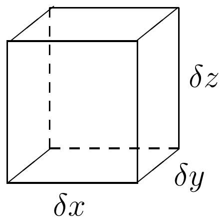
Ch 2

Our aim is to derive the fundamental equations of meteorology from conservation laws for mass, momentum, and energy.

Consider two possible perspectives, both with respect to control volume:



1) Eulerian

- rigid control volume, fixed in space (though still may be rotating with Earth)
- must consider flow through boundaries of control volume, thus material inside may be replaced
- easier for solving problems

2) Lagrangian

- control volume moves with air parcel
- control volume stretches and shrinks with motion
- "easier" for derivations
- can be trouble for solving problems, e.g. the fluid element can become highly distorted

2.1 Total differentiation

If a variable is a function of space and time, say $T(x, y, z, t)$, then the total (or exact) differential of T is

$$dT = \left(\frac{\partial T}{\partial x}\right)_{y,z,t} dx + \left(\frac{\partial T}{\partial y}\right)_{x,z,t} dy + \left(\frac{\partial T}{\partial z}\right)_{x,y,t} dz + \left(\frac{\partial T}{\partial t}\right)_{x,y,z} dt$$

Consider the finite-difference form of the above equation (replace d's with δ's), divide both sides by δt and take the limit as δt goes to zero. Because the derivative with respect to t is

$$\frac{dT}{dt} = \lim_{\delta t \to 0} \frac{\delta T}{\delta t},$$

we can write

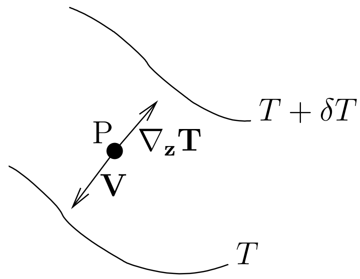
$$\frac{DT}{Dt} = \frac{\partial T}{\partial x}\frac{Dx}{Dt} + \frac{\partial T}{\partial y}\frac{Dy}{Dt} + \frac{\partial T}{\partial z}\frac{Dz}{Dt} + \frac{\partial T}{\partial t}$$

where we have replaced the small d's with big D's to remind ourselves that this is the time rate of change following the motion. The indices on the partial derivatives are usually suppressed. Now with $u = Dx/Dt$, and so forth

$$\frac{\partial T}{\partial t} = \frac{DT}{Dt} - \mathbf{U} \cdot \nabla T.$$

Recall **U** is our notation for the three dimensional velocity vector. The partial derivative indicates the *local* time rate of change at a fixed location (ie Eulerian). It equals the time rate of change following the air parcel (e.g, which might be warming via condensation) plus *advection*. The advection accounts for flow across the control volume boundaries.

The figure at right shows an example of warm air advection, or positive advection, at point P in the horizontal plane. Note that the gradient points towards higher values. The little z indicates the gradient is only in the horizontal plane (with z fixed).



Warm Air Advection at point P

2.1.1 Total differential of a vector in a rotating system

Our goal is to formally derive the momentum equation for atmospheric flow. It follows from Newton's 2nd law, recall

$$\frac{D_a\mathbf{U}_a}{Dt} = \sum \mathbf{F},$$

which applies in an inertial reference frame. The tiny a refers to absolute frame, which is another name for the inertial frame. The $\mathbf{F}$ only includes the fundamental forces, no apparent forces allowed. Unfortunately this equation is inconvenient for Earth dwellers. We see and feel acceleration with respect to our rotating reference frame. Thus we want to replace $D_a\mathbf{U}_a/Dt$ with $D\mathbf{U}/Dt$. When we do, we must also include the "apparent forces" that we treated phenomenologically in Ch 1.

First we derive a relationship between the total derivative of a *generic vector*, $\mathbf{A}$, in inertial and rotating reference frames, to relate $D_a\mathbf{A}/DT$ to $D\mathbf{A}/DT$. Both of these derivatives tell us the rate of change of $\mathbf{A}$ following the motion, but as you shall see, the derivatives depend on the coordinate system used to follow the motion (ie relative to Earth's rotation or relative to an absolute or fixed coordinate system).

