Atmospheric Science 441, Homework 7

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1. An air column at 60°N with $\zeta_\theta = 0$ initially stretches from the surface at 1000 hPa up to 250 hPa. This air column remains vertically upright and moves isentropically due east until it is over a mountain barrier where the ground is at the 700 hPa level.

(a) Assuming the top of the column remains at the 250 hPa level, what is its absolute vorticity and relative vorticity as it passes the mountain?

(b) Suppose the column continues to move due east and that on the lee side, where the surface is again at the 1000-hPa level, the top of the column has subsided to 500 hPa. What is its absolute vorticity and relative vorticity in this location?

2. A westerly zonal flow (flow from 270°) between the surface and the tropopause at 45°N rises adiabatically over a north-south oriented mountain barrier. Before striking the mountain the westerly wind increases linearly toward the south at a rate of 10 m s^{-1} per 1000 km (at all vertical levels). The crest of the mountain range is at 800-hPa and the tropopause, located at 300 hPa, remains undisturbed.

(a) What is the initial relative vorticity of the air?

(b) What is its relative vorticity when it reaches the crest if it is deflected 5 latitude toward the south during the forced ascent and the surface pressure upstream of the mountains was 1000 hPa?

(c) If the current develops a laterally uniform speed of 20 m s^{-1} during its ascent to the crest, what is the radius of curvature of the streamlines at the crest?

3. Starting from the horizontal momentum equations in z-coordinates,

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x}, \qquad \frac{Dv}{Dt} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y},$$

we found that the vertical vorticity equation took the form

$$\frac{D\zeta}{Dt} = \ldots + \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right).$$

Here the ... include the stretching and the tilting terms and the advection of planetary vorticity by the north-south winds. The final term on the right is the "baroclinic" or solenoidal term.

Show that the baroclinic term is not present if we derive the corresponding relation for the change in the vertical vorticity component in isobaric coordinates. That is, show that starting from

$$\frac{Du}{Dt} - fv = -\left(\frac{\partial \Phi}{\partial x}\right)_p, \qquad \frac{Dv}{Dt} + fu = -\left(\frac{\partial \Phi}{\partial y}\right)_p,$$

the accelerations arising from height gradients never change the vertical vorticity on a constant pressure surface

$$\zeta_p = \left(\frac{\partial v}{\partial x}\right)_p - \left(\frac{\partial u}{\partial y}\right)_p.$$

Hint: Do not bother deriving all the terms in the isobaric vertical vorticity equation, just focus on those related to pressure/height gradients and leave the others grouped as "...".

4. Suppose that, over the square domain $0 \le x \le L$, $0 \le y \le L$, all fields are periodic in both x and y. This means that for any field such as the u component of the velocity, and for fixed $y_0 \in [0, L]$, $u(0, y_0) = u(L, y_0)$ and for any fixed $x_0 \in [0, L]$, $u(x_0, 0) = u(x_0, L)$. Show that the integral of ζ over this domain is zero.

Extra Credit: 5. Prove that the global average of the isentropic relative vorticity over any isentropic surface that does not intersect the ground must be zero. (*Hint;* assuming that ζ_θ is the component of relative vorticity perpendicular to an isentropic surface, use the relation between circulation and vorticity in your proof.)