Angular Momentum Presentation
Week 2

from Iskenderian and Salstein, *Monthly Weather Review* (1998)

2. Definitions and data sources

The global atmospheric angular momentum about the earth's axis can be expressed as

$$\mathrm{AAM} = M_r + M_\Omega, \tag{1}$$

where

$$M_r = \frac{a^3}{g} \iiint u \cos^2\phi \, d\phi \, d\lambda \, dp \tag{2}$$

is the entire atmosphere's angular momentum associated with its motion relative to the rotating solid earth, a is the earth's radius, g is acceleration due to gravity, u is zonal wind, and the integral is performed over all latitudes ϕ, longitudes λ, and pressures p. The angular momentum associated with the rotation of the atmosphere's mass is

$$M_{\Omega} = \frac{a^4\Omega}{g} \iint p_s \cos^3\phi \, d\phi \, d\lambda, \qquad (3)$$

where Ω is the mean rotation rate of the earth and p_s the surface pressure (Rosen 1993).

The conservation of angular momentum states that changes in AAM are related to the surface torque by the relationship

$$\frac{d}{dt}\mathrm{AAM} = T_m + T_f, \qquad (4)$$

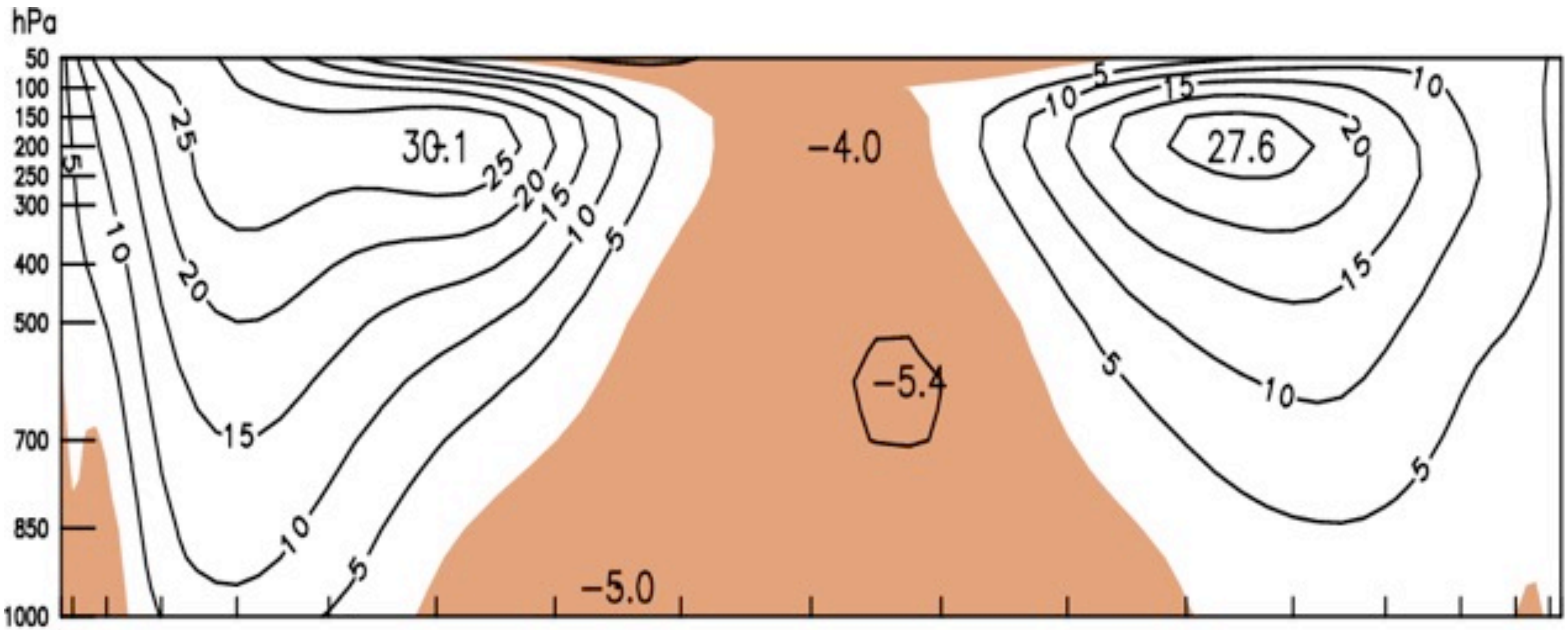
where the mountain torque (T_m) and friction torque (T_f) are defined through the following relationships (White 1991):

$$T_m = -a^2 \iint p_s \frac{\partial H}{\partial \lambda} \cos\phi \, d\phi \, d\lambda, \qquad (5)$$

$$T_f = a^3 \iint \tau \cos^2\phi \, d\phi \, d\lambda. \qquad (6)$$

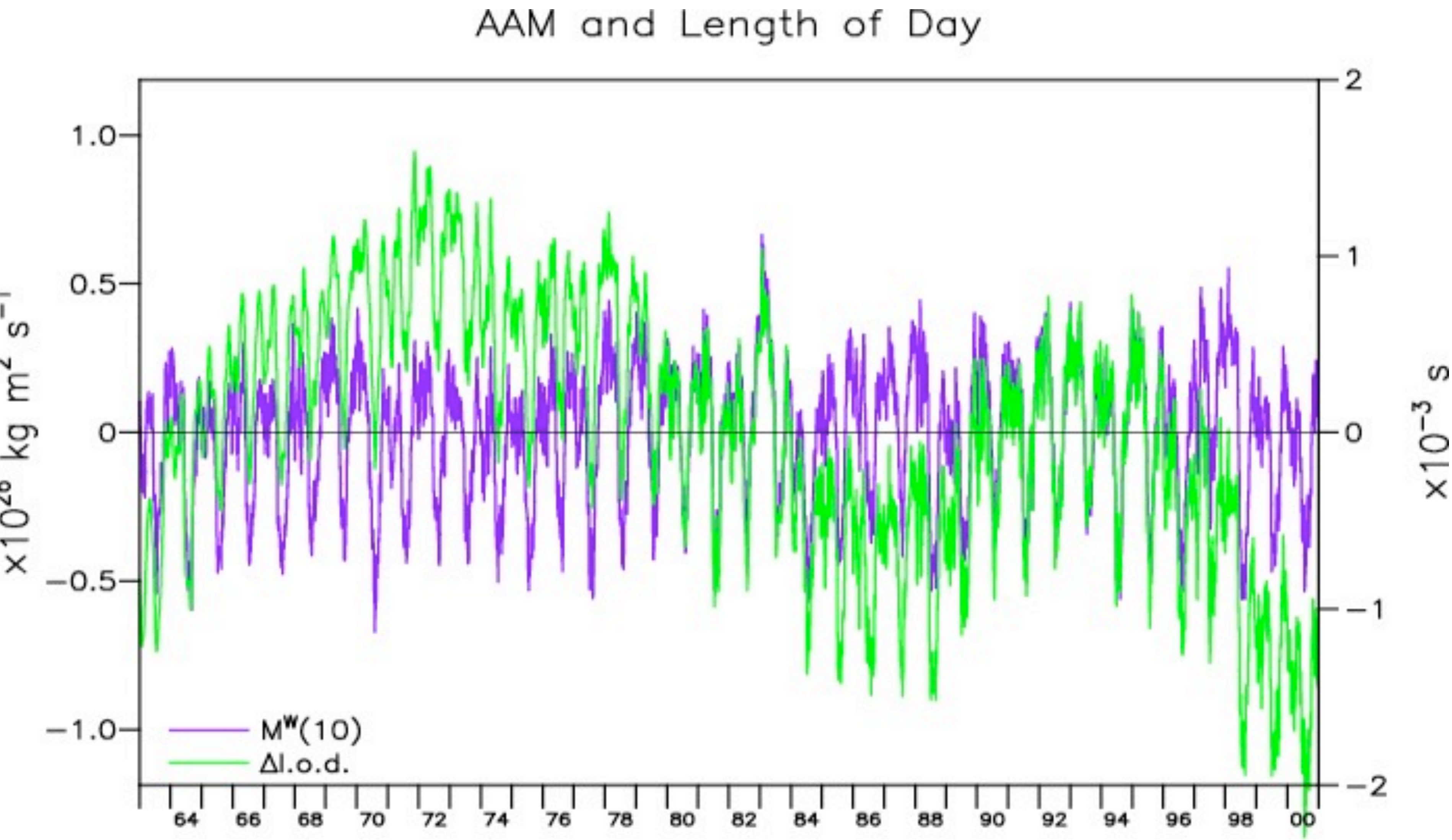
Here, τ is surface stress and H indicates the height of the sloping topography.

zonally averaged zonal wind $[u]$ (m s^{-1})



90°S 0° 90°N

Courtesy of David Salstein



$$\Delta M = k \times \Delta(L.O.D.)$$

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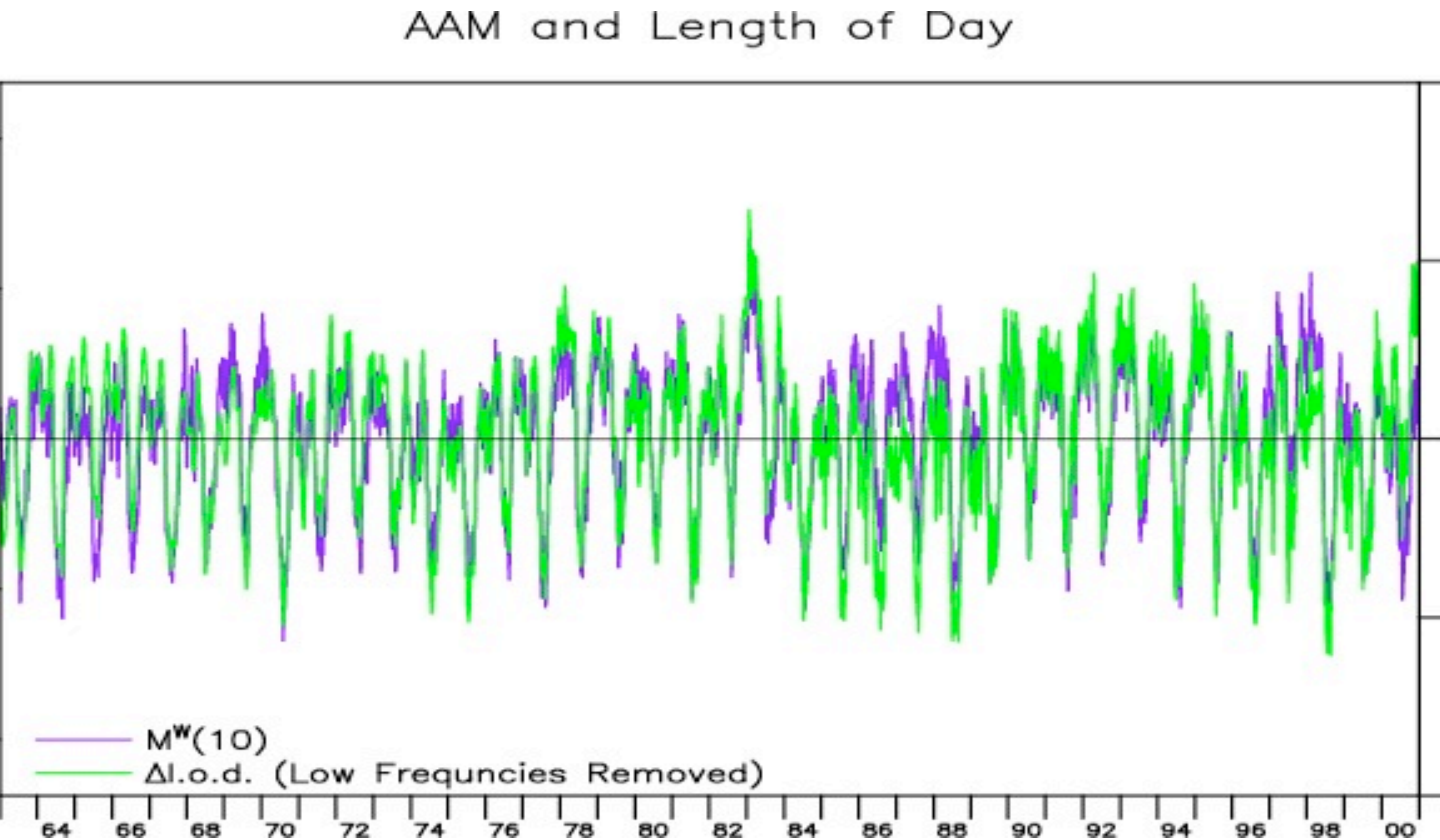

Filtered

Courtesy of David Salstein

Raw

$\Delta M = k \times \Delta(L.O.D.)$

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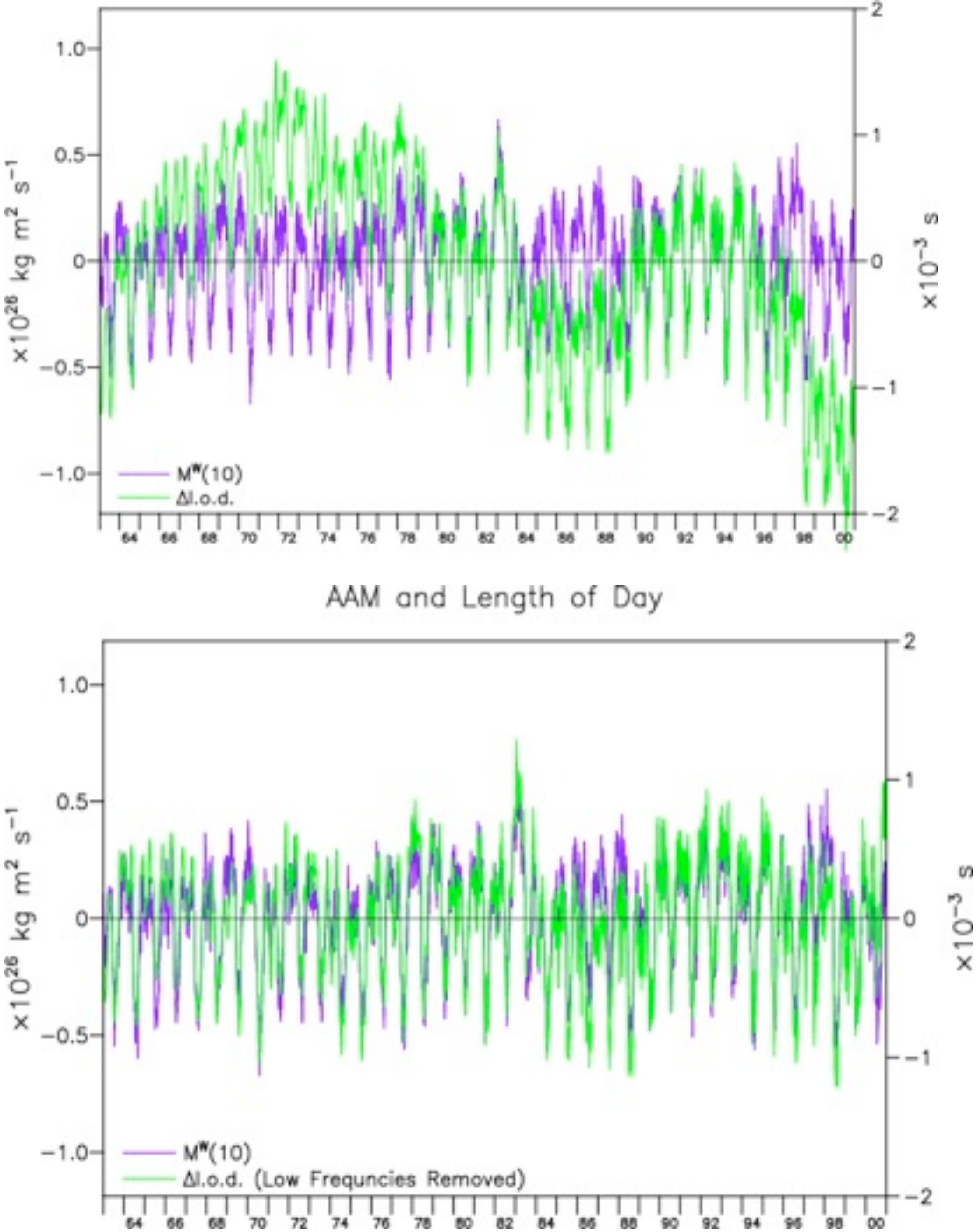