In our familiar Cartesian coordinate system rotating with Earth

$$\mathbf{A} = \mathbf{i}A_x + \mathbf{j}A_y + \mathbf{k}A_z. \tag{1}$$

When we take the time derivative following the motion from the inertial frame, we must use the product rule

$$\frac{D_a\mathbf{A}}{Dt} = \underbrace{\mathbf{i}\frac{DA_x}{Dt} + \mathbf{j}\frac{DA_y}{Dt} + \mathbf{k}\frac{DA_z}{Dt}}_{\frac{D\mathbf{A}}{Dt}} + A_x\frac{D_a\mathbf{i}}{Dt} + A_y\frac{D_a\mathbf{j}}{Dt} + A_z\frac{D_a\mathbf{k}}{Dt}. \tag{2}$$

We can drop the little a's in the first three terms because we are taking the derivatives of scalars. The last three terms arise because $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ change direction in space with rotation.

Please be aware that the $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ coordinates are as I defined last week, but in this section they are fixed in the Earth reference frame. Hence ($\frac{D\mathbf{i}}{Dt} = 0$ here, but this will not be the case most of the time!

It is good for your character and possibly your depth of understanding to derive the following relations:

$$\frac{D_a\mathbf{i}}{Dt} = \mathbf{\Omega} \times \mathbf{i}, \ \frac{D_a\mathbf{j}}{Dt} = \mathbf{\Omega} \times \mathbf{j}, \text{ and } \frac{D_a\mathbf{k}}{Dt} = \mathbf{\Omega} \times \mathbf{k} \tag{3}$$

where Earth's rotation in the rotating frame is

$$\mathbf{\Omega} = \mathbf{j}\Omega\cos\phi + \mathbf{k}\Omega\sin\phi$$

(beware Holton has the cosine and sine backwards).

Finally, putting it all together

$$\frac{D_a\mathbf{A}}{Dt} = \frac{D\mathbf{A}}{Dt} + \mathbf{\Omega} \times \mathbf{A}. \tag{4}$$

2.2 The vectorial form of the momentum equation in rotating coordinates

Refer to the goal stated at the beginning of the previous section. The next step is to apply Eq 4 to the position vector $\mathbf{r}$

$$\frac{D_a\mathbf{r}}{Dt} = \frac{D\mathbf{r}}{Dt} + \mathbf{\Omega} \times \mathbf{r}.$$

There is no distincting between $\mathbf{r}$ and $\mathbf{r}_a$, but $D_a\mathbf{r}/Dt \equiv \mathbf{U}_a$ and $D\mathbf{r}/Dt \equiv \mathbf{U}$, so

$$\mathbf{U}_a = \mathbf{U} + \mathbf{\Omega} \times \mathbf{r}, \tag{5}$$

which indicates that the absolute velocity of an object on Earth equals the velocity relative to Earth plus the velocity due to Earth's rotation.

Now apply Eq 4 to $\mathbf{U}_a$:

$$\frac{D_a\mathbf{U}_a}{Dt} = \frac{D\mathbf{U}_a}{Dt} + \mathbf{\Omega} \times \mathbf{U}_a.$$

Substitute Eq 5 into the right hand side of the previous equation gives:

$$\begin{aligned} \frac{D_a\mathbf{U}_a}{Dt} &= \frac{D}{Dt}(\mathbf{U} + \mathbf{\Omega} \times \mathbf{r}) + \mathbf{\Omega} \times (\mathbf{U} + \mathbf{\Omega} \times \mathbf{r}) \\ &= \frac{D\mathbf{U}}{Dt} + 2\mathbf{\Omega} \times \mathbf{U} - \Omega^2\mathbf{R}. \end{aligned} \tag{6}$$

where $\mathbf{R}$, as in Ch 1, is a vector normal to the axis of rotation with magnitude $a\cos\phi$, a is Earth's radius and ϕ is the latitude. The second term on the right is the Coriolis force and the third term is the centrifugal force.

