Chapter 3
The total energy balance

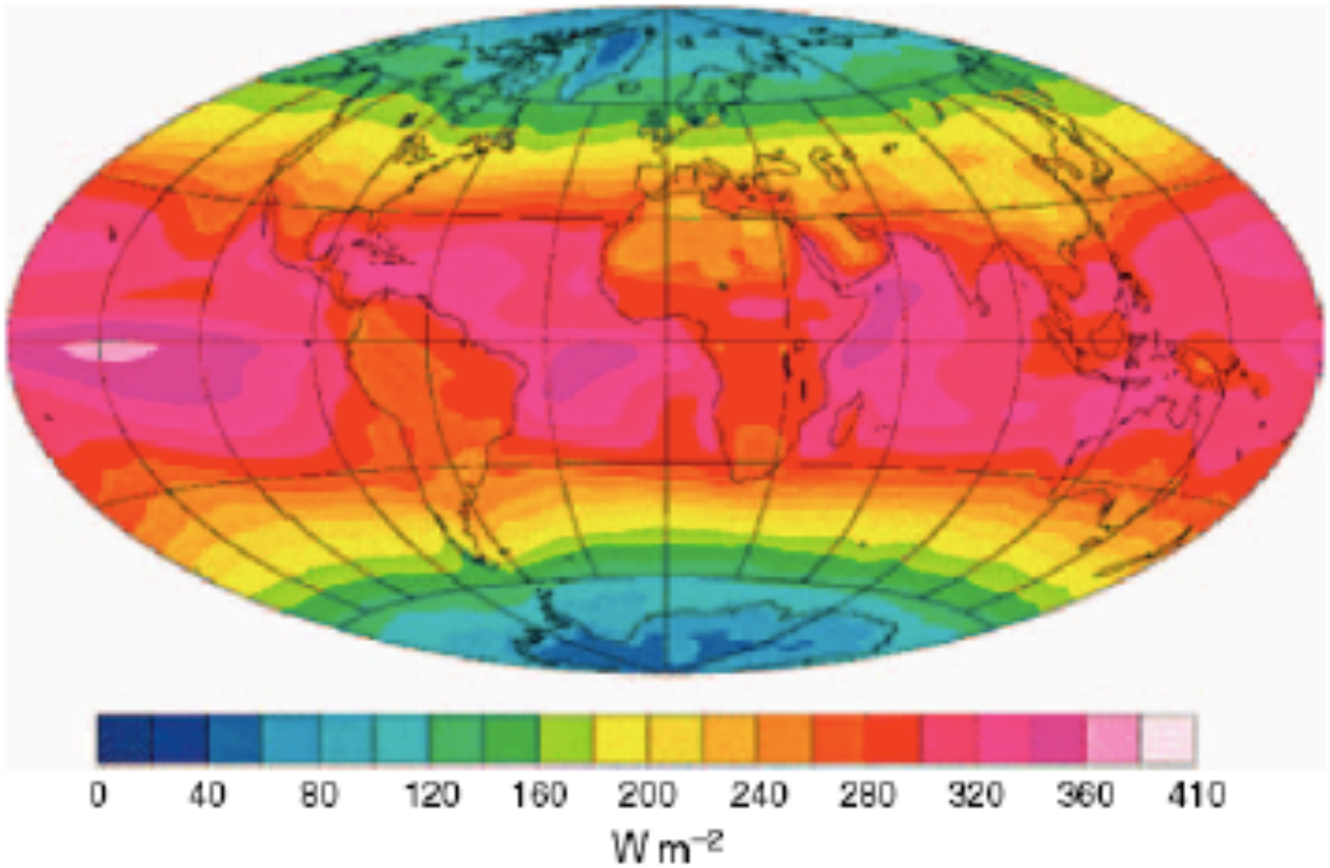
The balance requirement

How it is satisfied

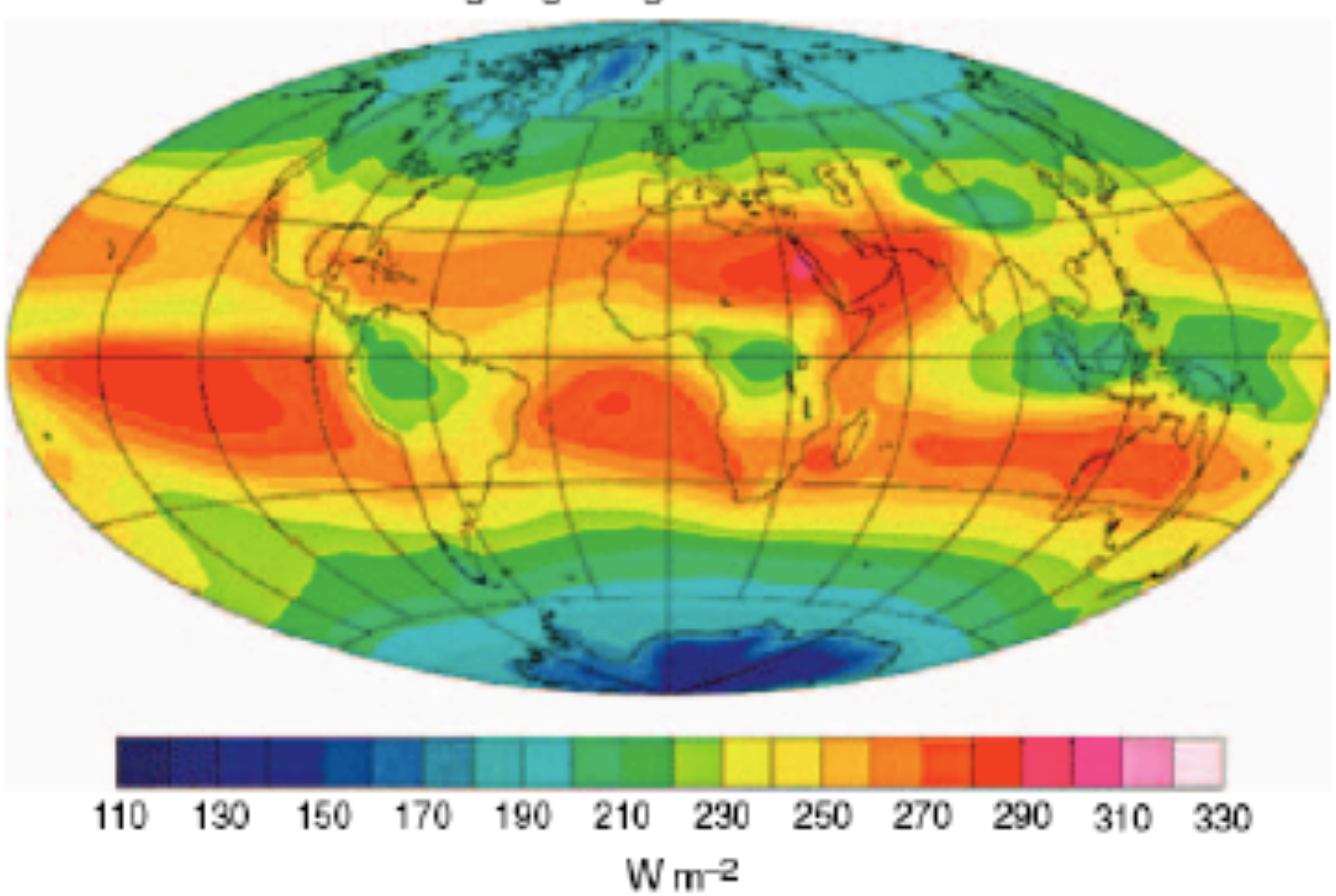
- role of the thermohaline circulation
- role of the wind-driven circulation
- role of the stationary waves
- role of the transient eddies
- role of the hydrologic cycle