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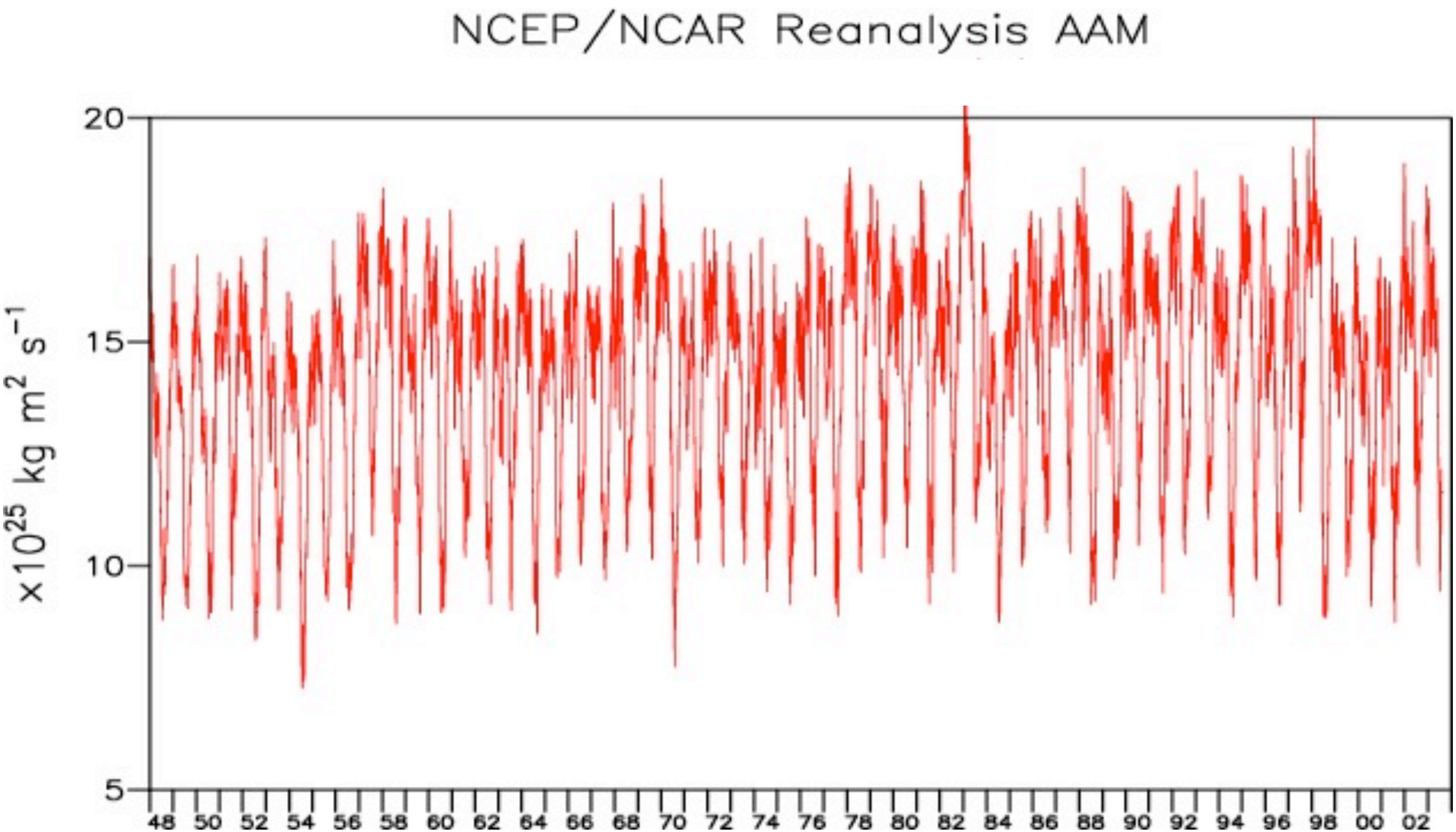
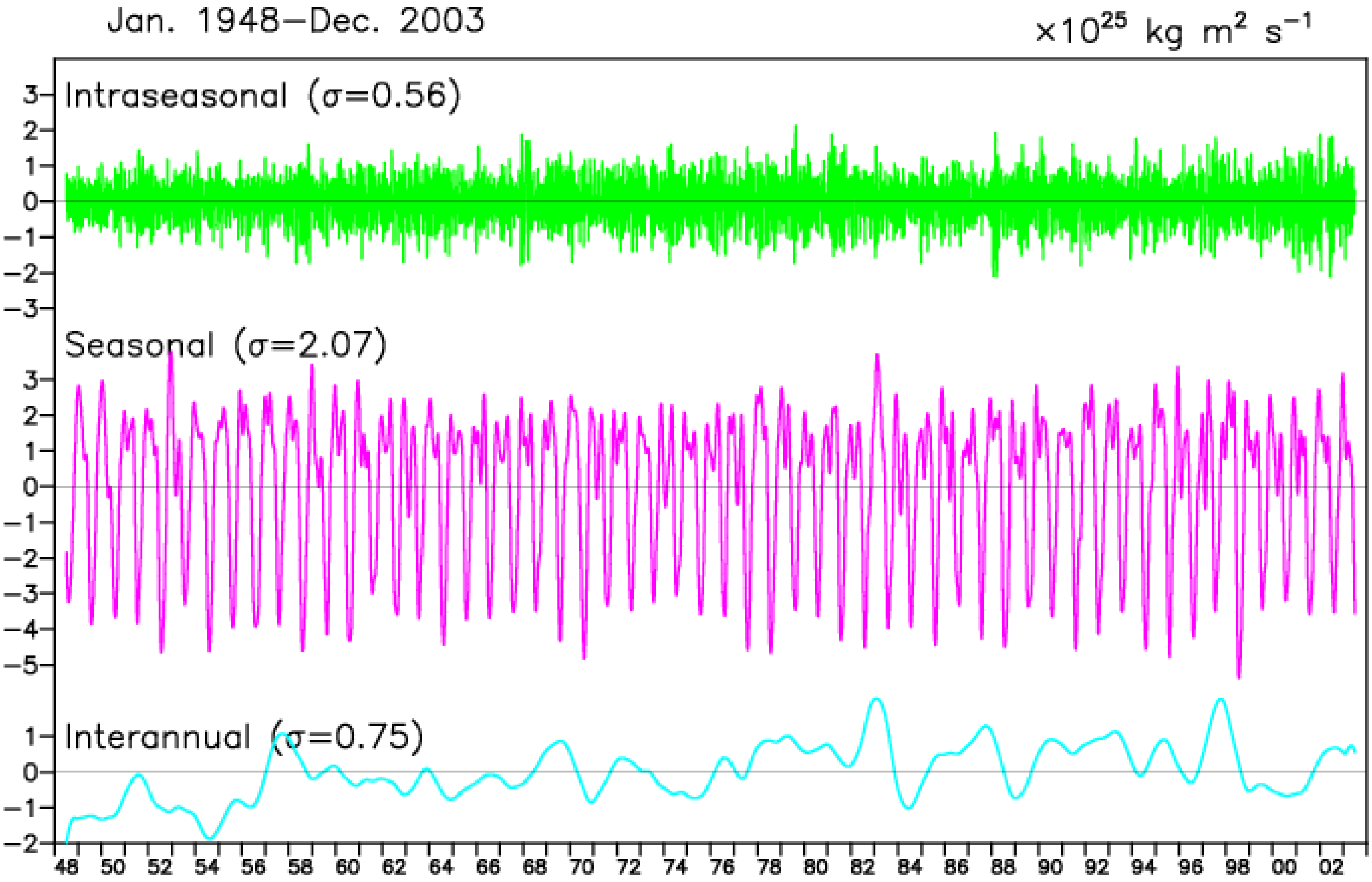


Note large seasonal cycle in angular momentum: factor of 2 between Jan/Feb and Jul/Aug

Courtesy of David Salstein

Breakdown into frequency bands



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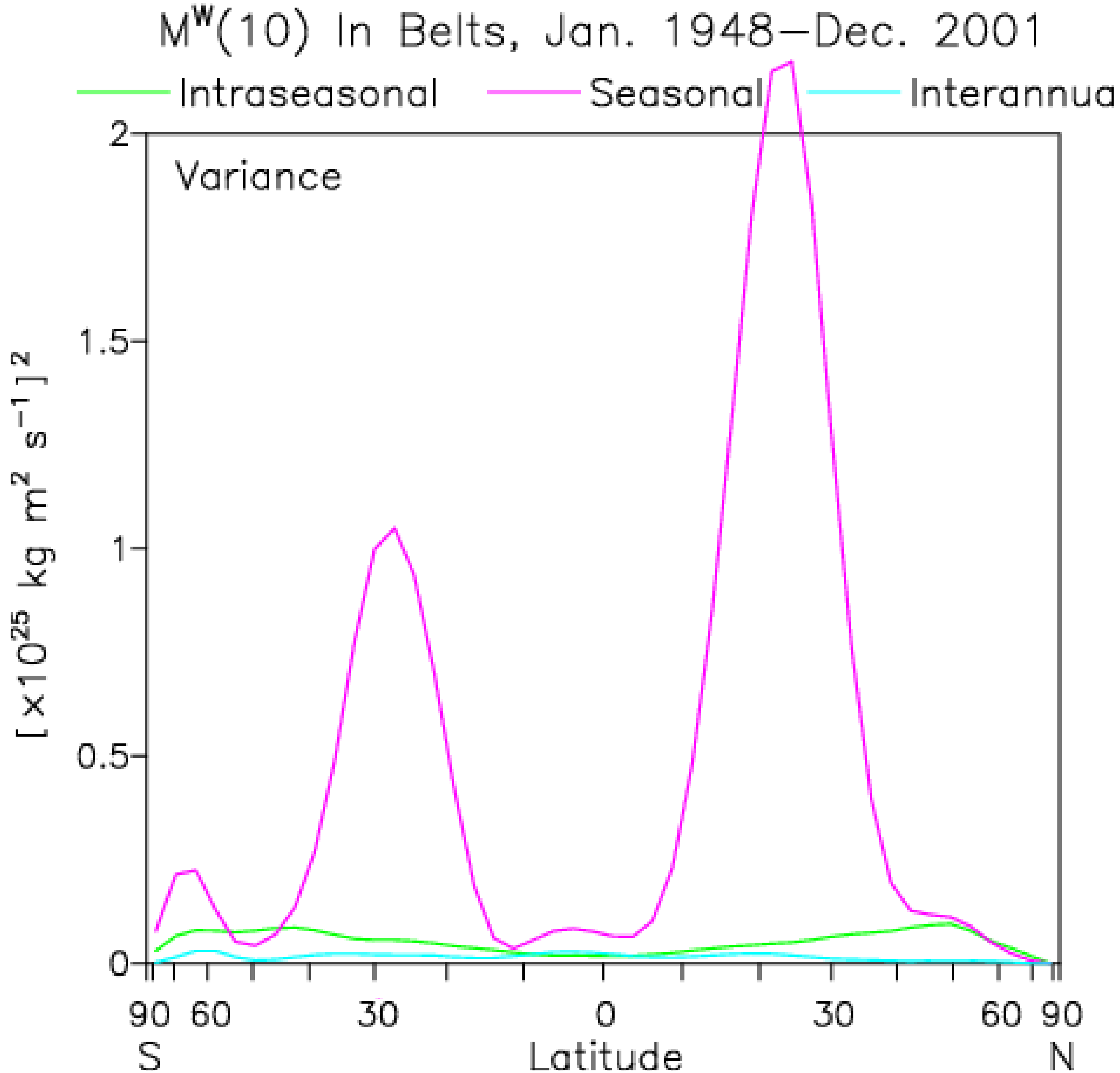

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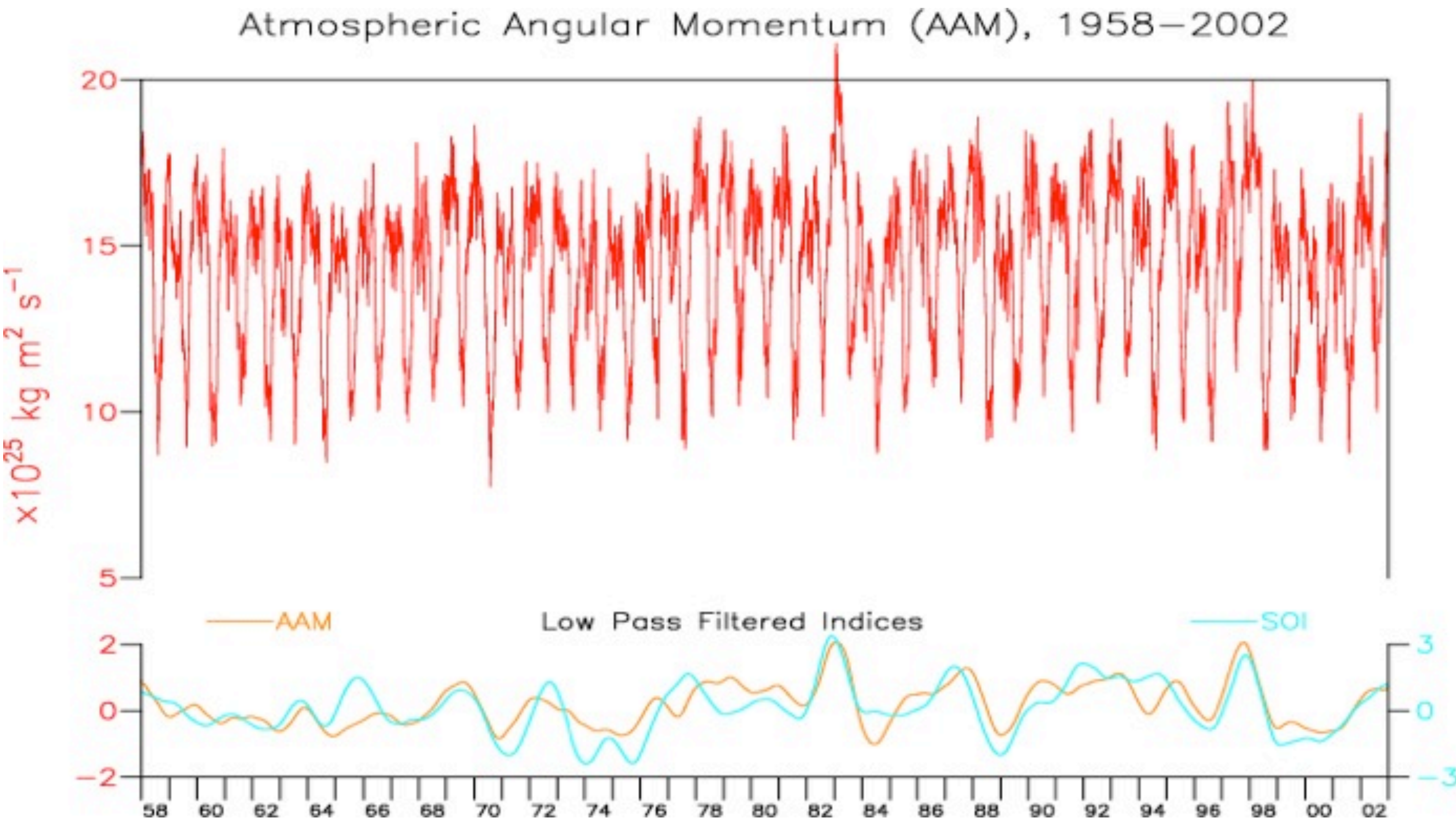
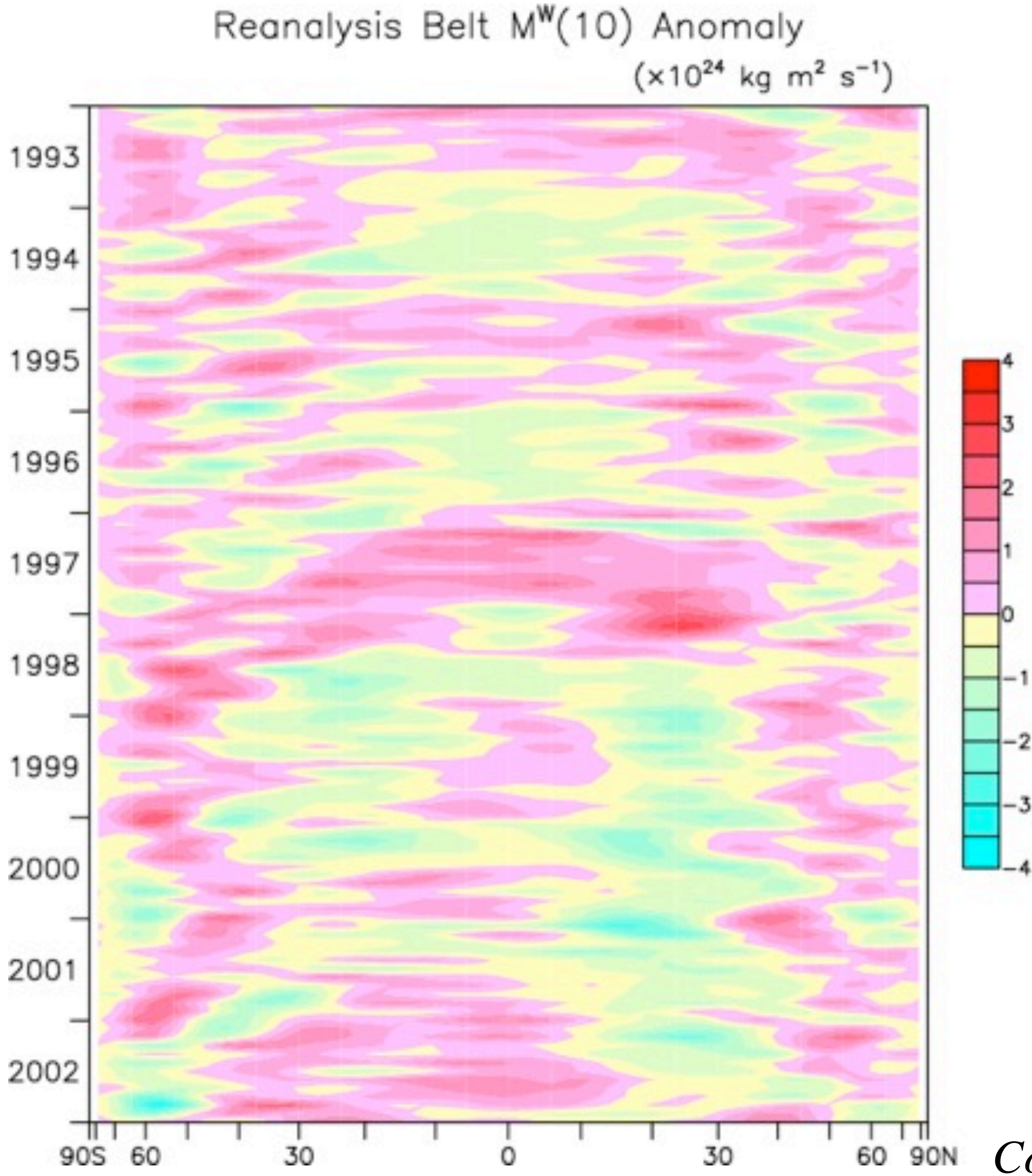


SOI = –Southern Oscillation Index; peaks correspond to El Nino events

Courtesy of David Salstein



Positive AAM anomalies in tropics coincide with El Nino events (e.g., 1997-98)

