Atmospheric Science 536—Winter 2010

Homework 1

1. *Compare pressure perturbations in a tornado and a mesoscyclone.* Suppose that the winds in both the mesocyclone and a tornado are in cyclostrophic balance. Then counterclockwise circular flow at speed V and radius r satisfies

$$\frac{V^2}{r} = \frac{1}{\rho_0}\frac{\partial p}{\partial r},$$

where ρ_0 is the density assumed constant in the spirit of the Boussinesq approximation, and p is the pressure.

(a) Based on Fig. 10 of Zeigler et al. (2001), one might estimate that winds are circling around a mesocyclone at 15 m s^{-1} at a radius of 2 km. Suppose the wind speed increases linearly with radius. What is the pressure difference (in Pa) between the center of the mesocyclone and surrounding points at a radius of 2 km? Based on Fig. 2 assume the environmental temperature and pressure are $T = 25$ C and $p = 900$ hPa.

(b) Now suppose that in the same environmental conditions, the tangential wind speeds in the tornado are 80 m s^{-1} at a radius of 100 m. Assuming once again that the tangential wind speed varies linearly with radius, what is the pressure difference between the center of the tornado and surrounding points 100 m away from the center?

(c) Given that the wind increases linearly with radius, does the pressure change between the center of a circular vortex and a point R where the tangental wind speed is S depend on the distance between the center and R?

2. *Vorticity generation associated with changes in fluid density.*

(a) Letting ρ_0 be a constant reference density, the Boussinesq momentum equation may be written

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} + \frac{1}{\rho_0}\nabla p' = -g\frac{\rho'}{\rho_0}\mathbf{k},$$

where $p = \overline{p}(z) + p'(x, y, z, t)$, $\rho = \overline{\rho}(z) + \rho'(x, y, z, t)$ and

$$\frac{\partial \overline{p}}{\partial z} = -g\overline{\rho}.$$

Show that the value of ρ' has no influence on the rate of change of the vertical vorticity

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}.$$

(b) The unapproximated momentum equation may be written (with the same hydrostatically balanced mean state removed) as

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} + \frac{1}{\rho}\nabla p' = -g\frac{\rho'}{\rho}\mathbf{k},$$

in which case the vertical vorticity equation becomes

$$\frac{D\zeta}{Dt} = -\zeta\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right) + \frac{1}{\rho^2}\left(\frac{\partial \rho'}{\partial x}\frac{\partial p'}{\partial y} - \frac{\partial \rho'}{\partial y}\frac{\partial p'}{\partial x}\right).$$

The first two terms on the right are the stretching and tilting terms, and need not be considered for this analysis. Derive the third "solenoidal" term, which describes how density perturbations can assist in the production of vertical vorticity in the unapproximated equations.

(c) Physically interpret the solenoidal term. Consider the case where $\rho' = \rho'(x)$ with $\partial\rho'/\partial y = 0$ and $p' = p'(y)$ with $\partial p'/\partial x = 0$.

Due Tuesday January 19th.