Now substitute Eq. 6 into Newton's 2nd law to arrive at the momentum equation with our more familiar $D\mathbf{U}/Dt$:

$$\frac{D\mathbf{U}}{Dt} = -2\mathbf{\Omega} \times \mathbf{U} - \frac{1}{\rho}\nabla p + \mathbf{g} + \mathbf{F_r} \tag{7}$$

(recall that g includes the centrifugal force).

2.3 Component Equations in spherical coordinates

Here our ($\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$) coordinate system on Earth moves with the parcel. Thus $\mathbf{U} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$ and

$$\frac{D\mathbf{U}}{Dt} = \mathbf{i}\frac{Du}{Dt} + \mathbf{j}\frac{Dv}{Dt} + \mathbf{k}\frac{Dw}{Dt} + u\frac{D\mathbf{i}}{Dt} + v\frac{D\mathbf{j}}{Dt} + w\frac{D\mathbf{k}}{Dt}. \tag{8}$$

Note that here $D\mathbf{i}/\mathbf{Dt}$ is not equal to $D_a\mathbf{i}/\mathbf{Dt}$ from section 2.1.1 (Eq 3). Not only is it a different derivative, but the $\mathbf{i}$ in section 2.1.1 didn't move in the Earth frame, while here it does. In fact, it is the motion of $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ in the Earth frame that makes $D\mathbf{i}/Dt$, $D\mathbf{j}/Dt$, and $D\mathbf{k}/Dt$ nonzero, causing the curvature effect that we learned about in chaper one. For example,

$$\frac{D\mathbf{i}}{Dt} = \frac{u}{a\cos\phi}(\mathbf{j}\sin\phi - \mathbf{k}\cos\phi)$$

The first term on the r.h.s. of Eq. (7) is separated into components by first writing

$$-2\mathbf{\Omega} \times \mathbf{U} = -2\Omega \left| \begin{array}{ccc|cc} \mathbf{i} & \mathbf{j} & \mathbf{k} & \mathbf{i} & \mathbf{j} \\ 0 & \cos\phi & \sin\phi & 0 & \cos\phi \\ u & v & w & u & v \end{array} \right|$$

The x-component is $(-2\Omega w\cos\phi + 2\Omega v\sin\phi)\mathbf{i}$

Finally the complete x-component of the momentum equation is

$$\frac{Du}{Dt} - \frac{uv\tan\phi}{a} + \frac{uw}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\Omega v\sin\phi - 2\Omega w\cos\phi + F_{rx}$$

see Holton for the other two components

2.4 Scale analysis

Characteristic Scales - all order of magnitude only

$U \sim$	10 m/s	horizontal velocity
$W \sim$	1 cm/s	vertical velocity
$L \sim$	10^3 km	storm width
$H \sim$	10 km	tropopause height
$\delta P/\rho \sim$	10^3 m^2/s^2	horizontal pressure variation
$L/U \sim$	10^5 s	advective time scale

Couple of useful constants: $\nu \sim 10^{-5}$ m^2/s, $f_o \sim 10^{-4}$ s^{-1}, and Earth's radius $a \sim 10^4$ km

x-Eq	$\frac{Du}{Dt}$	$-\frac{uv\tan\phi}{a}$	$+\frac{uw}{a}$	$= -\frac{1}{\rho}\frac{\partial p}{\partial x}$	$+2\Omega v\sin\phi$	$-2\Omega w\cos\phi$	$+F_{rx}$
scales	U^2/L	U^2/a	UW/a	$\delta P/(\rho L)$	f_oU	f_oW	$\nu U/H^2$
(m s^{-2})	10^{-4}	10^{-5}	10^{-8}	10^{-3}	10^{-3}	10^{-6}	10^{-12}

2.4.1 Geostrophic Approximation and Geostrophic wind

Scale analysis tells us that for midlatitude synoptic scale dynamics we can let

$$-fv \approx -\frac{1}{\rho}\frac{\partial p}{\partial x} \text{ and } fu \approx -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

which is an approximation.