Courtesy of David Salstein

Mountain and friction torques

$$T_{mountain} = -R^2 \int\int p_s \frac{\partial H}{\partial \lambda} \cos\phi d\phi d\lambda$$

$$T_{friction} = R^3 \int\int \tau \cos^2\phi d\phi d\lambda$$

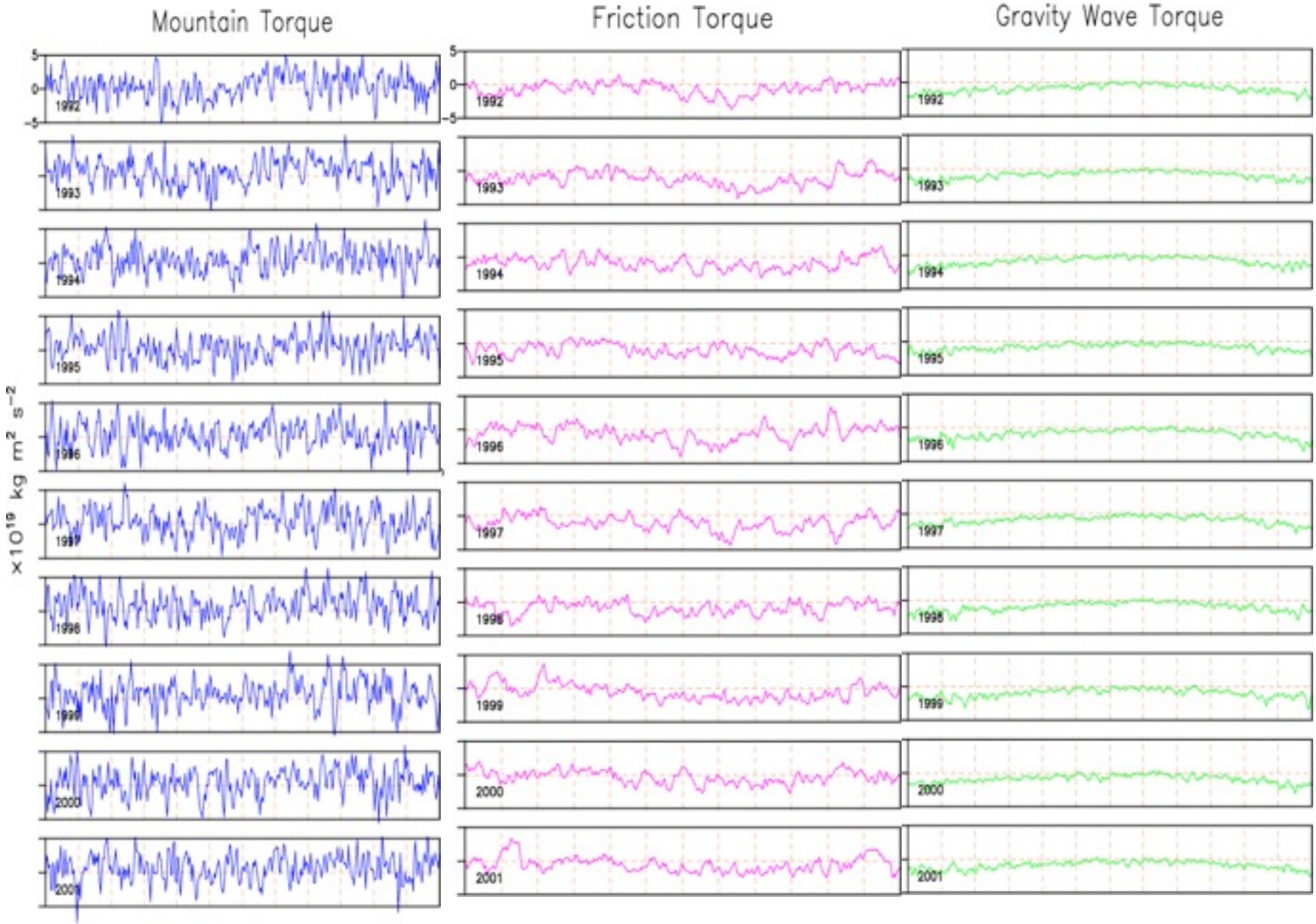
$T_{gravity-wave} =$ Frictional related to sub-grid scale Action in the atmospheric model

R=Earth radius, $\mathbf{p_s}$=surface pressure, **H**=topographic height

τ=frictional stress, related to winds and roughness (model)

ϕ=longitude λ=latitude

Courtesy of David Salstein



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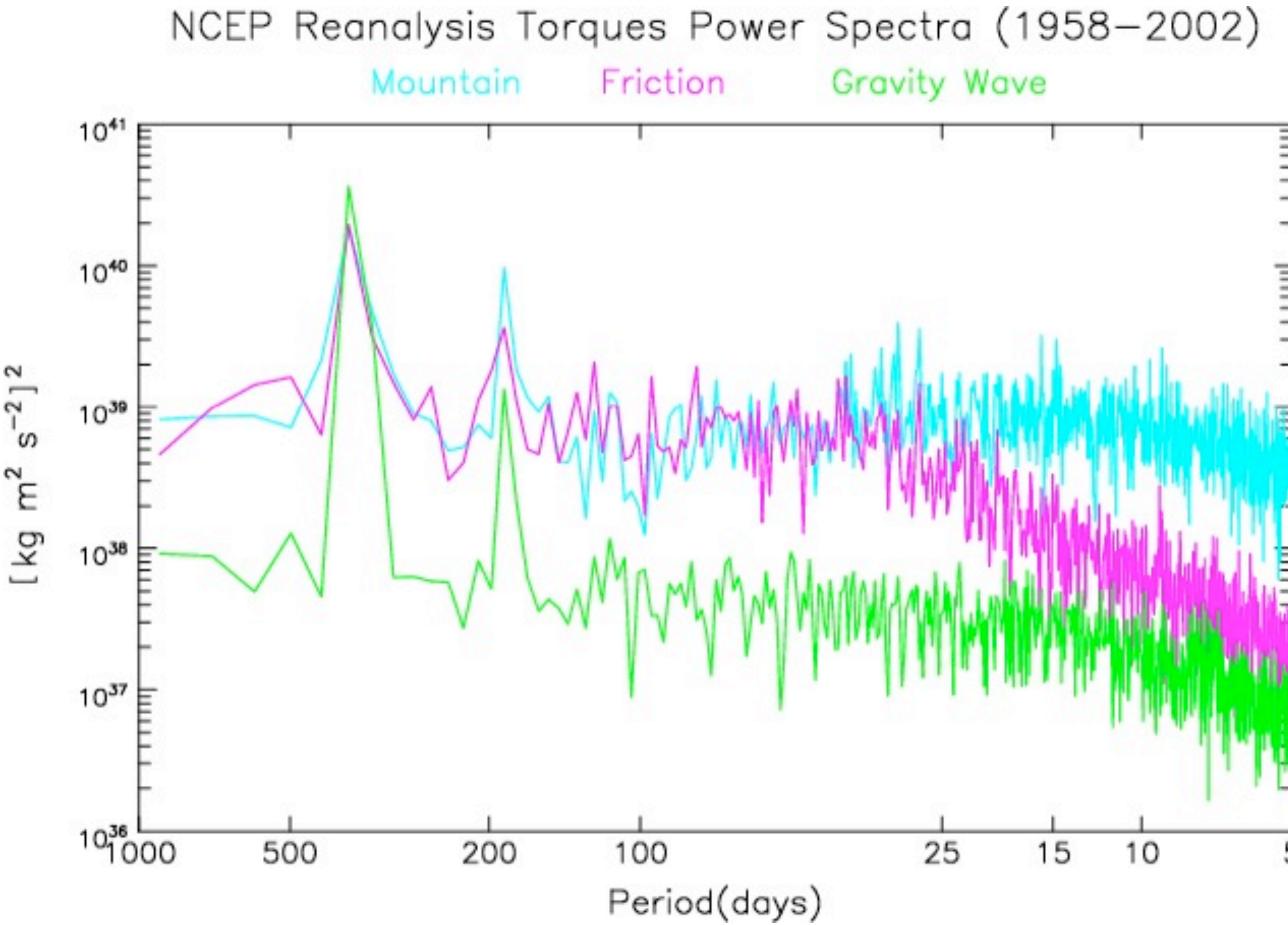
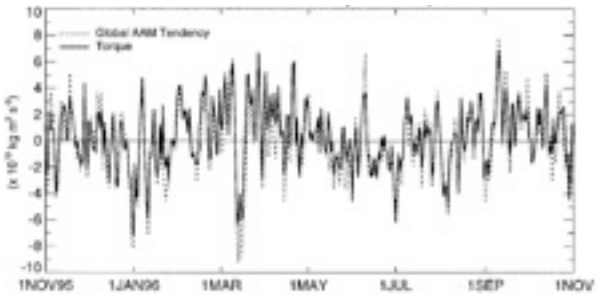
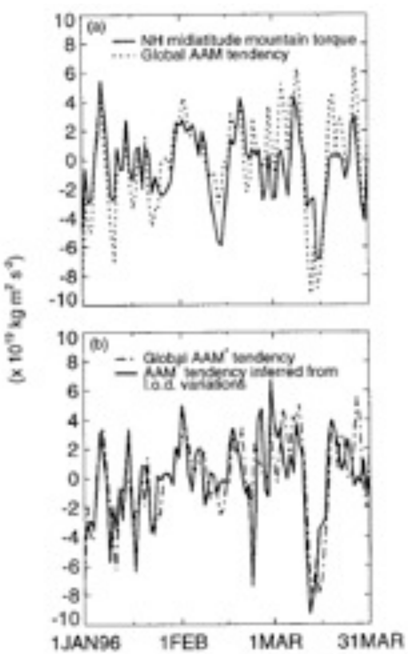


Courtesy of David Salstein

Regional Sources of Mountain Torque Variability and High-Frequency Fluctuations in Atmospheric Angular Momentum

HAIG ISKENDERIAN AND DAVID A. SALSTEIN





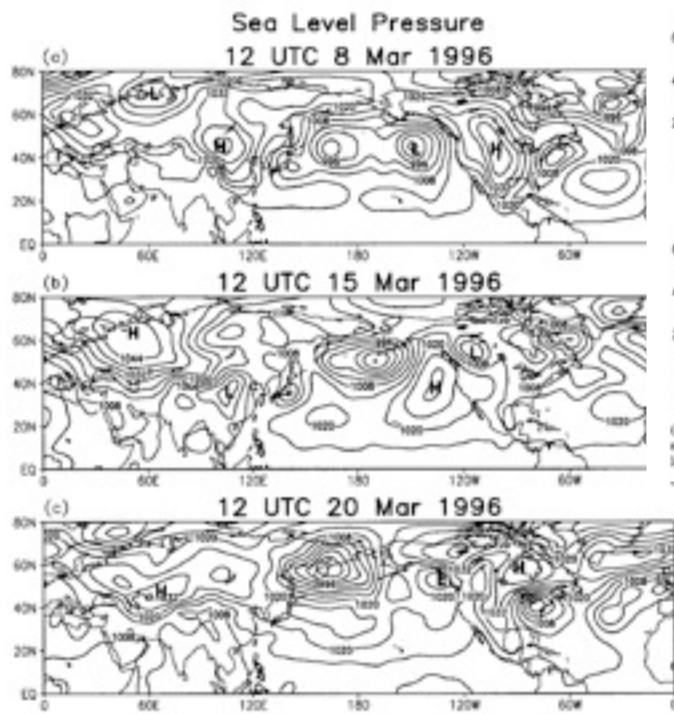


FIG. 8. Sea level pressure at 1200 UTC for (a) 8 March, (b) 15 March, and (c) 20 March 1996. Contours are every 6 hPa. Highs and lows discussed in the text are indicated by the letters H and L.

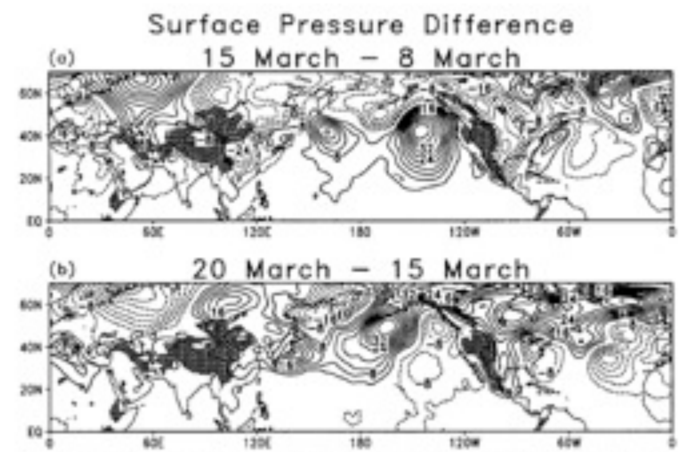
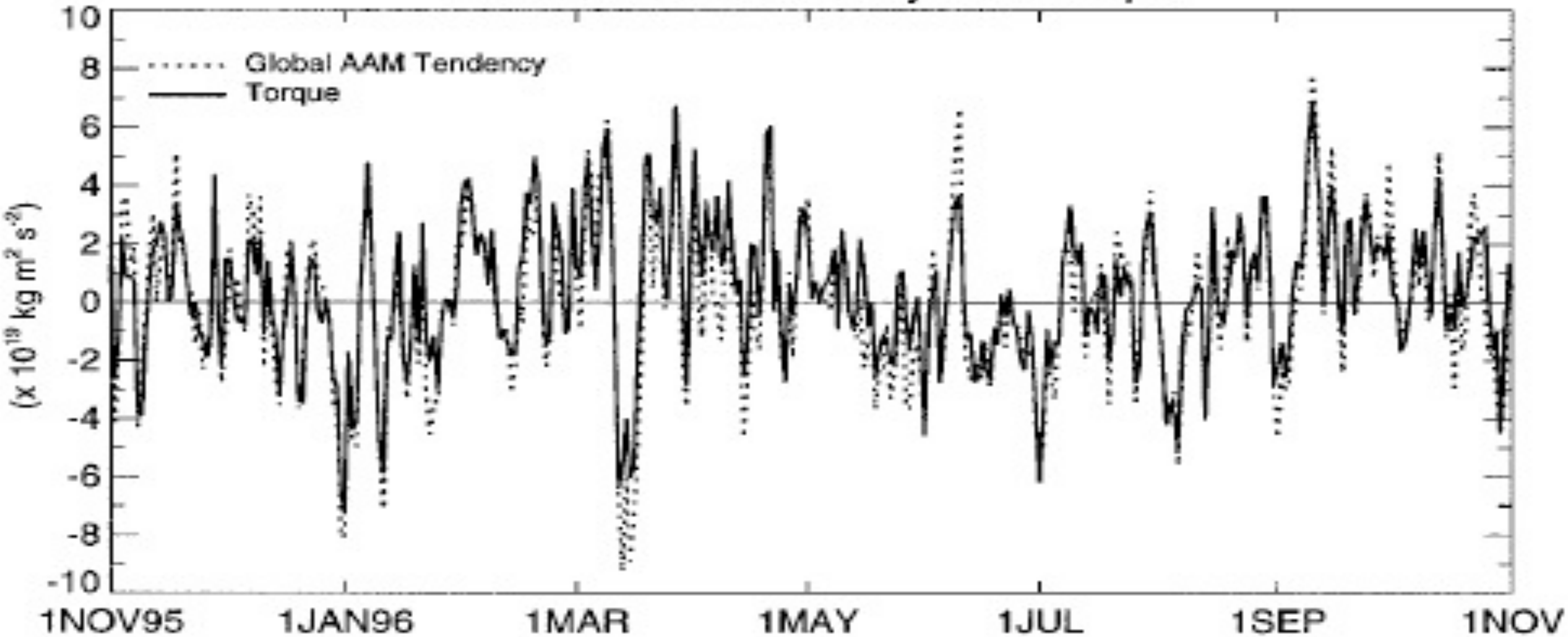
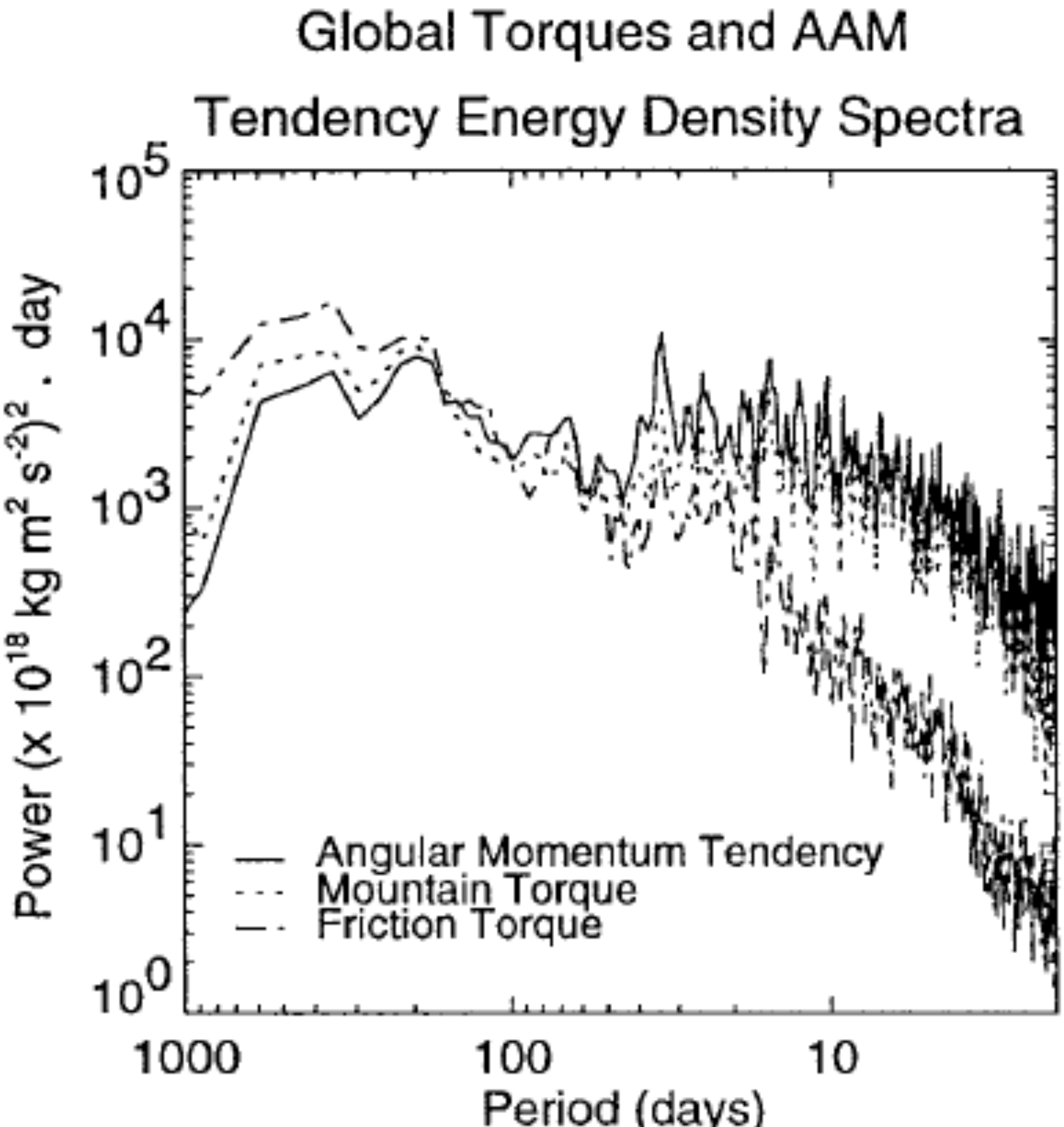