Two kinds of eddy heat transports

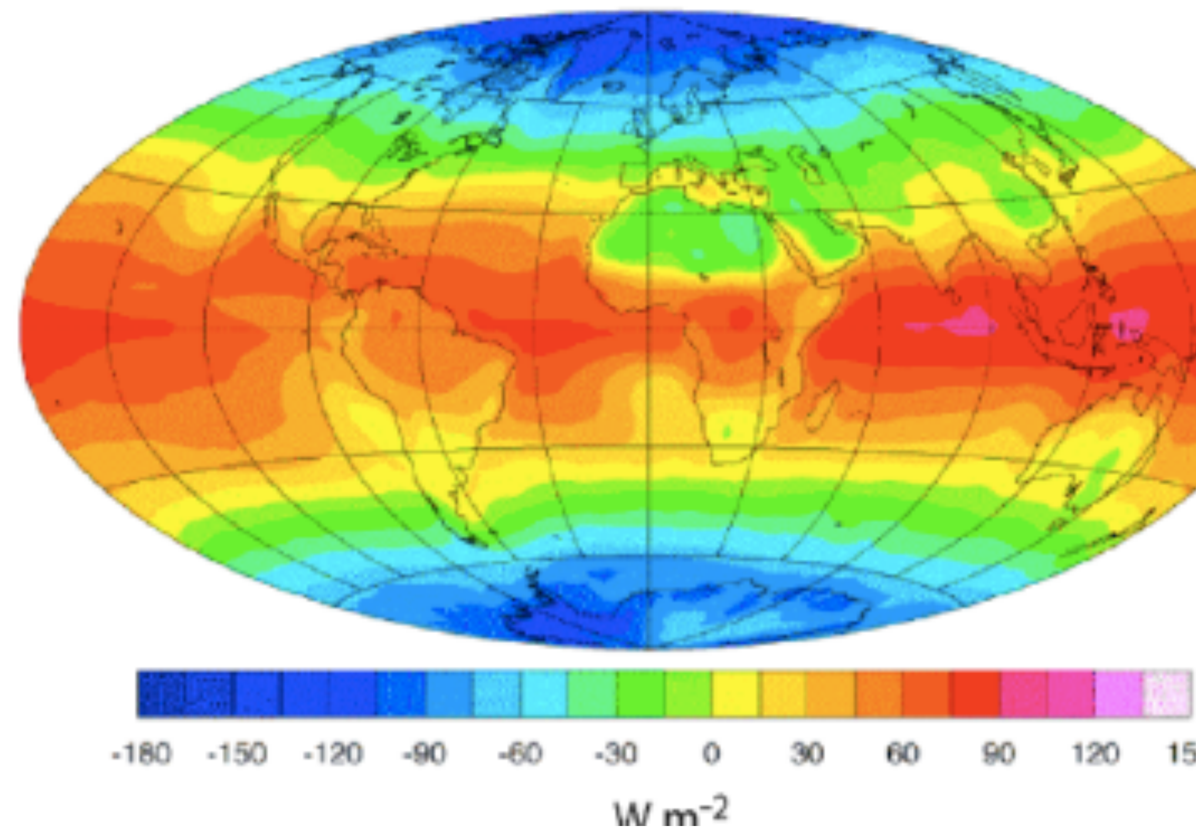
Absorbed Solar Radiation