We can rewrite these equations as exact for a construct known as the geostrophic wind, which we DEFINE $\mathbf{V}_g = u_g\mathbf{i} + \mathbf{v_g j}$ with

$$-fv_g = -\frac{1}{\rho}\frac{\partial p}{\partial x} \text{ and } fu_g = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

or

$$\mathbf{V}_g = \mathbf{k}\times\frac{1}{\rho f}\nabla p$$

Let me repeat this subtle point. The geostrophic approximation is an approximate for the wind in general, but it is exact for this thing that doesn't really exist, known as the geostrophic wind.

2.4.2 Approximate Prognostic Equations: The Rossby Number

We need time derivatives in our equations to make a prediction, so we often keep the acceleration term even though it is small:

$$\frac{Du}{Dt} \approx fv - \frac{1}{\rho}\frac{\partial p}{\partial x} = f(v - v_g)$$

$$\frac{Dv}{Dt} \approx -fu - \frac{1}{\rho}\frac{\partial p}{\partial y} = -f(u - u_g)$$

or

$$\frac{D\mathbf{V}}{Dt} \approx -f\mathbf{k}\times\mathbf{V} - \frac{1}{\rho}\nabla p = -f\mathbf{k}\times(\mathbf{V} - \mathbf{V_g})$$

The Rossby number tells us the magnitude of the accleration to the Coriolis force:

$$\left|\frac{D|\mathbf{V}|}{Dt}\right| \Big/ |f\mathbf{k}\times\mathbf{V}| = U/(f_oL) \equiv R_o$$

2.4.3 The Hydrostatic Approximation

Recall that hydrostatic balance:

$$g = -\frac{1}{\rho}\frac{dp}{dz}$$

is specifically for $p = p(z)$, an atmosphere at rest. The hydrostatic approximation refers to the partial differential form

$$g = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

which is necessary if $p = p(x, y, z, t)$, as is the case in general. A scale analysis of the vertical momentum equation shows this is true, but it is a weak analysis to justify this approximation.

Instead we can use ananalysis that expands the pressure and density in two parts: a "basic state" and a "perturbation" from the basic state.

$$p = p_o(z) + p'(x, y, z, t)$$

$$\rho = \rho_o(z) + \rho'(x, y, z, t)$$

The basic state pressure, $p_o(z)$, is also called the "standard pressure". It is the horizontal and time mean as a function of level. The basic state density $\rho_o(z)$ is defined so that $p_o(z)$ and $\rho(z)$ are in exact hydrostatic balance:

$$\frac{dp_o}{dz} + g\rho_o = 0.$$

The primed variables usually only represent smaller scale variations: It is assumed that $\rho'/\rho_o << 1$ and $p'/p_o << 1$.

Expanding hydrostatic balance with these variables is known as a perturbation analysis. After some work, you will find

$$\frac{\partial p'}{\partial z} + g\rho' = 0.$$

They key to deriving this result is to ignore all terms that are products of variables with two primes. Carefully read section 2.4.3 and convince yourself that the perturbation part of the pressure and density also is approximately in hydrostic balance, which is a much stronger statement than just showing that the basic state components are in approximate hydrostatic balance.

2.5 The Continuity Equation

This section will be taught with a worksheet, so no lecture notes are provided. Refer to the text and the worksheet for this topic.

2.6 The Thermodynamic Equation

The first law of thermodynamics

$$de = dq - dw$$

states that the change in internal energy e of a system is equal to the heat added to the system minus the work done by the system on its environment.