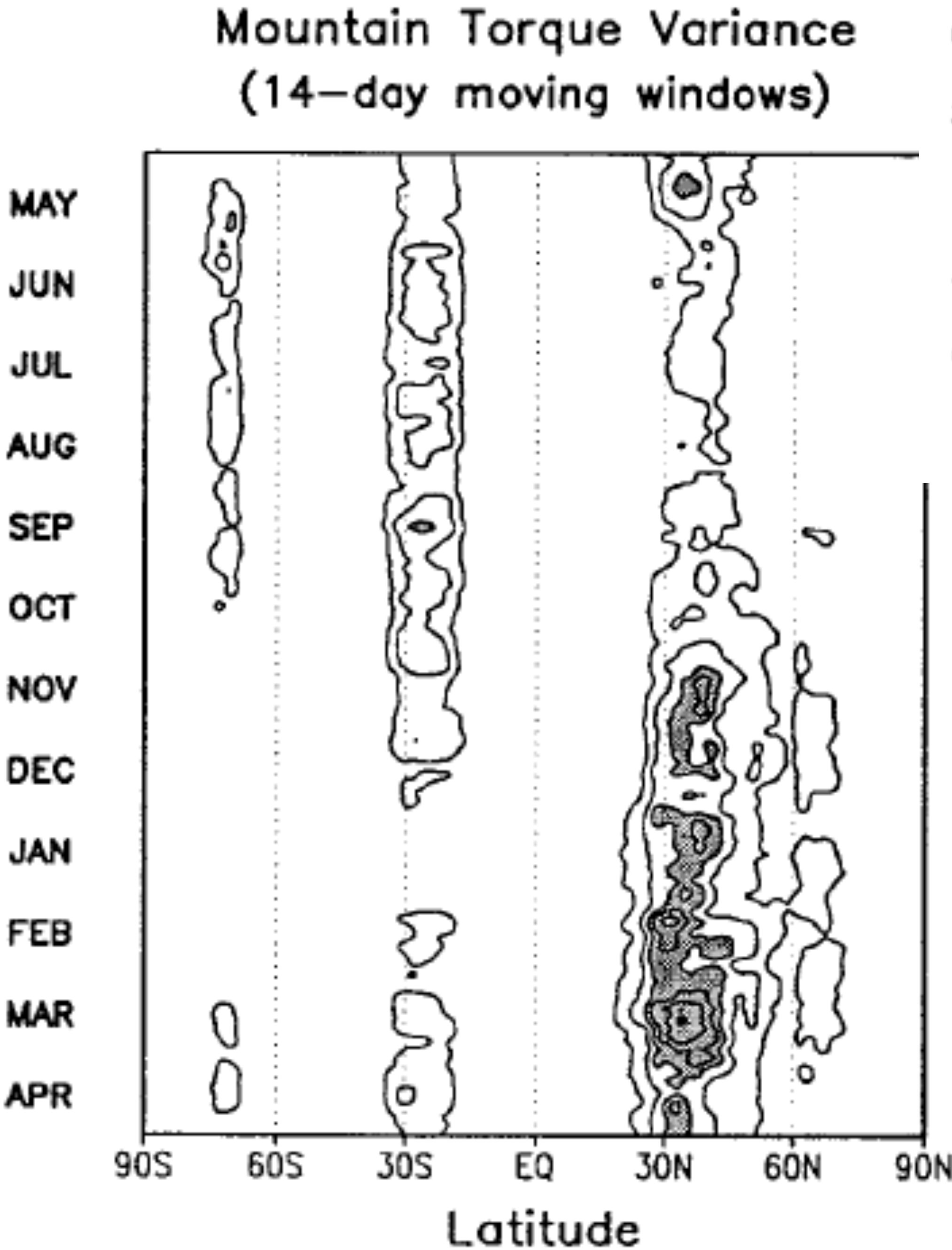
FIG. 9. (a) Difference in surface pressure between 1200 UTC 15 March and 1200 UTC 8 March 1996 (every 4 hPa contoured, zero line omitted). Positive contours are solid and negative dashed. (b) Same as (a) except between 1200 UTC 20 March and 1200 UTC 15 March. Shading indicates surface elevation of at least 1000 m.

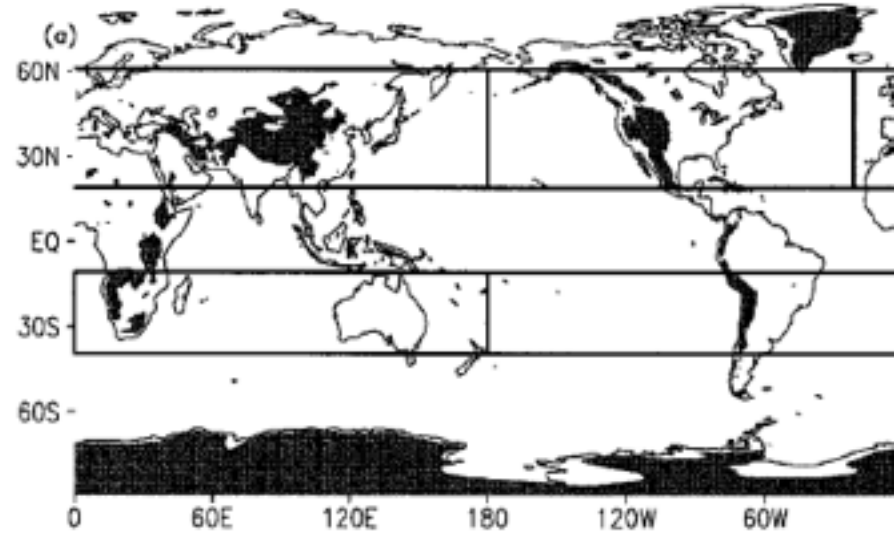
Global AAM Tendency and Torque





Global Torques and AAM
Tendency Energy Density Spectra
Power (x 10^18 kg m^2 s^-2)^2 . day
10^5
10^4
10^3
10^2
10^1
10^0
1000
100
10
Period (days)
Angular Momentum Tendency
Mountain Torque
Friction Torque


Mountain Torque Variance
(14–day moving windows)
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
JAN
FEB
MAR
APR
90S
60S
30S
EQ
30N
60N
90N
Latitude


(a)
60N
30N
EQ
30S
60S
0
60E
120E
180
120W
60W

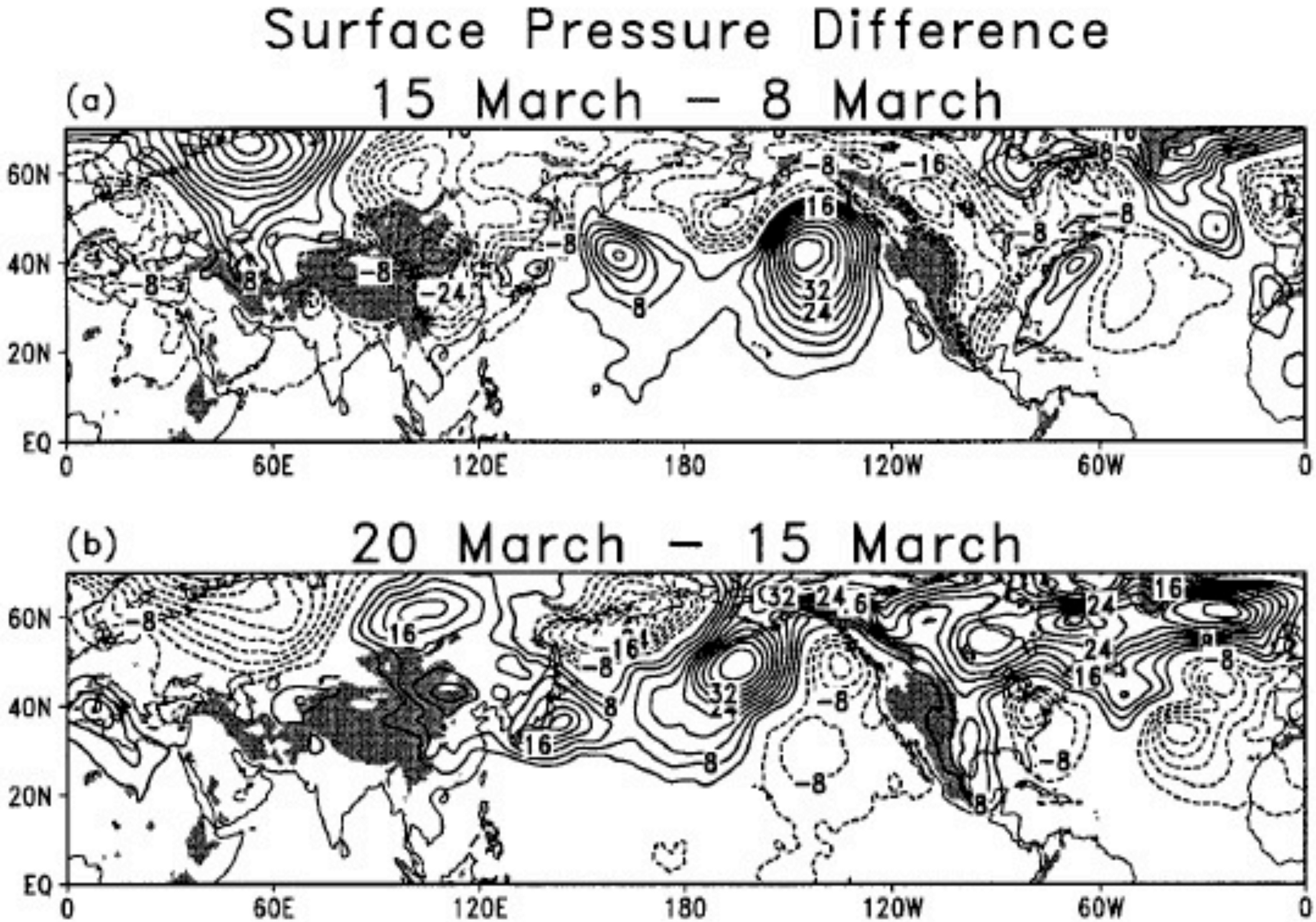
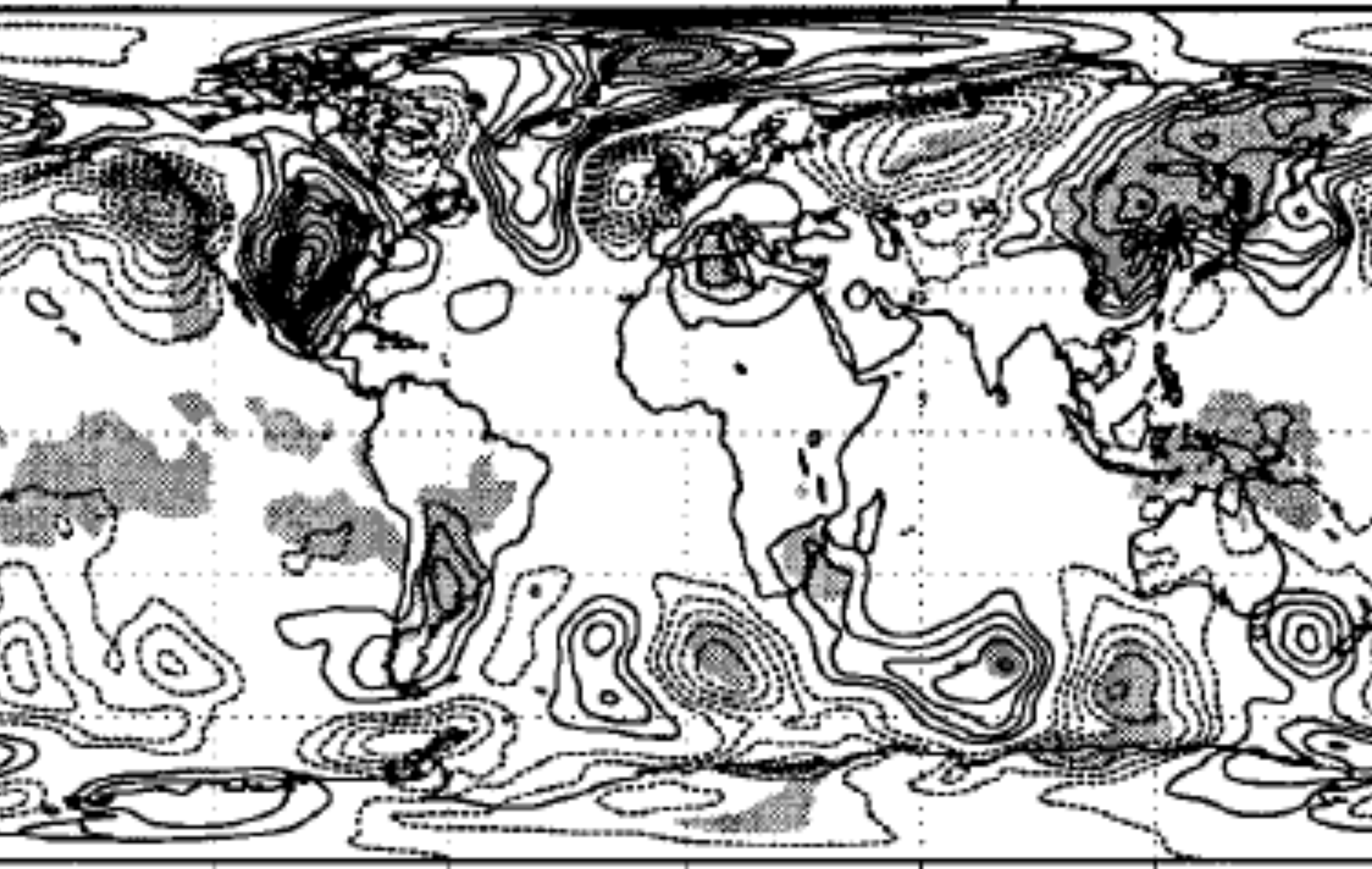


FIG. 9. (a) Difference in surface pressure between 1200 UTC 15 March and 1200 UTC 8 March 1996 (every 4 hPa contoured, zero line omitted). Positive contours are solid and negative dashed. (b) Same as (a) except between 1200 UTC 20 March and 1200 UTC 15 March. Shading indicates surface elevation of at least 1000 m.

b) November to April



For steady state:
Integrated over the globe
Net torque on atmosphere = 0

In the absence of mountains
net frictional torque = 0

if the surface winds are nonzero, there must be regions of easterlies and westerlies

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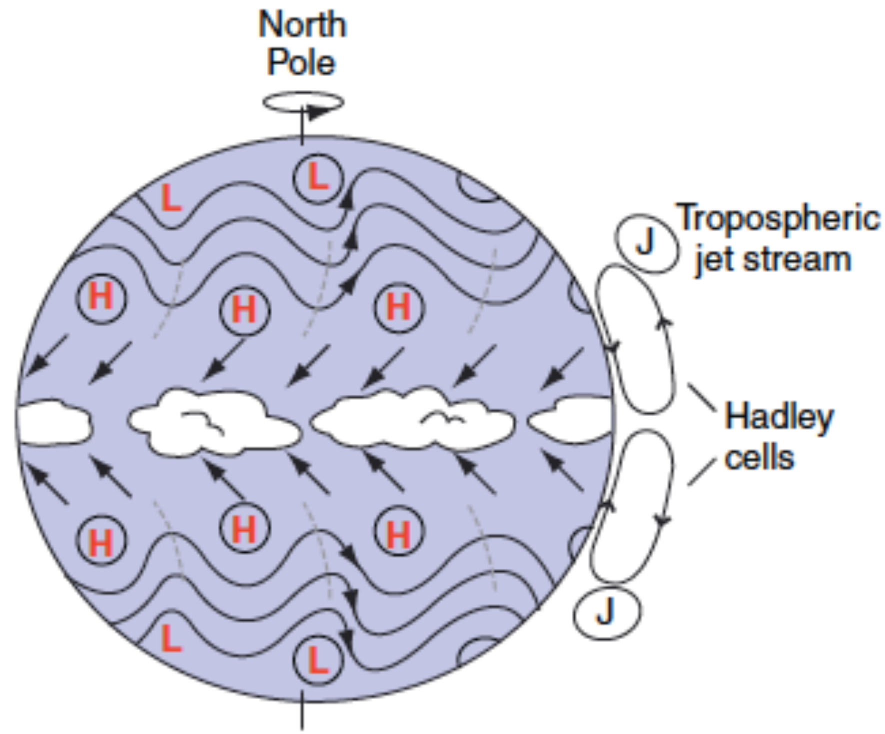