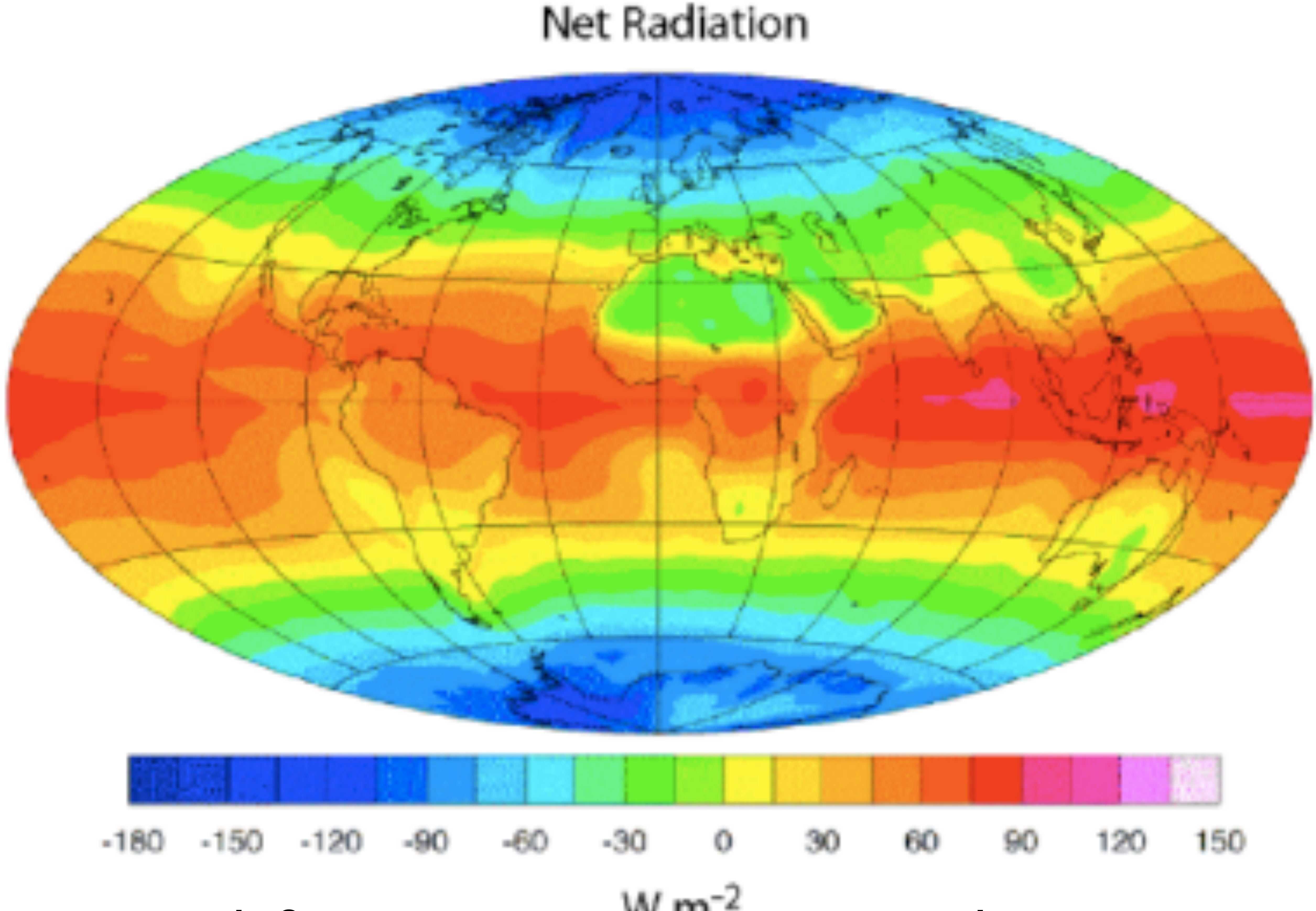


Outgoing Longwave Radiation

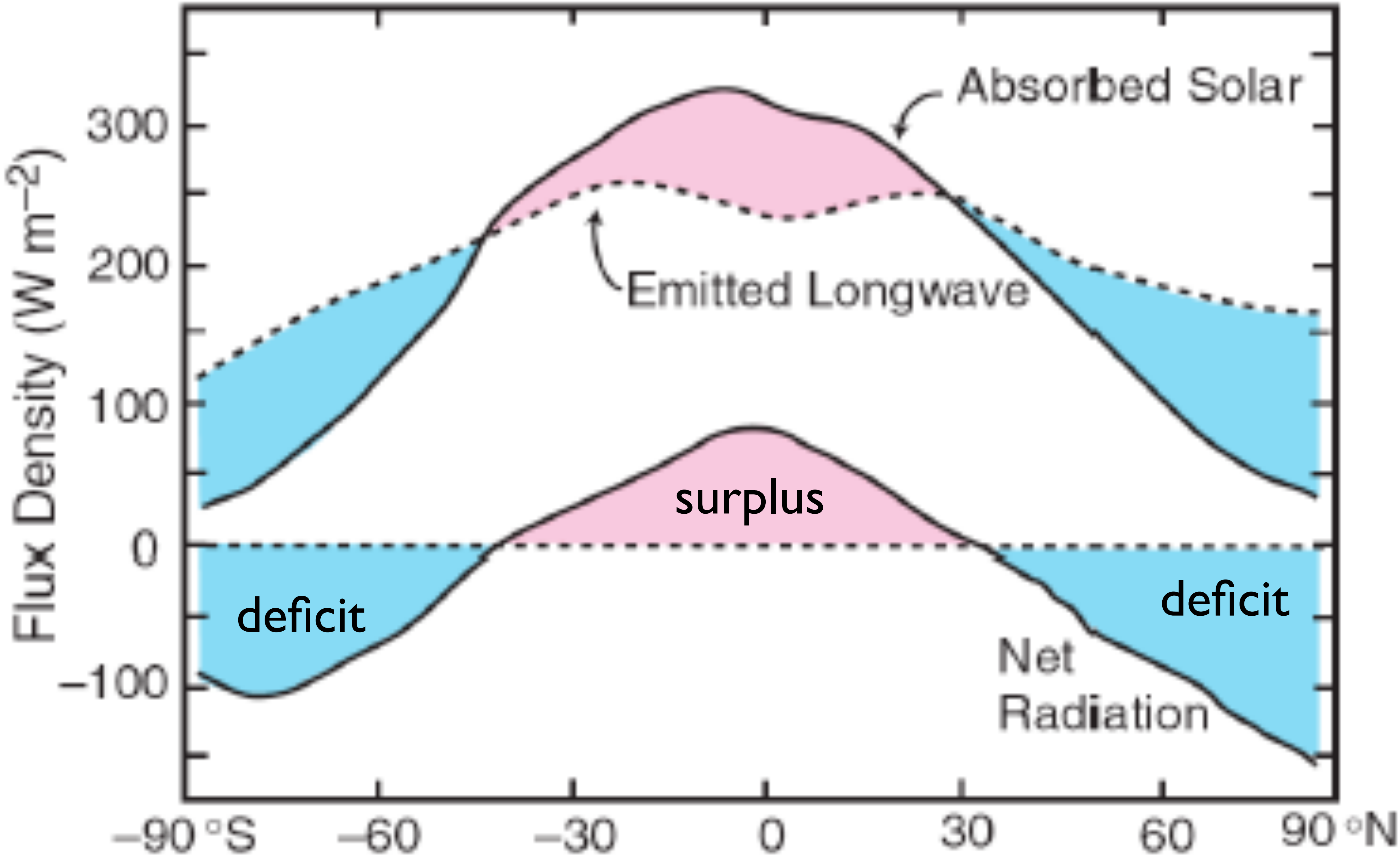