We aim to write an equation that conserves total thermodynamic energy and then cancel some terms to make it look like the first law.

The total thermodynamic energy in a Lagrangian fluid element of volume δV:

$$\rho[e + 0.5\mathbf{U} \cdot \mathbf{U}]\delta V$$

is the kinetic energy of molecules plus the kinetic energy of the macroscopic flow. Its Lagrangian rate of change equals the work done by the environment (thermal plus mechanical) on the system plus heat added, which is simply ρJ.

The rate at which work is done by the external forces on the parcel comes from the pressure force and gravity (the Coriolis force is perpendicular to $\mathbf{U}$ so it does no work):

$$-[\nabla\cdot(p\mathbf{U}) + \rho g w]\delta V.$$

Before combining terms it is helpful to take the dot product of $\mathbf{U}$ with the momentum equation to show that the rate of change of the kinetic energy of the macroscopic flow equals the work done by the pressure gradient force and gravity:

$$\rho\frac{D}{Dt}\left(\frac{1}{2}\mathbf{U}\cdot\mathbf{U}\right) = -\mathbf{U}\cdot\nabla p - \rho g w. \tag{9}$$

Now combining terms together gives

$$\rho\frac{De}{Dt} = -p\nabla\cdot\mathbf{U} + \rho J$$

The continuity equation allows us to write

$$\frac{1}{\rho}\nabla\cdot\mathbf{U} = -\frac{1}{\rho^2}\frac{D\rho}{Dt} = \frac{D\alpha}{Dt},$$

so we arrive at the thermodynamic energy equation (or TDE):

$$c_v\frac{DT}{Dt} + p\frac{D\alpha}{Dt} = J$$

where $\alpha = 1/\rho$.

$c_v\frac{DT}{Dt}$	change in internal (thermal) energy of dry air
$p\frac{D\alpha}{Dt}$	rate of work done by the fluid system (per unit mass) on its environment by thermal processes (as opposed to mechanical). This is mainly due to vertical motion and the consequent expansion or contraction of a parcel. For example, a rising parcel expands and does work on its environment, so it cools.
J	"diabatic heating", the heating done by external means, such as radiation or through phase change. The units are W/kg.

2.7 A more useful form of the Thermodynamic Energy Equation

The middle term is not very convenient so rewrite it using the ideal gas law

$$p\alpha = RT$$

Take D/Dt of both sides

$$\alpha \frac{Dp}{Dt} + p\frac{D\alpha}{Dt} = R\frac{DT}{Dt}$$

and substitute it into the T.D.E. above:

$$c_v \frac{DT}{Dt} + R\frac{DT}{Dt} - \alpha\frac{Dp}{Dt} = J$$

Recall $c_p = c_v + R$, so

$$c_p \frac{DT}{Dt} - \alpha\frac{Dp}{Dt} = J, \tag{10}$$

which should be the equation you turn to first!

Sample problem (work Holton 2.5 on your own, it's similar)

What is the vertical velocity of an air parcel that is heated diabatically at the rate of 10^{-1} W/kg if the air parcel is warming at the rate of 0.9 K/hr? (Hint: The vertical velocity is constant. $c_p = 1004$ J/kg/K.)

Eq. 10 can be modified so each term only depens on a single state variable. If we substitute for ρ from the gas law and divide by T, we get

$$c_p \frac{D\ln T}{Dt} - R\frac{D\ln p}{Dt} = \frac{J}{T} = \frac{Ds}{Dt}, \tag{11}$$

where s in entropy. The T.D.E. written in this way indicates that vertical motion in the atmosphere changes the thermodynamic state in a reversible way. A parcel moving vertically will change temperature due to compression or expansion with pressure change, but when returned to its original level, it will have done no work on its environment. If $J = 0$, its temperature will return to the original temperature too.

