AMATH/ATMOS 505, OCEAN 511—Autumn 03

Homework 5

1. By construction, the velocity fields in the two-dimensional (x-z) linear shallow water waves derived in Kundu and Cohen (p. 203) are irrotational.

(a) Determine whether the deep water and shallow-water approximations preserve the property that the velocity field be irrotational.

(b) It is possible to derive both linear and nonlinear versions of the equations governing shallow-water flow by making approximations to the full the momentum and mass continuity equations. Would it would be a good idea to try to represent the velocity field in such a derivation as the gradient of a velocity potential? Explain.

2. Suppose that a parcel in a Boussinesq incompressible fluid is attached to a slanting rod by a frictionless coupling, and that the rod is slanted off vertical such that the angle between the z-axis and the rod is ϕ. Assume that pressure gradient forces are negligible and that the Brunt-Väisälä frequency is constant. Show that if a parcel is displaced from its equilibrium level, it will oscillate along the slanted rod at a frequency $N \cos \phi$.

3. Repeat the derivation of equation (7.147, p. 245) in Kundu and Cohen under the assumption that the perturbation pressure and density are in exact hydrostatic balance in the set of equations (7.140-7.144). Use your result to obtain the dispersion relation for hydrostatic internal gravity waves in the case where N is constant throughout the fluid. How does your dispersion relation compare to the "long-wave limit" ($k^2 + l^2 << m^2$) of the nonhydrostatic dispersion relation:

$$\omega^2 = \frac{N^2(k^2 + l^2)}{(k^2 + l^2 + m^2)} \quad ?$$

4. Consider the case of two-dimensional, small-amplitude internal waves in the unbounded x-z plane. Suppose that the mean wind is zero and the Brunt-Väisäla frequency is constant throughout the fluid. Make the Boussinesq approximation, but not the hydrostatic approximation. The perturbation energy, averaged over one wavelength, is defined as

$$E = \frac{1}{2}\rho_0 \overline{(u'^2 + w'^2 + b'^2/N^2)}.$$

The first two terms represent kinetic energy, the third represents potential energy (with $b' = -g\rho'/\rho_0$). The overbar denotes an average over one wavelength (any 2π variation in the phase of the wave).

(a) Show that E is evenly divided between kinetic and potential energy. Does this result hold if we examine the perturbation kinetic and potential energies at a single point without averaging over a wavelength?

(b) How do these results compare to the partitioning of perturbation energy between KE and PE in a surface gravity wave?

(c) Show that the perturbation energy flux $(p'u', p'w')$ satisfies

$$(\overline{p'u'}, \overline{p'w'}) = E\vec{c_g}$$

Due Friday December 5th