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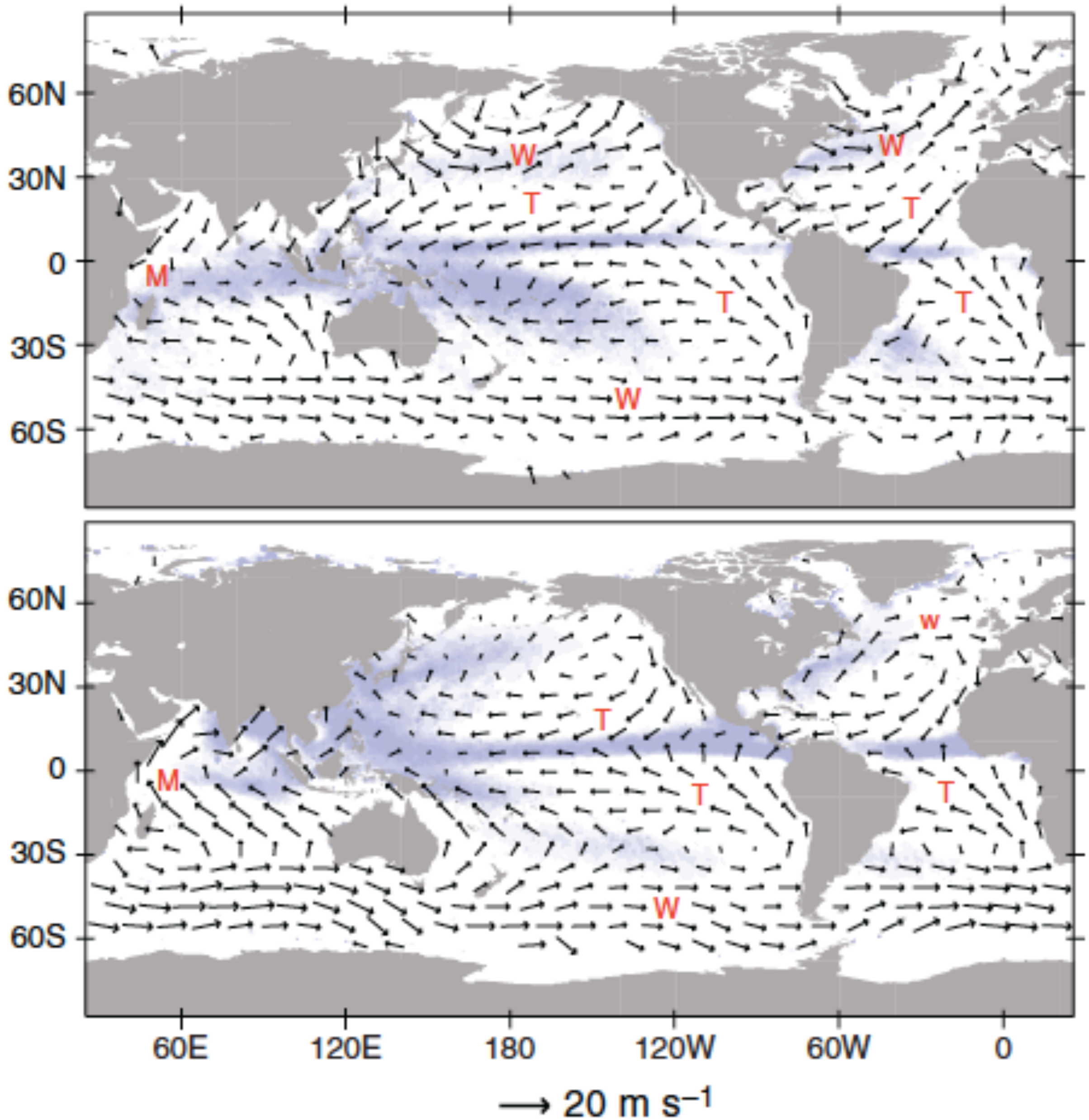
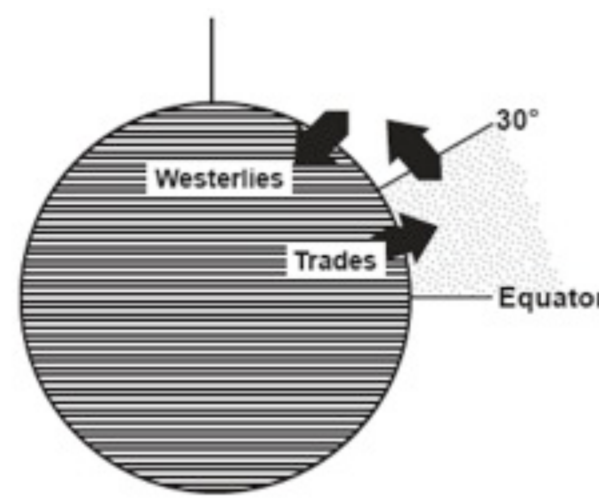
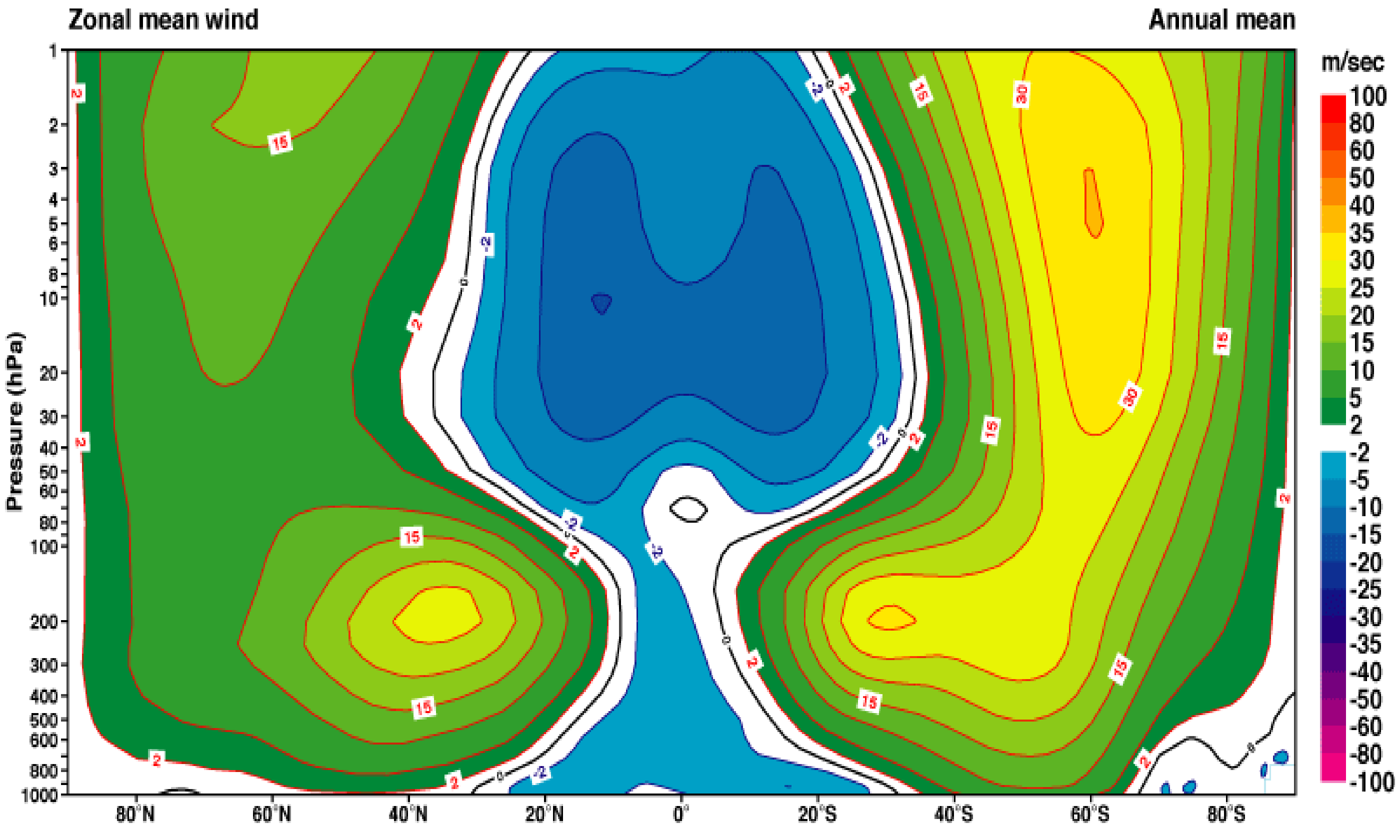
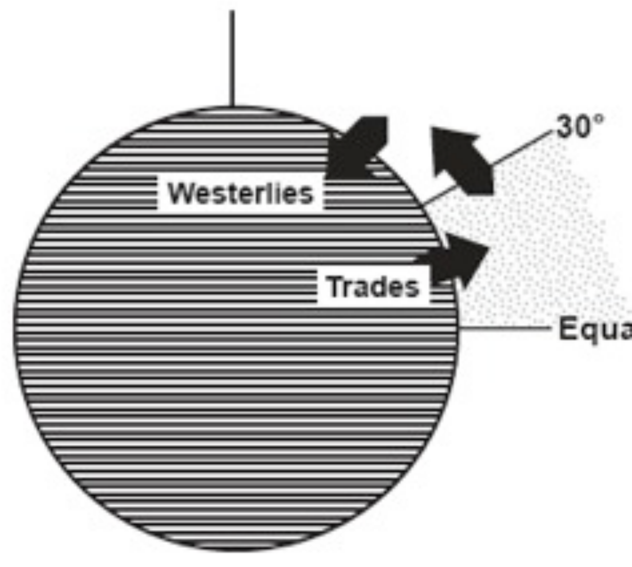
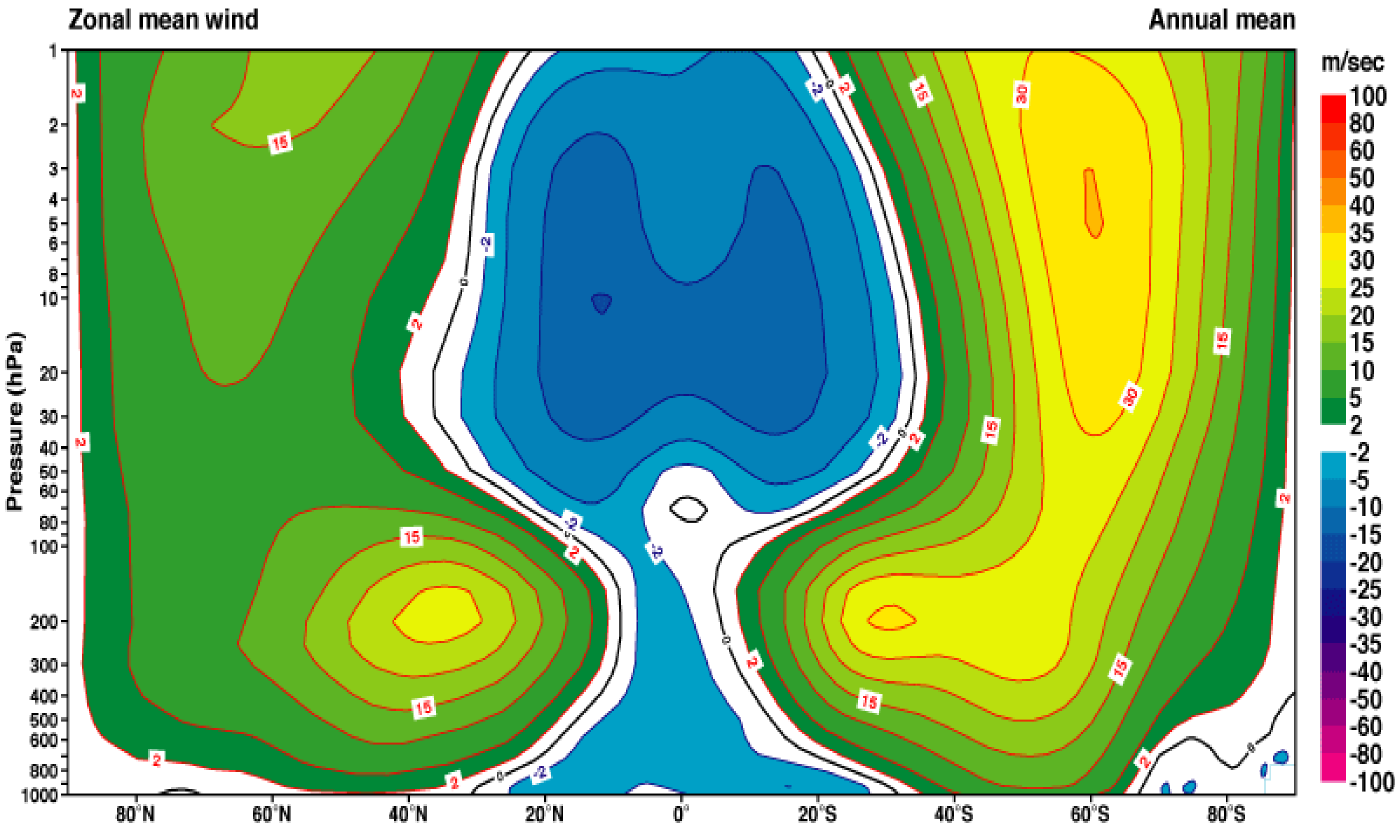

W
T
M
60N
30N
0
30S
60S
60E
120E
180
120W
60W
0
w
⟶ 20 m s⁻¹

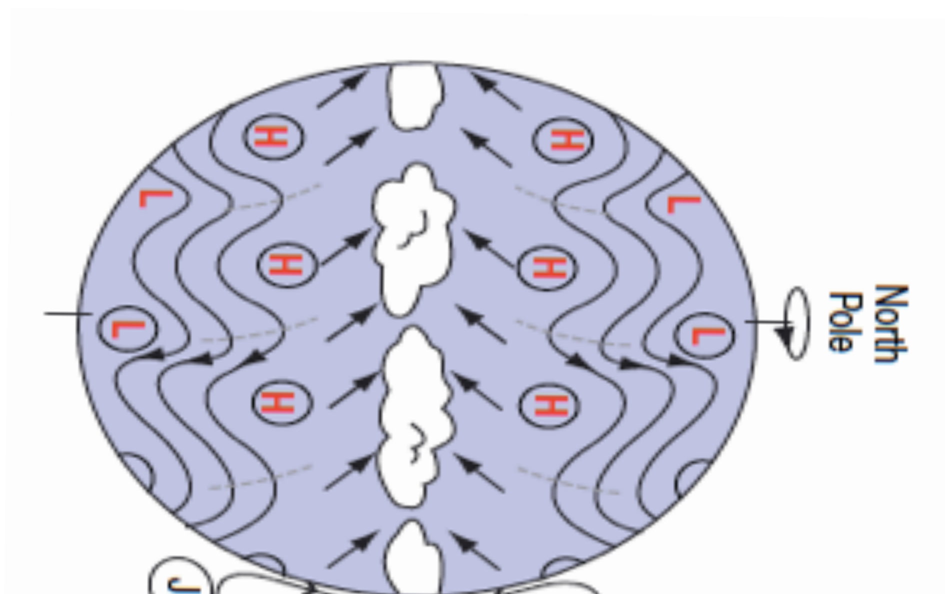

Westerlies
Trades
30°
Equator


Zonal mean wind
Annual mean
Pressure (hPa)
1
2
3
4
5
6
8
10
20
30
40
50
60
80
100
200
300
400
500
600
800
1000
80°N
60°N
40°N
20°N
0°
20°S
40°S
60°S
80°S
m/sec
100
80
60
50
40
35
30
25
20
15
10
5
2
-2
-5
-10
-15
-20
-25
-30
-35
-40
-50
-60
-80
-100

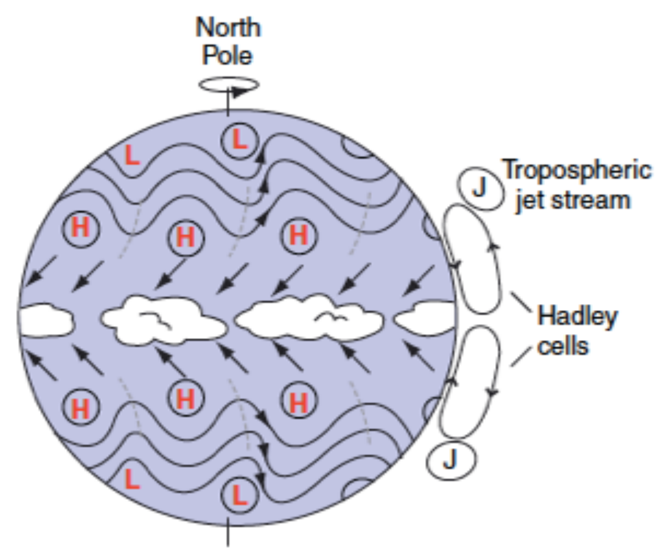

30°
Westerlies
Trades
Equa



Zonal mean wind
Annual mean
m/sec
Pressure (hPa)
100
80
60
50
40
35
30
25
20
15
10
5
2
-2
-5
-10
-15
-20
-25
-30
-35
-40
-50
-60
-80
-100
80°N
60°N
40°N
20°N
0°
20°S
40°S
60°S
80°S



North Pole



North Pole
Tropospheric jet stream
Hadley cells

$$-2\pi R_E^3 \int_{eq}^{30^\circ} [\overline{\tau_x}] \cos^2 \phi d\phi = \frac{2\pi R_E \cos\phi}{g} \int_{30^\circ} [\overline{mv}]\, dp$$

Torque equatorward of 30 = Transport across 30°

Another balance requirement:

$$-2\pi R_E^3 \int_{eq}^{30^\circ} [\overline{\tau_x}] \cos^2\phi \, d\phi = \frac{2\pi R_E \cos\phi}{g} \int_{30^\circ} [\overline{mv}] \, dp$$

Torque equatorward of 30 = Transport across 30°

Poleward flux of AAM across 30° latitude in both hemispheres

Another balance requirement:

$$-2\pi R_E^3 \int_{eq}^{30^\circ} [\overline{\tau_x}] \cos^2\phi d\phi = \frac{2\pi R_E \cos\phi}{g} \int_{30^\circ} [\overline{mv}]\, dp$$