2.7.1 Potential temperature

The potential temperature θ is defined as the temperature a parcel of air would have if it were expanded or compressed adiabatically to standard pressure p_s. Adiabatically means $J = 0$ in Eq 11 above, which after integrating from (T, p) to (θ, p_s) gives:

$$\theta(z) = T(z)(p_s/p)^{R/c_p}$$

At any level z there is a temperature $T(z)$ and a corresponding potential temperature $\theta(z)$, where $\theta(z)$ is a construct, kind of like the geostrophic wind. You can compute θ for the atmosphere, but you can't measure it.

For adiabatic motion ($J = 0$), so $\theta(z)$ is a constant.

2.7.2 The adiabatic lapse rate

How does the temperature of a parcel vary as it moves up and down? First write down Poisson's Eq. $\theta = T\left(p_s/p\right)^{R/c_p}$, and take the total derivative with respect to z.

$$\frac{d\theta}{dz} = \frac{\partial\theta}{\partial T}\frac{dT}{dz} + \frac{\partial\theta}{\partial p}\frac{dp}{dz}$$

$$\frac{d\theta}{dz} = \left(\frac{p_s}{p}\right)^{R/c_p}\frac{dT}{dz} + \frac{RT}{c_p}\left(\frac{p_s}{p}\right)^{R/c_p-1}\left(\frac{-p_s}{p^2}\right)\frac{dp}{dz}$$

Use hydrostaticity and rearrange

$$\left(\frac{p_s}{p}\right)^{-R/c_p}\frac{d\theta}{dz} = \frac{dT}{dz} - \frac{RT}{pc_p}(-\rho g)$$

Substitute from Poisson's Eq and Ideal gas law again

$$\frac{T}{\theta}\frac{d\theta}{dz} = \frac{dT}{dz} + \frac{g}{c_p} \tag{12}$$

If the diabatic heating is zero $J = 0$, then $d\theta/dz = 0$ and

$$-\frac{dT}{dz} = g/c_p \equiv \Gamma_d \tag{13}$$

and Γ_d is the adiabatic lapse rate. It is about 10°K per 1 km for a dry atmosphere.

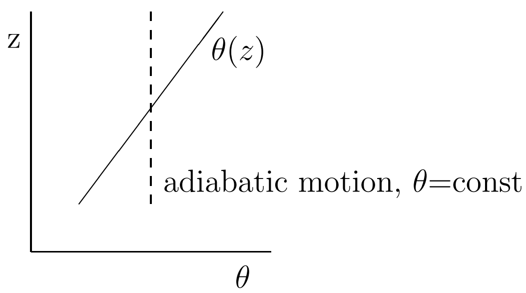
2.7.3 Static Stability

In general, J is nonzero and

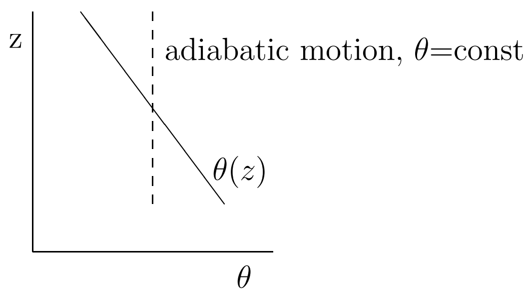
$$S \equiv \frac{T}{\theta}\,\frac{d\theta}{dz} = \Gamma_d - \Gamma,$$

where S is known as the static stability.

Stable: $\frac{d\theta}{dz} > 0$ so $\Gamma < \Gamma_d$



Unstable: $\frac{d\theta}{dz} < 0$ so $\Gamma > \Gamma_d$



A positively buoyant parcel is warmer (hence lighter) than its surroundings.

Statically stable — an upward adiabatic displacement causes a parcel to become negatively buoyant, so it tends to return towards its initial level.

Statically unstable — an upward adiabatic displacement causes a parcel to become positively buoyant, so it tends to continue to move upward.

