AMATH/ATMOS 505, OCEAN 511—Autumn 03

Take-Home Midterm (Homework 4)

Please work completely independently!!

1. Waves are defined as nondispersive if their phase speed is independent of their wavelength. Nevertheless, the dispersion that one actually observes in waves spreading out from a compact source is more obviously related to the group velocity, which describes the rate of energy transport by the waves. Show that *if the phase speed is independent of wavelength, the group velocity must also be independent of wavelength.* Limit your consideration to waves in two spatial dimensions with x and y wavenumbers k and ℓ, respectively, for which the phase speed is defined as

$$\frac{\nu}{\sqrt{k^2 + \ell^2}}.$$

(Informational note: the converse is not true; one can construct dispersion relations for which the group velocity is independent of wavelength, but the phase speed remains dispersive.)

2. Consider an inviscid homogeneous fluid (i.e., a fluid in which the density is constant in both space and time)

(a) Linearize the governing equations for such a fluid about a basic state at rest ($\overline{u} = \overline{v} = \overline{w} = 0$). Show your work and justify how you chose the basic state distribution of pressure $\overline{p}$.

(b) Eliminate the perturbation velocities (u', v', w') from this system of equations to obtain a single partial differential equation with p' as the only unknown variable. How does this equation compare to the equation for the velocity potential in a nondivergent flow?

3. Charlie is on a stationary ship in the middle of a deep ocean. Charlie notices swell (surface gravity waves), with period T_1, propagating past his ship from a particular direction. After an interval of time Δt, he notes that the swell is still arriving from the same direction, but now has a shorter period T_2. Charlie hypothesizes that both sets of swell originated from the same event, but due to wave dispersion, the initial disturbance gradually separated into sinusoidal waves in which the horizontal wave number varies slowly along the wave train. Following up on this insight, Charlie concludes that the distance x from his location to the region in which the event occurred is given by

$$x = \frac{g\Delta t}{4\pi} \left(T_2^{-1} - T_1^{-1}\right)^{-1}.$$

(a) Is Charlie correct? Show your derivation of this (or another) expression. (All credit is awarded for the derivation, not for the 50-50 chance of guessing whether Charlies is right.)

(b) If $T_1 = 30$ s, $T_2 = 10$ s, and $\Delta t = 2$ hours, how far away from Charlie's ship was the event that generated these waves.

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Due Friday November 21th