Torque equatorward of 30 = Transport across 30°

Poleward flux of AAM across 30° latitude in both hemispheres

$$\frac{2\pi R_E \cos\phi}{g} \int_{30^\circ} [\overline{mv}]\, dp = \frac{2\pi \Omega R_E^3 \cos^3\phi}{g} \int_{30^\circ} [\overline{v}]\, dx dp + \frac{2\pi R_E^2 \cos^2\phi}{g} \int_{30^\circ} [\overline{uv}]\, dx dp$$

Total | M_Ω | M_r

requires poleward mass flux

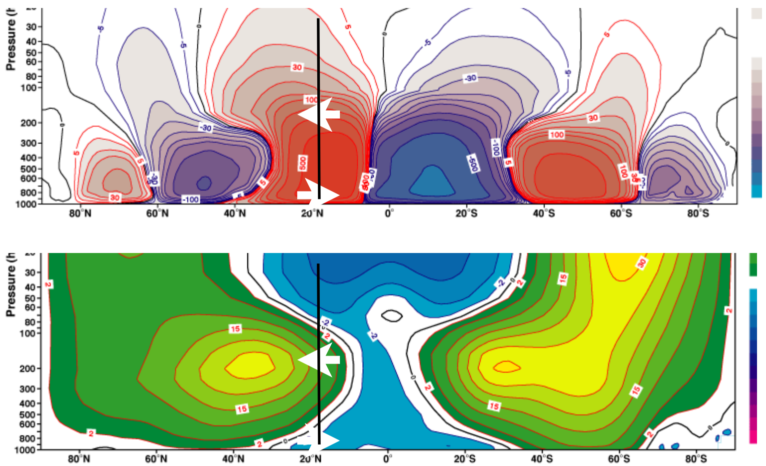
Decomposition of M_r

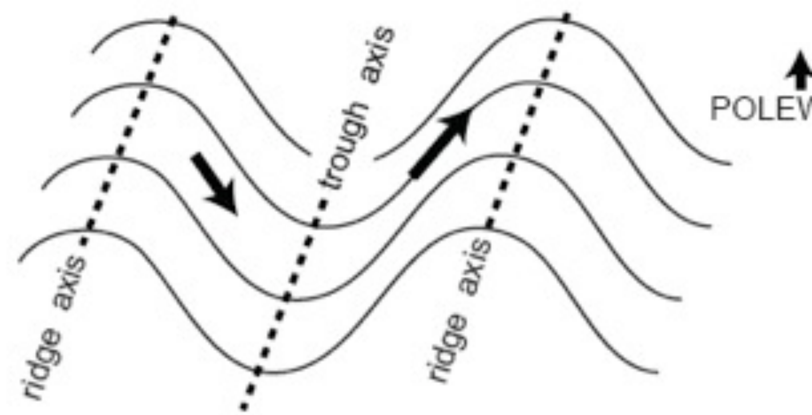
$$[\overline{uv}] = [\overline{u}]\,[\overline{v}] + \overline{[u]'\,[v]'} + [\overline{u}^*\overline{v}^*] + \left[\overline{u^{*\prime}v^{*\prime}}\right]$$

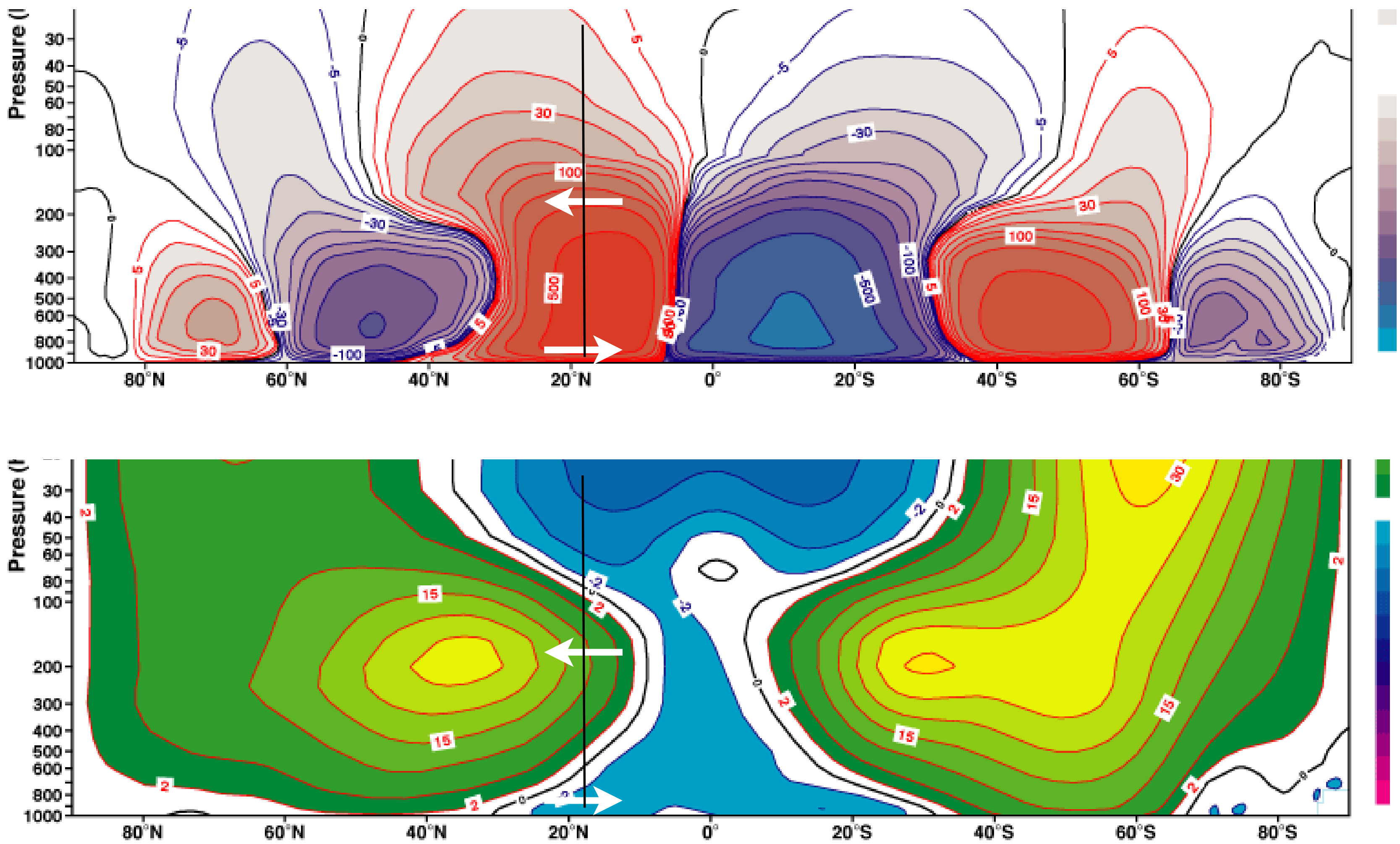
Steady MMC	transient MMC	steady eddy	transient eddy
		a.k.a. *stationary wave*	

Decomposition of M_r

$$[\overline{uv}] = [\overline{u}]\,[\overline{v}] + \overline{[u]'\,[v]'} + [\overline{u}^*\overline{v}^*] + \left[\overline{u^{*\prime}v^{*\prime}}\right]$$

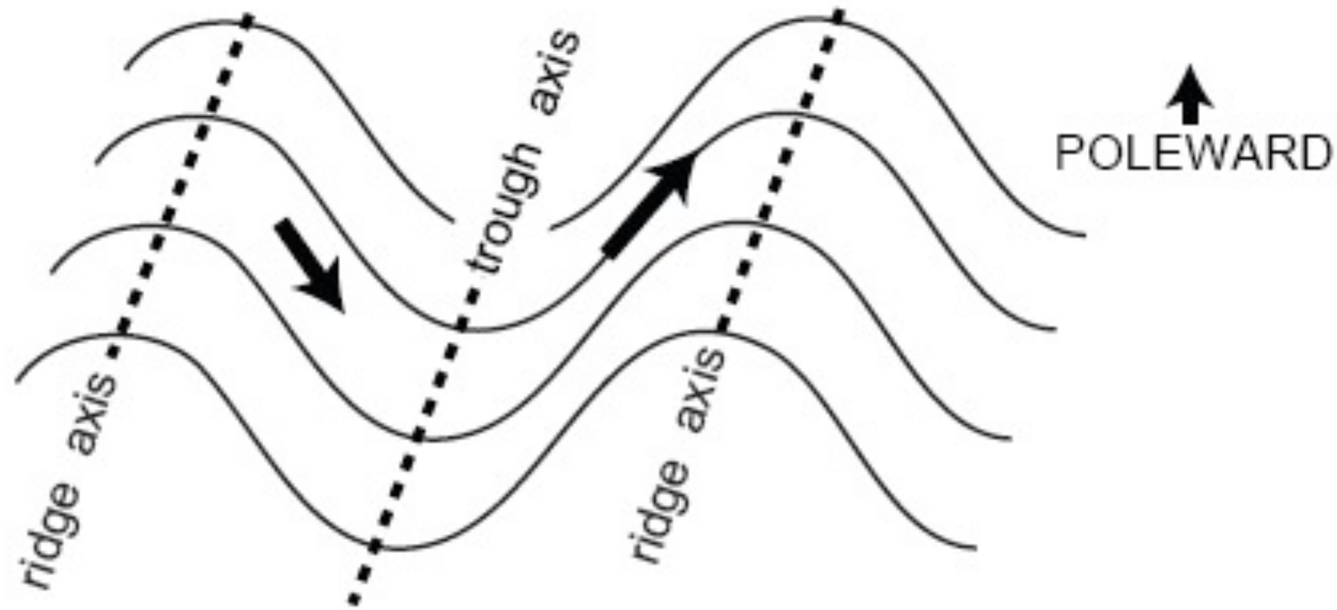




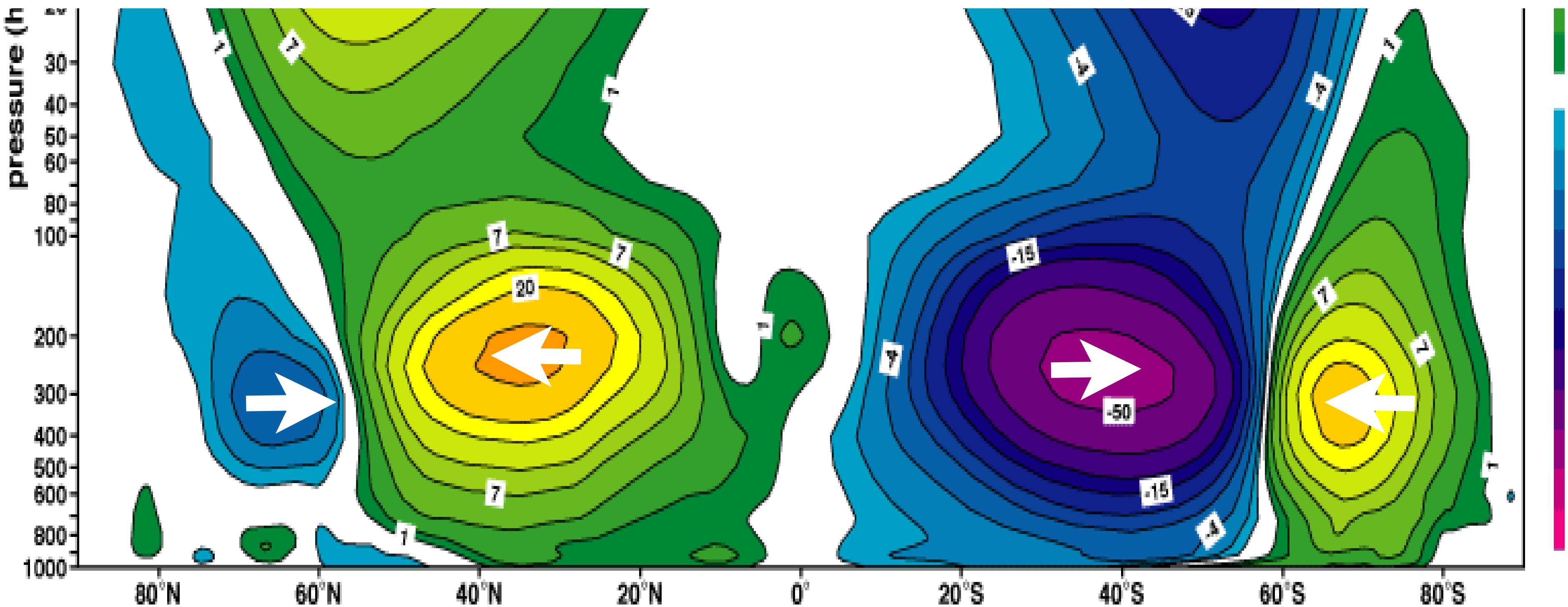


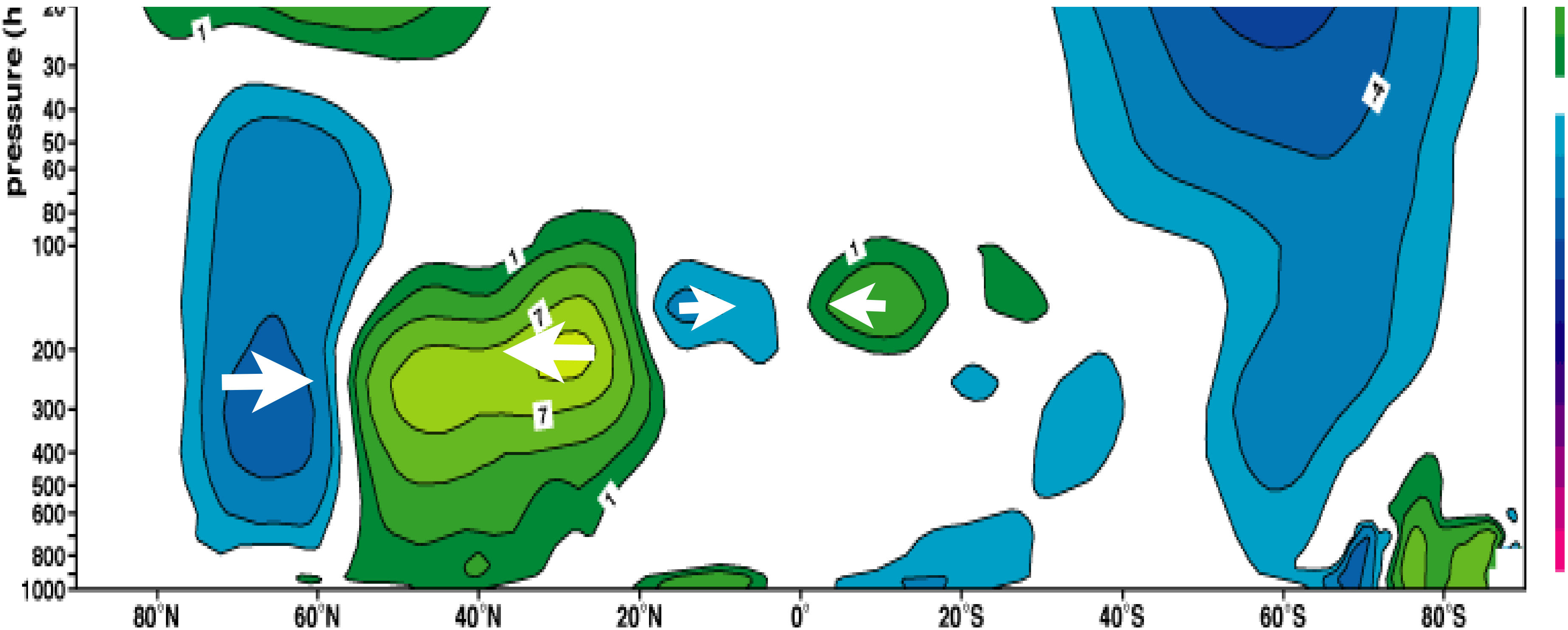
Pressure
30
40
50
60
80
100
200
300
400
500
600
800
1000
80°N
60°N
40°N
20°N
0°
20°S
40°S
60°S
80°S

