantity. Consider Figure A.1, one of the central figures of this book. It shows how the globally averaged surface temperature has varied with time from 1860 through 1995. Now look at Figure A.2, which has exactly the same information, but the aspect ratio of the graph has been changed: The time axis (left-to-right direction) is compressed while the temperature axis (up-down direction) is expanded.

By means of changing the aspect ratio of this figure, the temperature rise has been made more dramatic. The temperature increase of roughly 0.5°C observed over the past 130 years isn't any different in Figure A.2 than it is in Figure A.1, but the impact is quite different. Figure A.2 lends itself to a more alarming view of climate change, even though the information content is the same.

A.4 Some Basic Chemistry

We use some very basic concepts from chemistry in this book. As you are probably aware, an atom is composed of a nucleus around which electrons orbit as shown

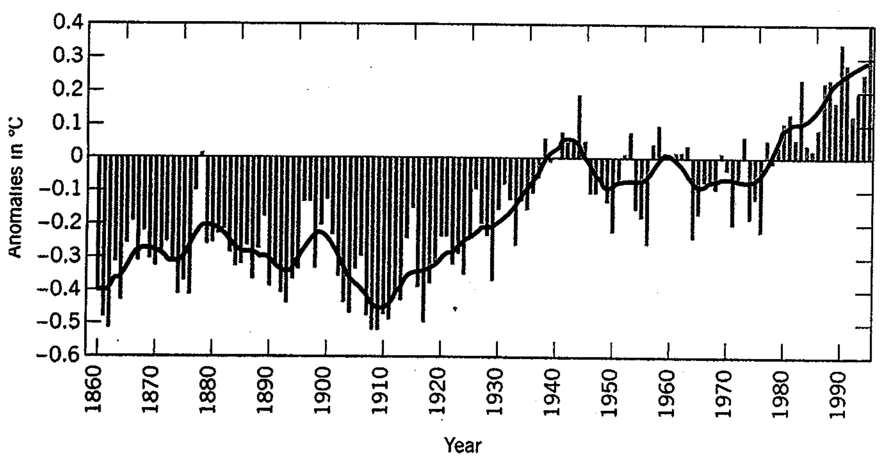

Figure A.1 Globally averaged surface temperature anomalies from 1860 to 1995 (in °C) relative to the 1961–1990 average.

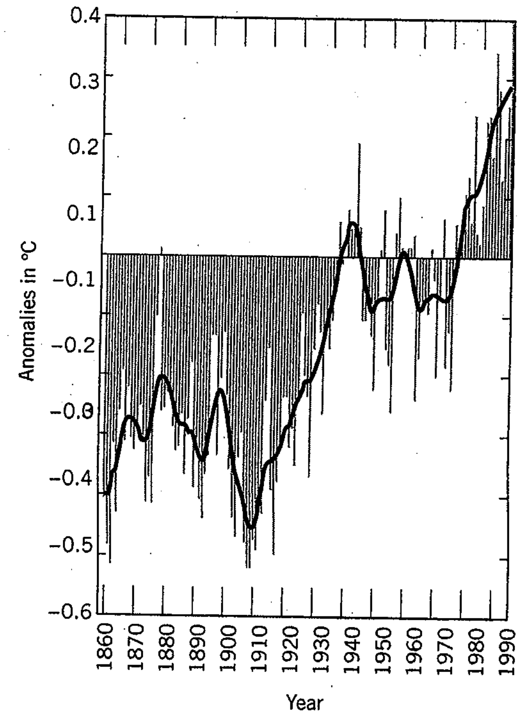

Figure A.2 Same as Figure A.1 but the aspect ratio has been changed.

schematically in Figure A.3. The mass (weight) of the electrons is negligible compared to the mass of the nucleus, which is composed of protons and neutrons. The mass of a proton is the same as that of a neutron. Consider now an oxygen atom. It has 8 protons and 8 neutrons in its nucleus with 8 electrons in orbit around the nucleus. Rather than drawing a diagram with a nucleus and 8 electrons circling around it, it is more convenient to use notation that summarizes the characteristic of the oxygen atom as ^{16}O. Here, the superscript indicates the number of neutrons and protons combined

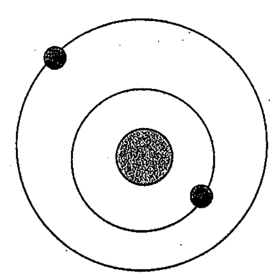

Figure A.3 Two electrons orbit around a nucleus composed of protons and neutrons.

in the nucleus. We could get away with writing it simply as O, except that there are three different forms of oxygen.

$$^{16}O \quad ^{17}O \quad ^{18}O$$

These different forms of oxygen are called oxygen *isotopes*. The first one is the most common form, while the other two are rarely present in amounts in excess of a few tenths of a percent. The differences between the three isotopes of oxygen are the number of neutrons found in the nucleus; the ^{18}O isotope has two additional neutrons, so that it is 12 percent heavier than the ^{16}O isotope.