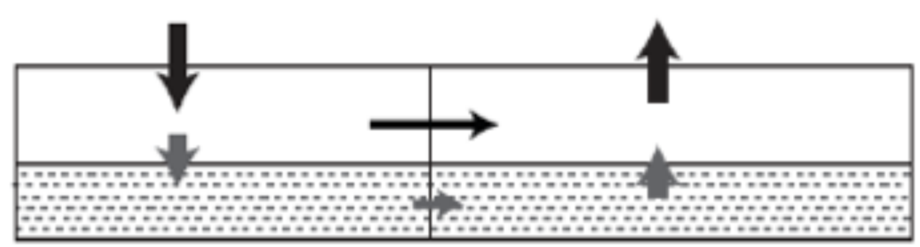
Net Radiation

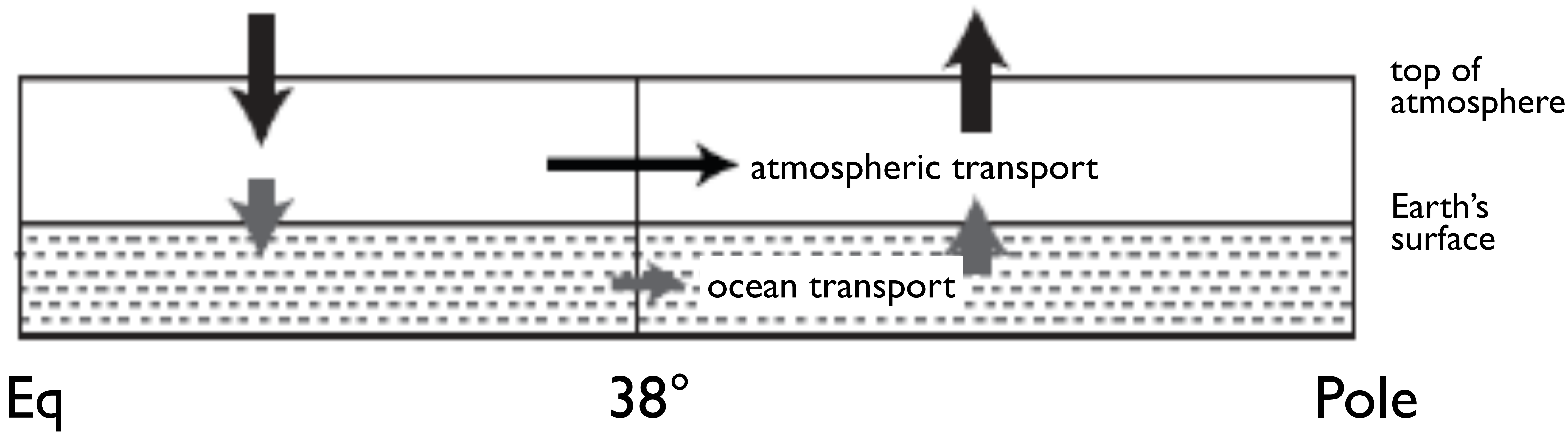


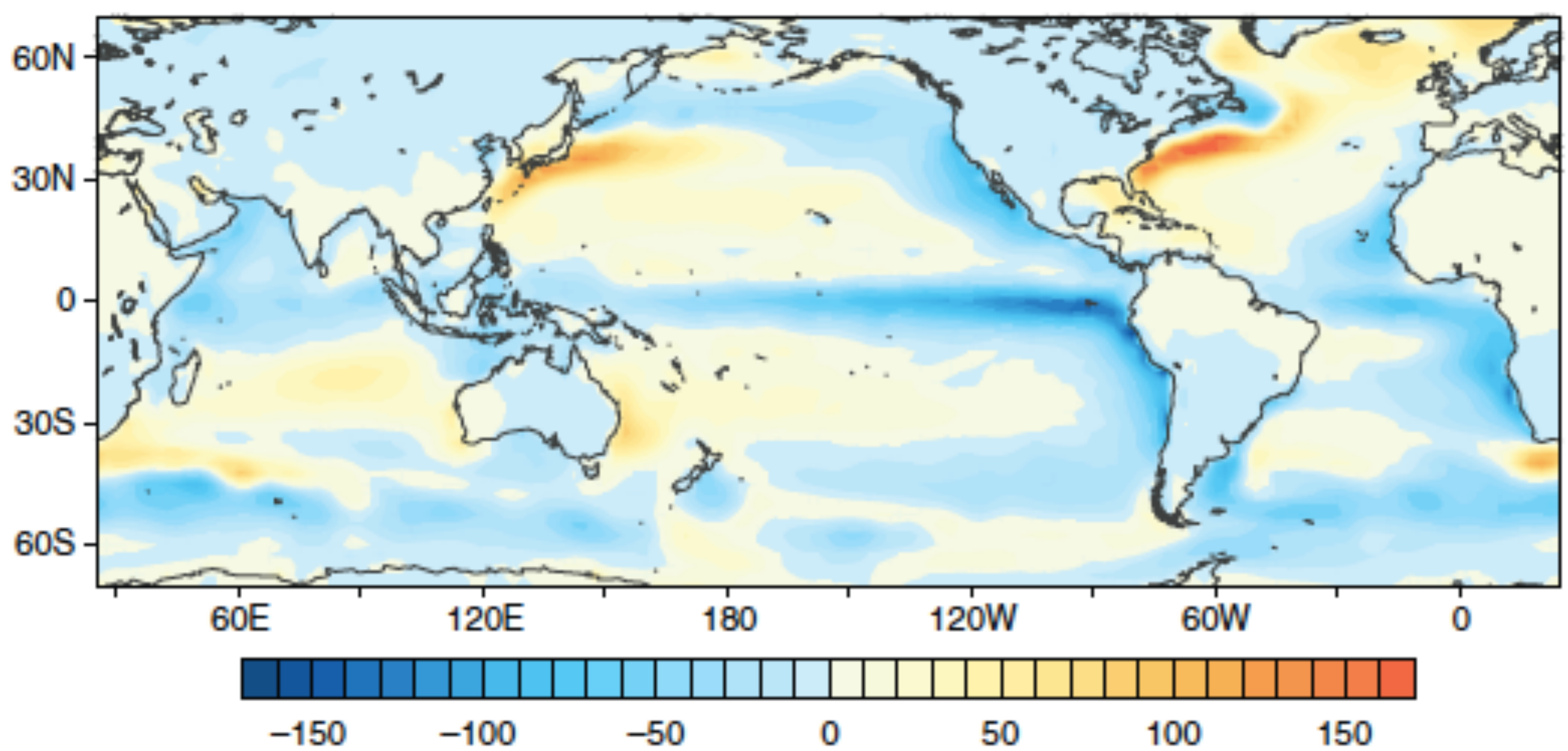