$$\int_0^{p_0} [u][v]\,dp$$



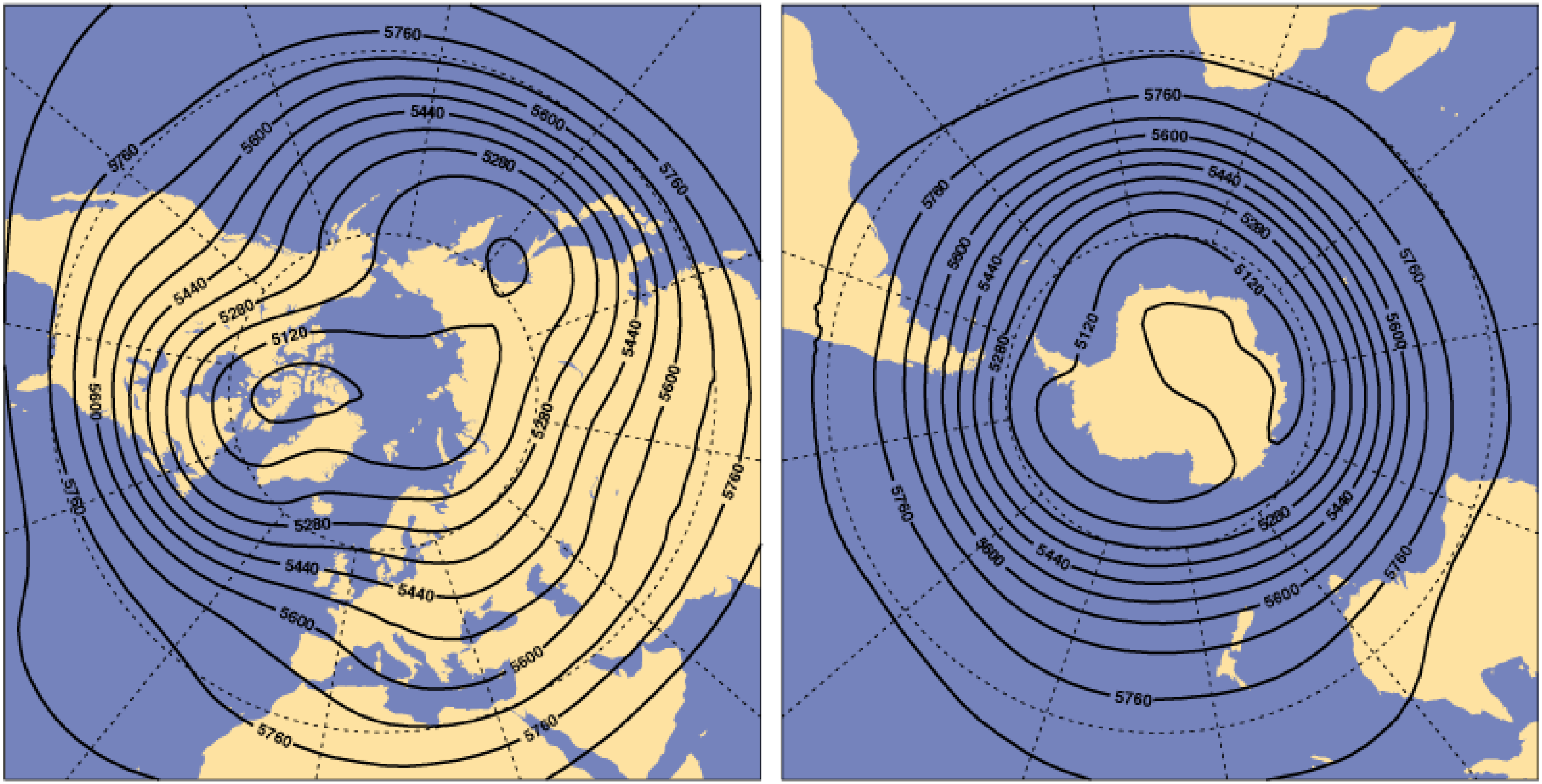
$$[u^* v^*]$$

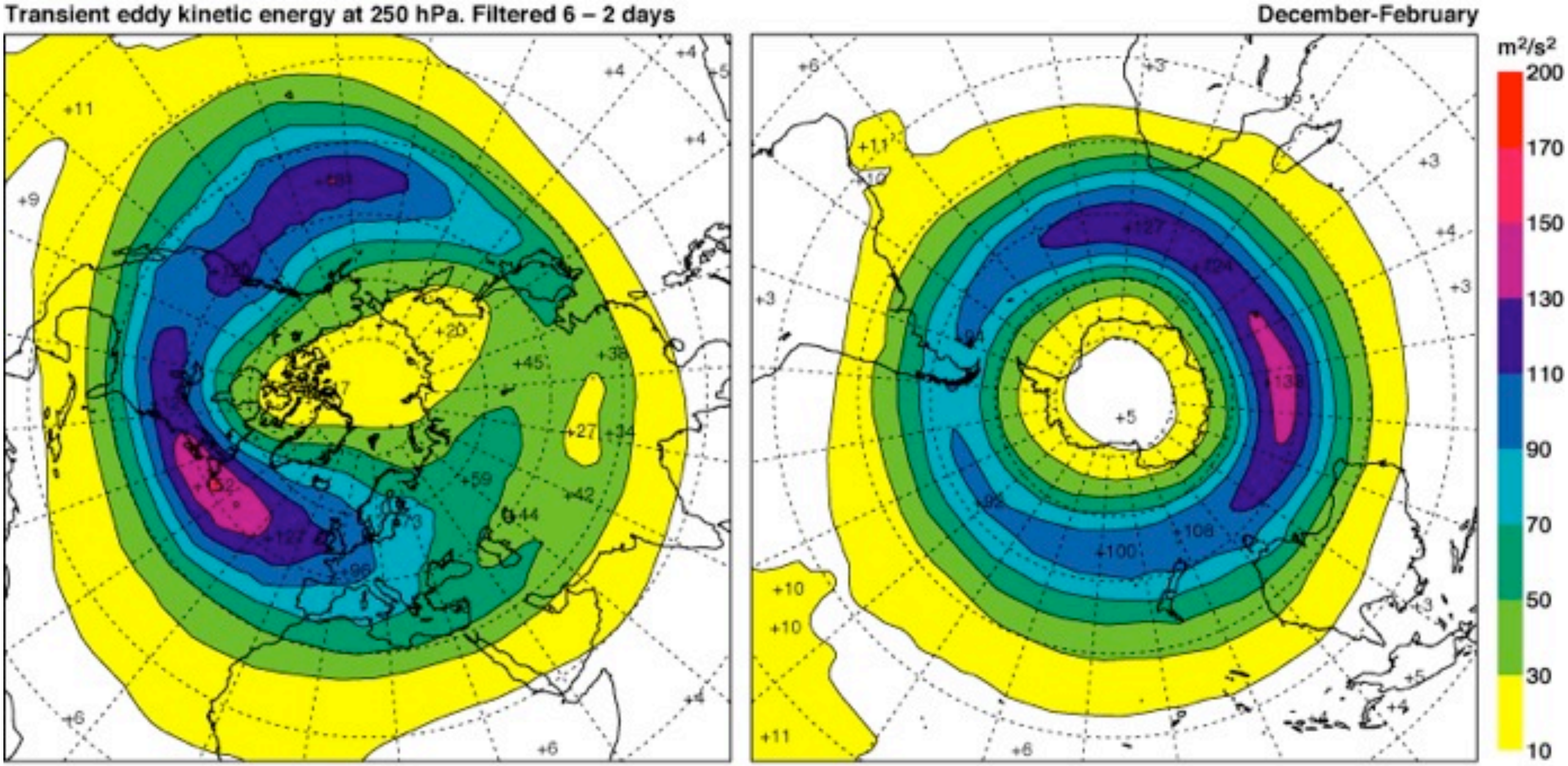

pressure (h
30
40
50
60
80
100
200
300
400
500
600
800
1000
80°N
60°N
40°N
20°N
0°
20°S
40°S
60°S
80°S
20
-15
-50
-4
7
1


pressure (h
30
40
50
60
80
100
200
300
400
500
600
800
1000
80°N
60°N
40°N
20°N
0°
20°S
40°S
60°S
80°S
1
7

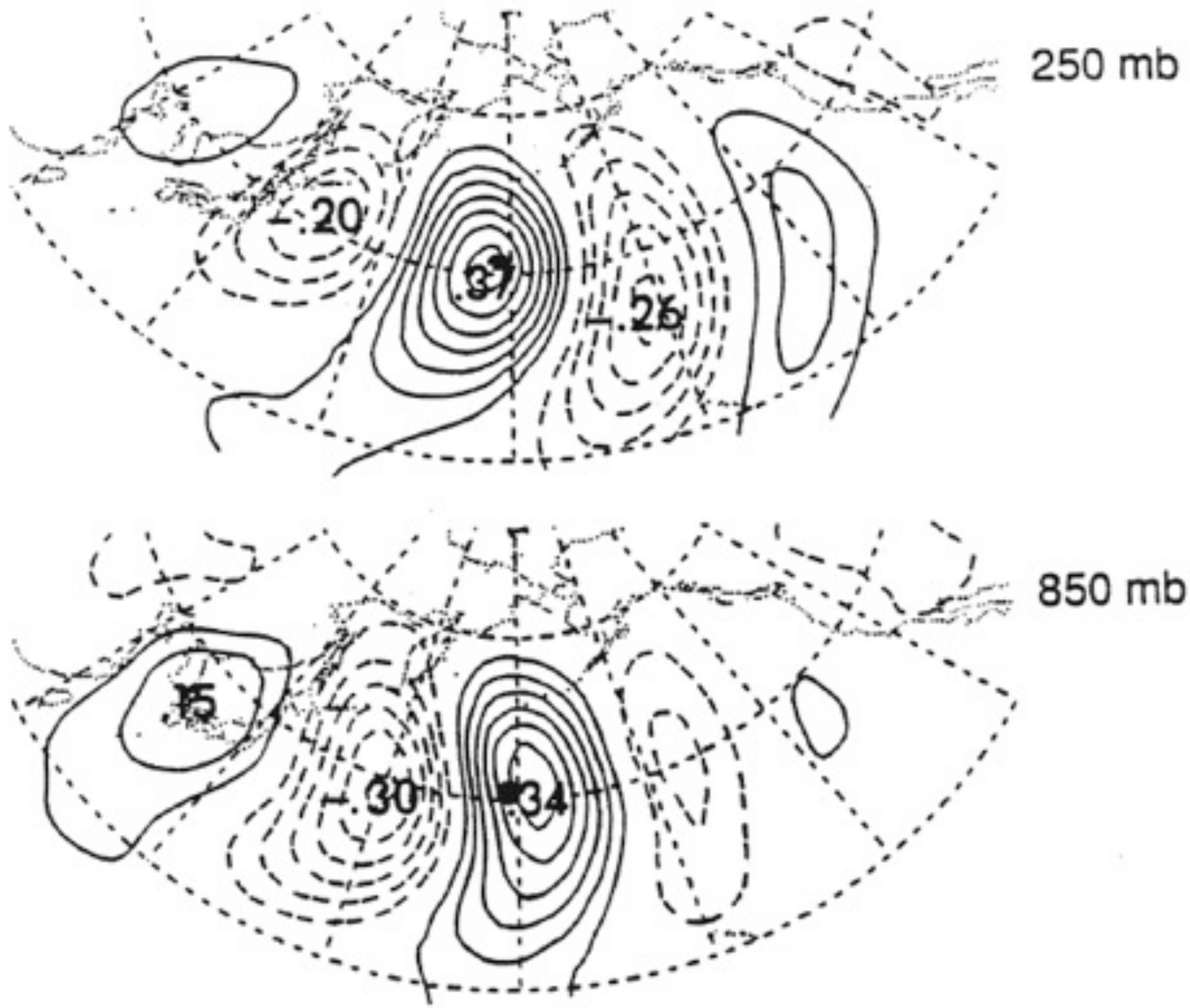
Geopotential height at 500 hPa

December-February





Transient eddy kinetic energy at 250 hPa. Filtered 6 – 2 days
December-February
m²/s²
200
170
150
130
110
90
70
50
30
10



one-point correlation maps; 500 hPa height; highpass filtered

Conclusions

In accordance with the balance requirements, there is a strong poleward flux of angular momentum across 30° latitude

The flux is greater during winter when the surface westerlies are stronger

The poleward flux across 30° is accomplished exclusively by the eddies

Transient eddies and stationary waves both contribute

Nearly all the flux occurs around the jet stream level (above 500 hPa)

Balance requirement for an upward flox of M equatorward of 30° and a downward flux poleward of 30°

Vertical transport of angular momentum

$$\frac{2\pi\Omega R_E^3}{g}\int -[\overline{\omega}]\cos^3\phi dy + \frac{2\pi R_E^2}{g}\int -[\overline{u\omega}]\cos^2\phi dy$$

M_Ω M_r

$$\frac{2\pi R_E^2}{g}\int -[\overline{\omega}]\left(\Omega R_E\cos\phi + [\overline{u}]\right)\cos^2\phi dy + \frac{2\pi R_E^2}{g}\int [\overline{u*\omega*}]\cos^2\phi dp$$

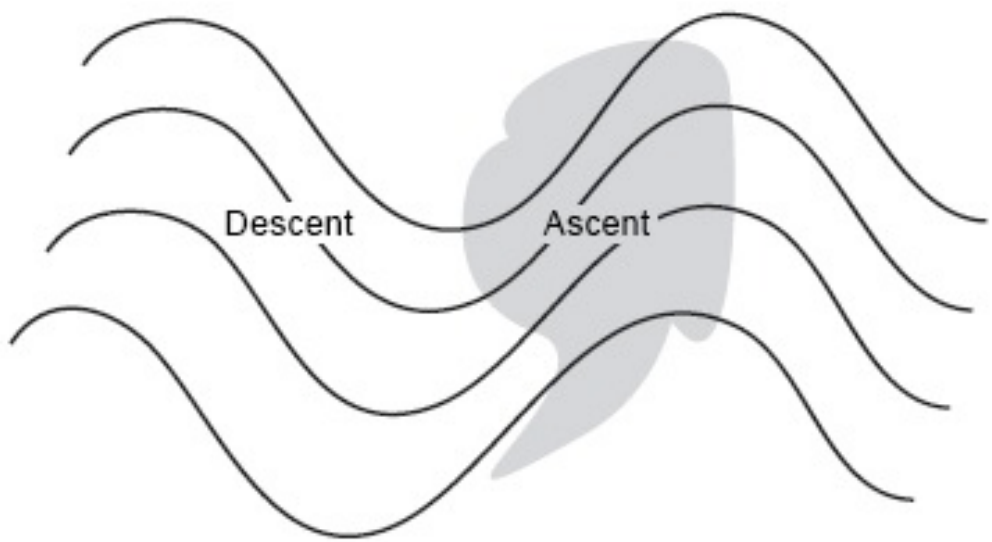
MMC term Eddy term

The eddies are not the answer

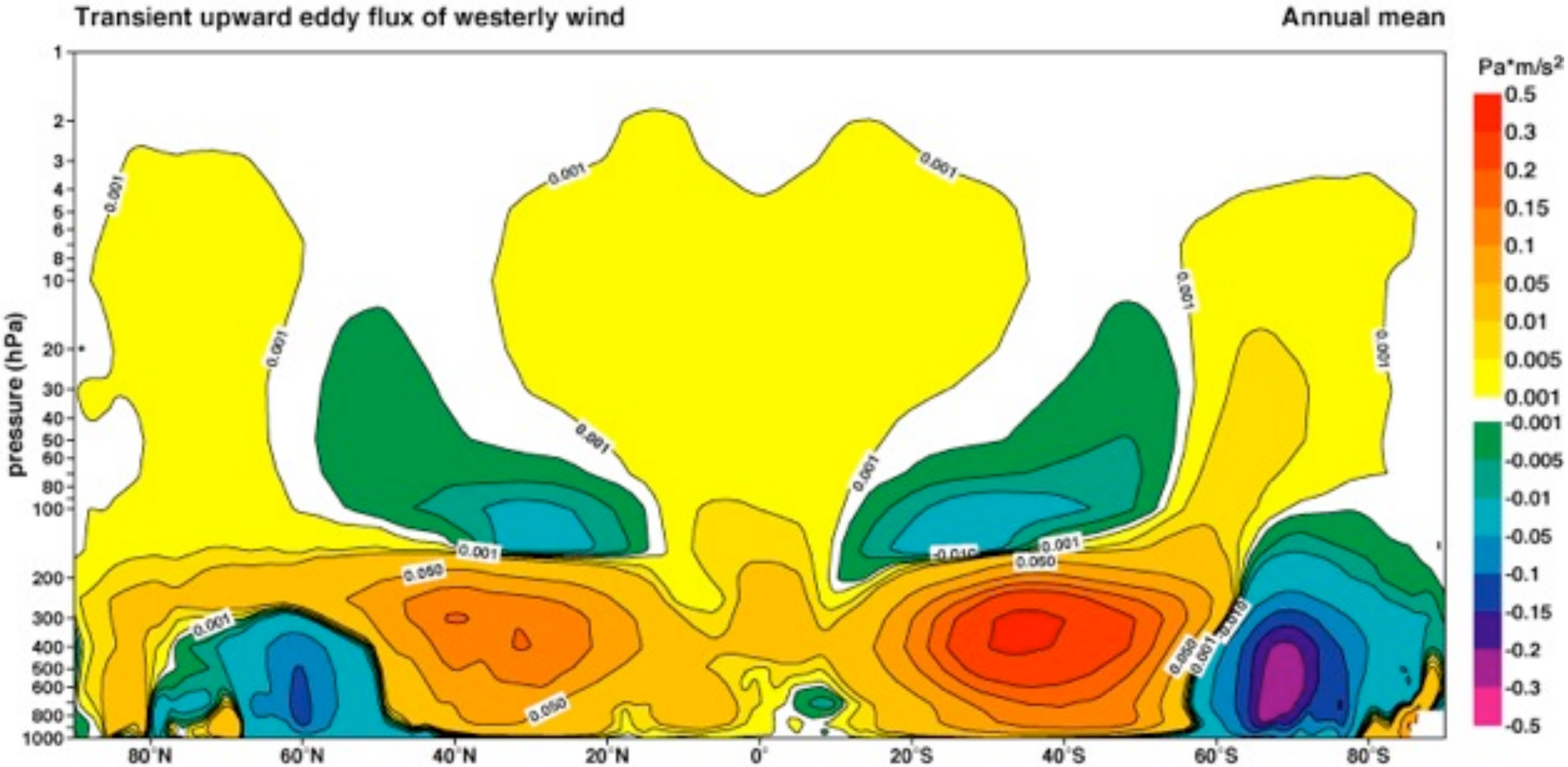
Scaling arguments: extratropical eddy fluxes are too small by a factor of *Ro*