A molecule is an aggregate of two or more atoms. The molecule carbon dioxide is made up of one carbon atom and two oxygen atoms. Either the molecule or the gas composed of the molecules can be denoted by the symbol CO_2. Note that with this notation, we don't say which isotope of oxygen is present or, for that matter, which isotope of carbon.

In a few places in this book we write chemical reactions that describe how different atoms and molecules react with one another. An example is the process of *photosynthesis* by which plants consume carbon dioxide (CO_2) and water (H_2O) and produce molecular oxygen (O_2) and a carbohydrate unit that is a fundamental part of glucose (sugar).

$$CO_2 + H_2O \rightarrow CH_2O + O_2$$

In the process of *respiration*, the reaction runs from right to left, putting the carbon atom in the carbohydrate unit back into a carbon dioxide molecule.

Most of the atmosphere consists of the gas nitrogen, whose molecule is denoted N_2. Some molecules of atmospheric nitrogen find their way into the ocean to constitute what is called dissolved nitrogen. Atmospheric carbon dioxide molecules also enter the ocean to constitute dissolved carbon dioxide. What distinguishes carbon dioxide from all other dissolved atmospheric gases in the ocean is that it is chemically active in water.

$$CO_2 + H_2O \rightarrow H^+ + HCO_3^-$$

The products of the reaction are a hydrogen ion (H^+, which is a hydrogen atom whose sole electron has been removed) and a bicarbonate ion (HCO_3^-, which has taken that electron). A small fraction of the bicarbonate ions undergo a further reaction, but this fact is of no importance to us here. In summary, what we have seen here is that the carbon in carbon dioxide dissolved in ocean water actually resides in bicarbonate ions.

In another relevant chemical reaction many marine plants and animals make shells.

$$Ca^{++} + 2HCO_3^- \rightarrow CaCO_3 + H_2O + CO_2$$

The calcium ion (Ca^{++}) is a calcium atom that has lost two of its (negatively charged) electrons and hence has a net positive charge of two units. The bicarbonate ion (HCO_3^-) consists of a hydrogen atom, a carbon atom, and three oxygen atoms, with one electron missing from the combination.

There is no net gain or loss of charge when, as in the above reaction, two bicarbonate ions combine with one calcium ion. The reaction yields calcium carbonate ($CaCO_3$), which is the material of the shell, plus water and carbon dioxide. After the organism that has made the shell dies, the shell may dissolve back into its original chemical constituents. In this case the reaction above proceeds from right to left instead of from left to right as we have shown it here.

Fractions

alias: ratio, division, "per", percent

formula:

$$y = \frac{a}{b} = \frac{\text{numerator}}{\text{deno min ator}}$$

Example:

What portion of the city budget went to schools?

total budget: $650,000,000
school budget: $ 50,000,000

answer: $\frac{\$50,000,000}{\$650,000,000} = 0.077$ (or 7.7%) [rounding to two significant figures]

Uses:
Notice that the answer above (7.7% of government spending went to schools) gives us a perspective on the system (in this case, the political system) that the two dollar amounts by themselves do not provide. This is often the case. That is, when we take the ratio of two quantities, we often derive an important system property that is not apparent from either of the quantities by themselves. Consider how this is the case in the following examples:

miles driven / time = speed (miles per hour)

cost of gasoline / miles driven = cost per mile

money earned / hours worked = pay rate ($/hr)

gallons of gas burned / miles driven = efficiency of car (mpg)

kilowatts of power produced / tons of coal burned = power plant efficiency

tons of gasoline used by the US / US population = average tons used per person

solar constant / area of earth = average intensity of radiation received by earth (W/m^2)

land area / earth area = fraction of the planet covered by land

solar energy reflected away / total solar energy impinging on planet = albedo of planet