

Homework 1

1. Do Problem 5, p. 22 of Kundu and Cohen.
2. Define the potential temperature for a perfect gas as

$$\theta = T \left(\frac{p}{p_0} \right)^{-R/c_p}.$$

Where $R = c_p - c_v$ is the gas constant. For perfect gases, the equation of state may be expressed $pv = RT$ (where v is specific volume) and the internal energy is $e = c_v T$. Using the first law of thermodynamics, show that entropy and potential temperature are related such that

$$S = c_p \ln \theta + \text{const.}$$

3. Show that if a pure water column with temperatures in the range $0 < T < 4^\circ\text{C}$ is stably stratified, then the entropy *decreases* with height. Note that this is the temperature range over which the density of water increases with increasing temperature, i.e., $(\partial v / \partial T)_p < 0$ over this temperature range. *Hint:* begin by considering $S(p, T)$, i.e., the equation of state relating entropy, pressure and temperature. One of Maxwell's thermodynamic relations

$$\left(\frac{\partial S}{\partial p} \right)_T = - \left(\frac{\partial v}{\partial T} \right)_p$$

may also be helpful.

4. Find equations of the form $f(x, y) = \text{const}$ for the streamlines in a steady 2D velocity field defined by the relations $u = x$ and $v = -y$. Also sketch the streamline pattern.

Due Friday October 10th