Tropical eddy fluxes are almost nonexistent

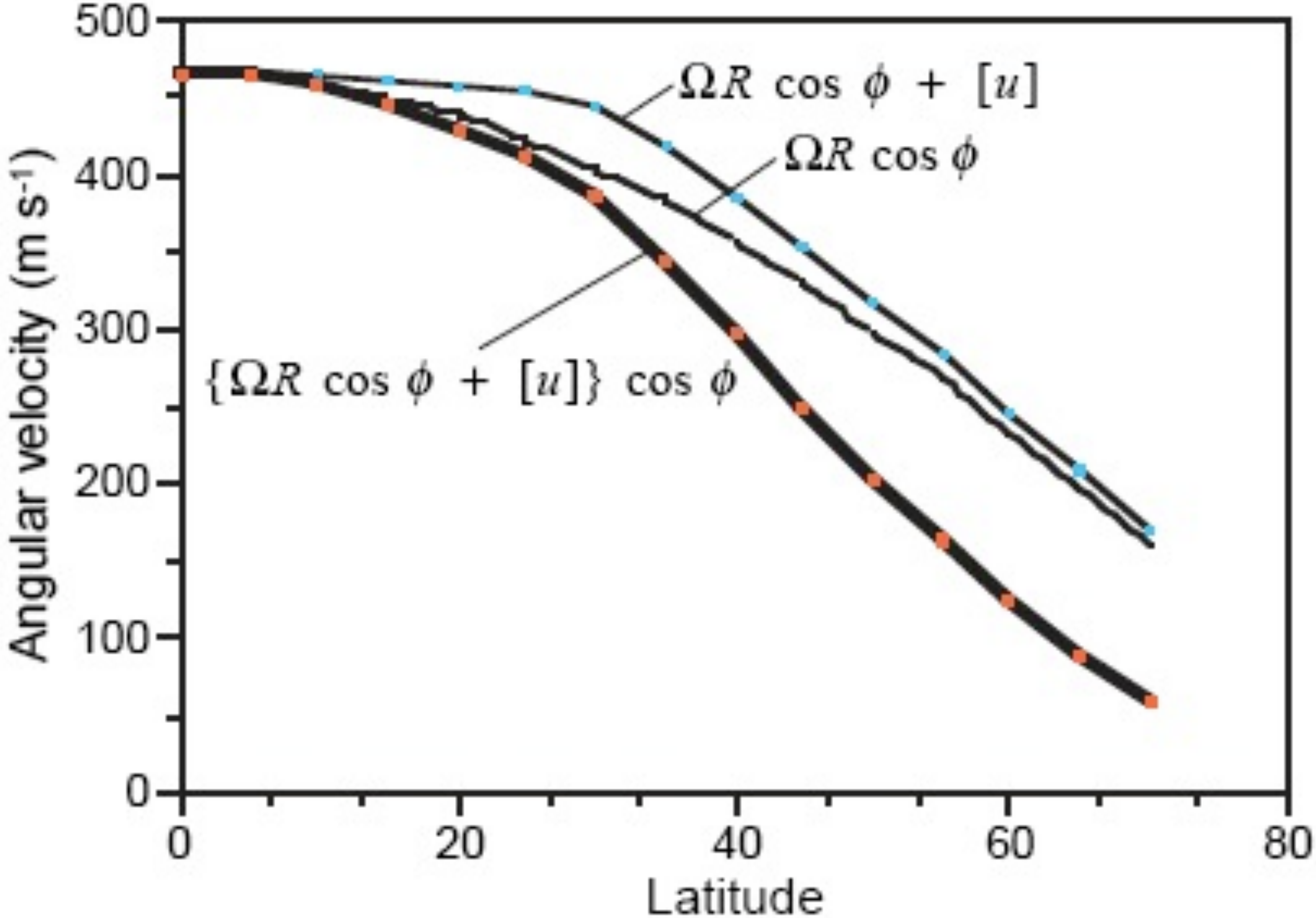
Extratropical eddy fluxes are upward; not downward



So it must be the MMC



Transient upward eddy flux of westerly wind
Annual mean
pressure (hPa)
Pa*m/s²
0.5
0.3
0.2
0.15
0.1
0.05
0.01
0.005
0.001
-0.001
-0.005
-0.01
-0.05
-0.1
-0.15
-0.2
-0.3
-0.5
80°N
60°N
40°N
20°N
0°
20°S
40°S
60°S
80°S



$\Omega R \cos \phi + [u]$
$\Omega R \cos \phi$
$\{\Omega R \cos \phi + [u]\} \cos \phi$
Angular velocity (m s^{-1})
Latitude
500
400
300
200
100
0
0
20
40
60
80

“Spin down” of the circulation in a teacup

The zonally averaged equation of motion

Spherical geometry

$$\frac{\partial [u]}{\partial t} = [v]\left(f - \frac{1}{\cos\phi}\frac{\partial}{\partial y}[u]\cos\phi\right) - [\omega]\frac{\partial [u]}{\partial p} - \frac{1}{\cos^2\phi}\frac{\partial}{\partial \phi}[u^* v^*]\cos^2\phi - \frac{\partial}{\partial p}[u^*\omega^*] + F_x$$

Cartesian geometry

$$\frac{\partial [u]}{\partial t} = [v]\left(f - \frac{\partial [u]}{\partial y}\right) - [\omega]\frac{\partial [u]}{\partial p} - \frac{\partial}{\partial y}[u^* v^*] - \frac{\partial}{\partial p}[u^*\omega^*] + F_x$$

Neglecting vertical advection by MMC; using G to represent eddies

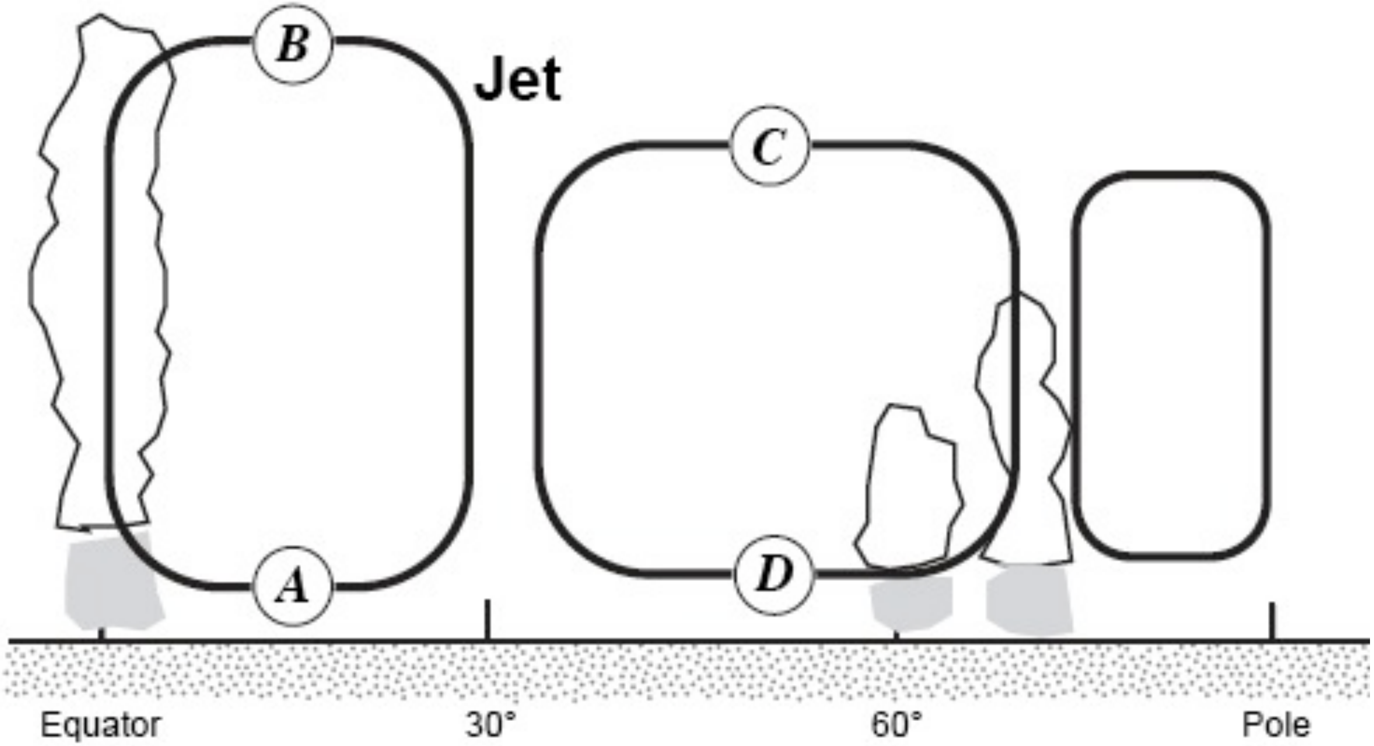
$$\frac{\partial [u]}{\partial t} = [v]\left(f - \frac{\partial [u]}{\partial y}\right) + G + F_x$$

$$\frac{\partial [u]}{\partial t} = [v]\left(f - \frac{\partial [u]}{\partial y}\right) + G + F_x$$

MMC | *dynamic stability* $\propto dM/dy$ | eddy forcing | frictional drag

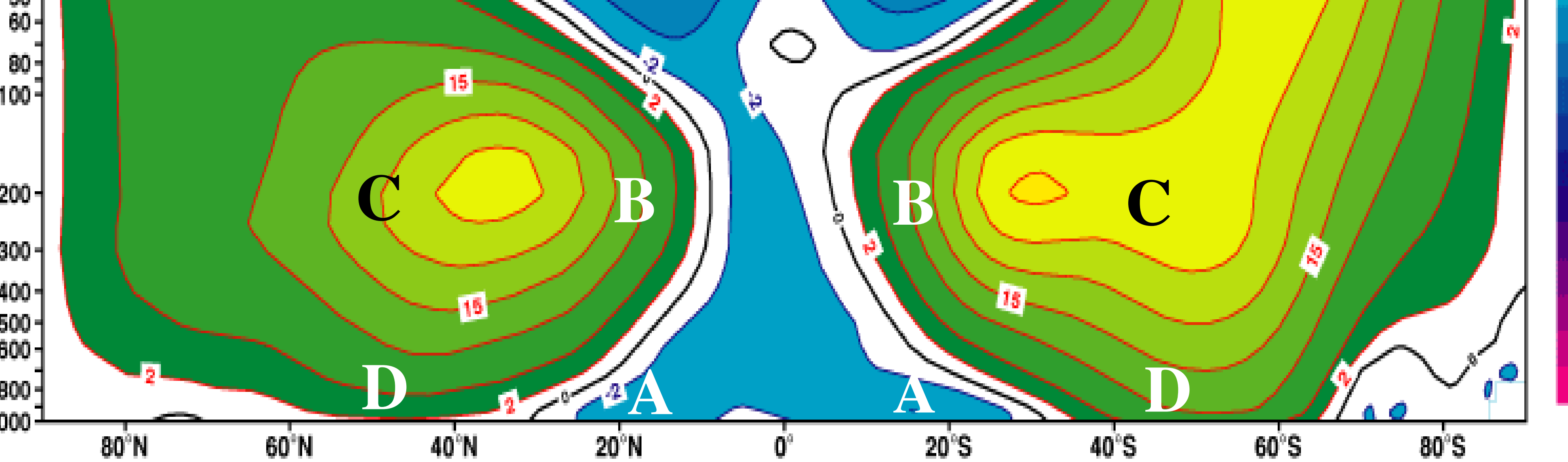
For long term "balance requirement" $d/dt = 0$

$$[v] = \frac{G + F_x}{f - \partial [u] / \partial y}$$



at A and D $\quad G = 0$

at B and C $\quad F = 0$



at B $d[u]/dy \sim 30 \text{ m s}^{-1}$ over 2000 km $\sim 1.5 \times 10^{-5}\text{s}^{-1}$

$$f \sim 4 \times 10^{-5}\text{s}^{-1}$$

$$f - d[u]/dy \sim 2.5 \times 10^{-5}\text{ s}^{-1}$$

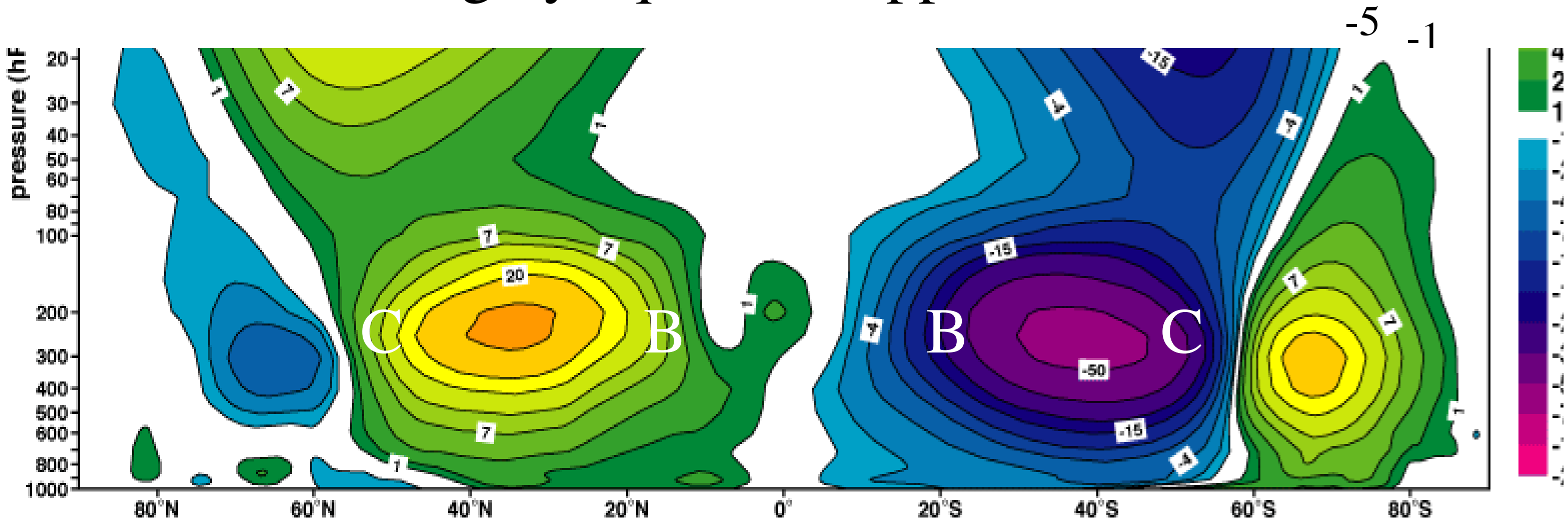
at C $\quad f - d[u]/dy \sim 10 \times 10^{-5}\text{s}^{-1}$ $\qquad$ 4 × larger than at B

Recalling that

$$[v] = \frac{G + F_x}{f - \partial [u] / \partial y}$$

and $F_x = 0$ at B and C

and G is roughly equal and opposite at B and C



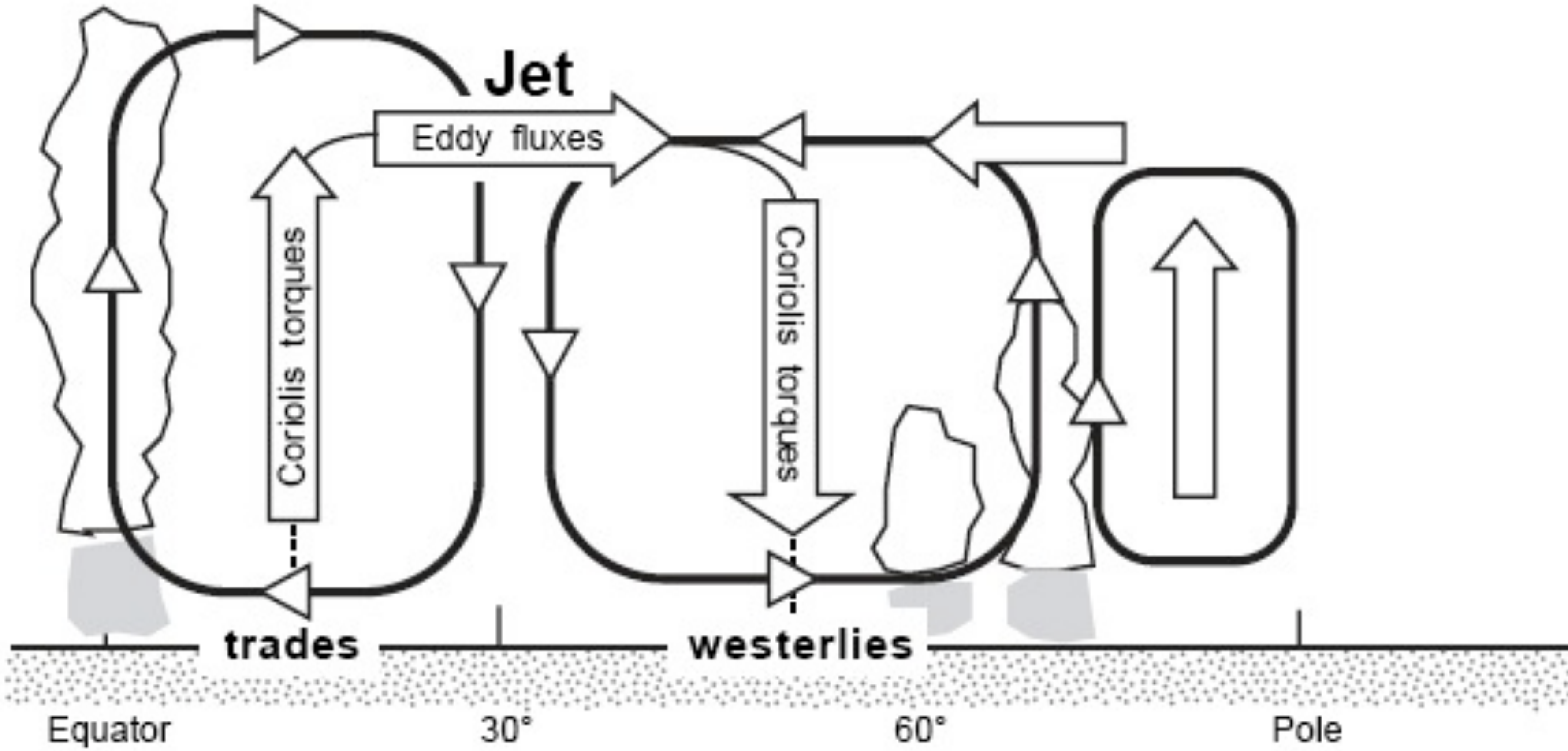
Recalling that

$$[v] = \frac{G + F_x}{f - \partial[u]/\partial y}$$

and $F_x = 0$ at B and C

and G is roughly comparable at B and C

it follows that [v] is ~ 4 × stronger at B than at C
i.e., that the Hadley cell is roughly 4 times as strong as the Ferrell cell


Jet
Eddy fluxes
Coriolis torques
Coriolis torques
trades
westerlies
Equator
30°
60°
Pole