

Homework 2

1. Do Problem 13, p. 75 of Kundu and Cohen. This important identity was derived by Lagrange in 1781. *Hint:* it helps to use equation (2.19) on p. 36.
2. Do Problem 5, p. 122 of Kundu and Cohen. Also explain what this relation tells us (together with appropriate energy equations already derived in Kundu and Cohen and in class) about the way that viscous dissipation changes the kinetic and internal energies of fluid systems.
3. Do Problem 6, p. 122 of Kundu and Cohen.
4. Do Problem 11, p. 123 of Kundu and Cohen.
5. The flow of an incompressible viscous density stratified fluid is governed by the equations.

$$\rho \frac{D\mathbf{u}}{Dt} + \nabla p = \rho \mathbf{g} + \mu \nabla^2 \mathbf{u}.$$

$$\frac{D\rho}{Dt} = 0.$$

$$\nabla \cdot \mathbf{u} = 0.$$

Here the unknown variables are \mathbf{u} , ρ and p , but there is no prognostic equation for p , i.e., no equation for $\partial p / \partial t$. Instead p is calculated by solving an elliptic partial differential equation (subject to boundary conditions that we will ignore). Derive this PDE for p by requiring that the pressure impose accelerations in the momentum equations such that the time-tendency of the divergence remains zero, i.e., such that

$$\frac{\partial(\nabla \cdot \mathbf{u})}{\partial t} = 0.$$

Hint: the resulting PDE is a Poisson equation for p in which the right-hand-side forcing is a function of \mathbf{u} and ρ .

Due Friday October 24th