

Box Model equation: $q = \frac{S * \tau}{V}$

1. The emissions rate of NO in downtown Metropolis is 5.8×10^{27} molecules of NO per day. If the lifetime of NO in downtown is $\frac{1}{4}$ day, calculate the concentration (molecules/cm³) of NO in the downtown area. Use a volume of 4.7×10^{16} cm³.

S = 5.8×10^{27} molecules NO/day

V = volume of the basin = 4.7×10^{16} cm³

$\tau = 1/4$ day

We can now calculate the concentration of NO by applying the above formula:

$$q = \frac{S * \tau}{V} = \frac{\left(5.8 \times 10^{27} \frac{\text{molecules NO}}{\text{day}}\right) * \left(\frac{1}{4} \text{ day}\right)}{\left(4.7 \times 10^{16} \text{ cm}^3\right)} = 3.09 \times 10^{10} \frac{\text{molecules NO}}{\text{cm}^3}$$

2. Convert your answer from #3 to a mixing ratio (ppbv). Assume that 1 cm³ of air contains 2.5×10^{19} molecules.

$$\text{Mixing Ratio} = \frac{\text{Concentration}}{\text{AirDensity}} = \frac{3.09 \times 10^{10} \frac{\text{molecules NO}}{\text{cm}^3}}{2.5 \times 10^{19} \frac{\text{molecules air}}{\text{cm}^3}} = 1.23 \times 10^{-9} \frac{\text{molecules NO}}{\text{molecule air}} = 1.23 \text{ ppbv NO}$$

3. The city of Metropolis has an approximate volume of 1.5×10^{18} cm³. The total SO₂ emissions in that basin are 1×10^{30} molecules of SO₂ per day and the residence time of SO₂ is $\frac{1}{2}$ a day. What is the concentration in molecules/cm³ of SO₂ in the city?

S = 1×10^{30} molecules SO₂/day

V = volume of the basin = 1.875×10^{18} cm³

$\tau = 1/2$ day

We can now calculate the concentration of SO₂ by applying the above formula:

$$q = \frac{S * \tau}{V} = \frac{\left(1 \times 10^{30} \frac{\text{molecules SO}_2}{\text{day}}\right) * \left(\frac{1}{2} \text{ day}\right)}{\left(1.5 \times 10^{18} \text{ cm}^3\right)} = 3.33 \times 10^{11} \frac{\text{molecules SO}_2}{\text{cm}^3}$$

4. The mixing ratio of carbon monoxide in Metropolis is 300 ppb. What is its concentration in molecules/cm³? Remember that 1 cm³ of air contains 2.5×10¹⁹ molecules, and that 1 ppb means “one part per billion” (billion=10⁹).

300 ppbv of CO means 300 molecules of CO in 10⁹ molecules of air (M.R.=300×10⁻⁹). Given that there are 2.5×10¹⁹ molecules of air per cm³ and we know that:

Concentration=Mixing Ratio * Air Density

$$\text{Concentration} = 300 \times 10^{-9} \frac{\text{molecules CO}}{\text{molecule air}} * 2.5 \times 10^{19} \frac{\text{molecules air}}{\text{cm}^3} = 7.5 \times 10^{12} \frac{\text{molecules CO}}{\text{cm}^3}$$

5. Let's assume that the residence time, τ , of carbon monoxide in the Metropolis basin is 1 day and that 14 Million people live in the basin. How many CO molecules does each person emit per day? The volume of the basin is 1.875×10¹⁸ cm³.

To solve this question transform the steady state box model equation to calculate the source rate, S , from the concentration, q , the volume V , and the residence time τ .

$$q = \frac{S * \tau}{V} \Rightarrow S = \frac{q * V}{\tau}$$

q = CO concentration = 7.5×10¹² molecules/cm³

V = volume of the basin = 1.875×10¹⁸ cm³

τ = 1 day

We can now calculate the source rate of CO by applying the above formula:

$$S = \frac{q * V}{\tau} = \frac{\left(7.5 \times 10^{12} \frac{\text{molecules CO}}{\text{cm}^3} \right) * (1.875 \times 10^{18} \text{ cm}^3)}{(1 \text{ day})} = 1.4 \times 10^{31} \frac{\text{molecules CO}}{\text{day}}$$

Each person the emits: $s = S/\text{population} = (1.4 \times 10^{31} \text{ molecules CO/day}) / (1.4 \times 10^7 \text{ persons}) = 1 \times 10^{24} \text{ molecules CO/day per person}$