Stable conditions promote **Buoyancy Oscillations**, as displaced parcels overshoot their equilibrium level on their return. The vertical momentum equation describes the motion of the parcel when we retain the acceleration term:

$$\frac{Dw}{Dt} = \frac{D^2}{Dt^2}(\delta z) = -g - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

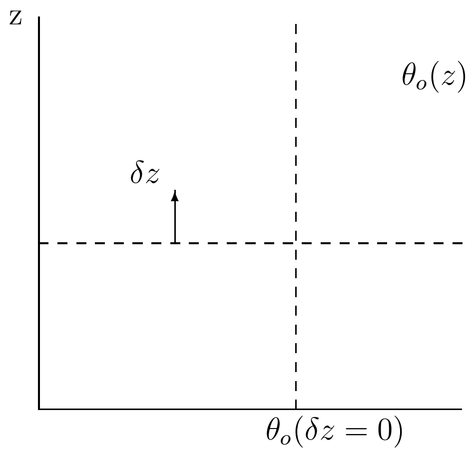
The δ in front of the z only reminds us that it is a small displacement.

Assume the parcel and environment do not exchange heat, so the parcel's T and ρ are distinct from the environment's (call them T_0 and ρ_0). However, the parcel pressure adjusts freely to its surroundings, so $p = p_0$. *The key to describing these oscillations (and working the problems in the textbook) is careful book keeping of these symbols.*

Assume hydrostatic balance for the environment:

$$\frac{\partial p}{\partial z} = \frac{\partial p_0}{\partial z} = -g\rho_0,$$

Environment Description



but keep the vertical acceleration in the parcel's vertical momentum equation:

$$\frac{D^2}{Dt^2}(\delta z) = g\left(\frac{\rho_0 - \rho}{\rho}\right),$$

where instead the acceleration equals the buoyancy force that a displaced parcel experiences. With the ideal gas law

$$\frac{D^2}{Dt^2}(\delta z) = g\left(\frac{\rho_0 - \rho}{\rho}\right) = g\frac{T - T_0}{T_0}$$

Now let the reference level be at $\delta z = 0$ so for the environment $\theta_0(\delta z) = T_0(\delta z)[p_0(0)/p_0(\delta z)]^{R/c_p}$ and for the parcel $\tilde{\theta}(\delta z) = T(\delta z)[p_0(0)/p_0(\delta z)]^{R/c_p}$, then

$$\frac{D^2}{Dt^2}(\delta z) = g\frac{T - T_0}{T_0} = g\frac{(\tilde{\theta} - \theta_0)}{\theta_0} = \frac{\theta}{\theta_0}.$$

where $\theta \equiv \tilde{\theta} - \theta_0$.

Recall that the parcel's potential temperature is constant, hence $\tilde{\theta}(\delta z) = \theta_0(0)$. The environments potential temperature can be expanded in a Taylor's series expansion about the equilibrium height $z = 0$ for the environment gives

$$\theta_0(\delta z) \approx \theta_0(0) + (d\theta_0/dz)\delta z.$$

Hence

$$\tilde{\theta} - \theta_0 \approx -(d\theta_0/dz)\delta z.$$

$$\frac{D^2}{Dt^2}(\delta z) \approx -g\frac{d\ln\theta_0}{dz}\delta z \equiv -N^2\delta z$$

where N^2 is a measure of the environment's stability and N is the frequency of the oscillations. It is known as the Brunt-Väisälä frequency.

The atmosphere only exibits oscillations when it is statically stable $(d\theta_o/dz > 0)$ If it is unstable, the parcel would keep going in the same direction when it is perturbed.

You may be surprised after trying so hard to convince you that hydrostatic balance is very accurate that I would deviate from it here. This is the only occasion where we consider vertical acceleration in this course and it is only to illustrate atmospheric stability. The Quasi-geostrophic approximation "filters" buoyancy oscillations out of our equations. This is reasonable since they do little to alter synoptic scale storms. You should know about them because they are present in data, and often must be removed before analyzing the data.