deficit surplus

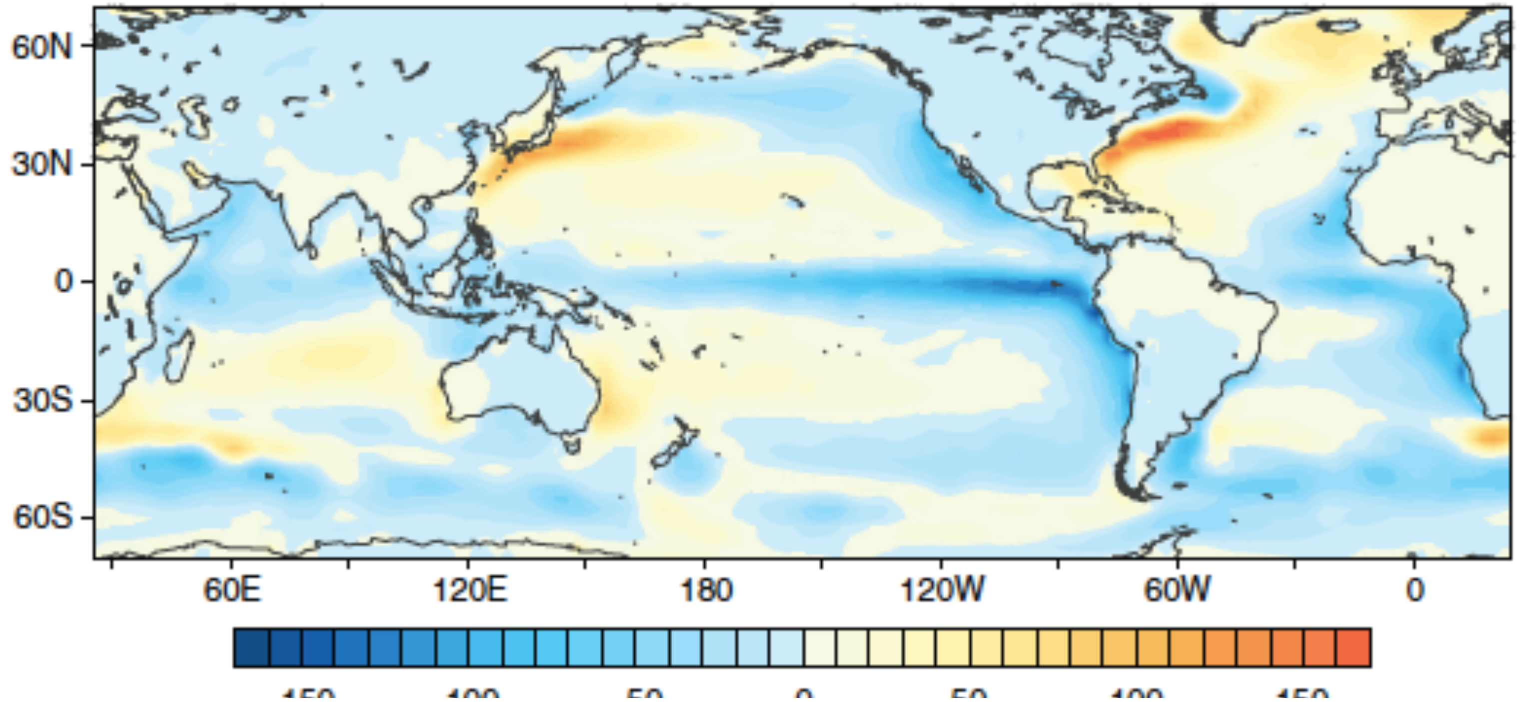

Absorbed Solar
Emitted Longwave
surplus
deficit
deficit
Net Radiation
Flux Density (W m^{-2})
300
200
100
0
−100
−90 °S
−60
−30
0
30
60
90 °N



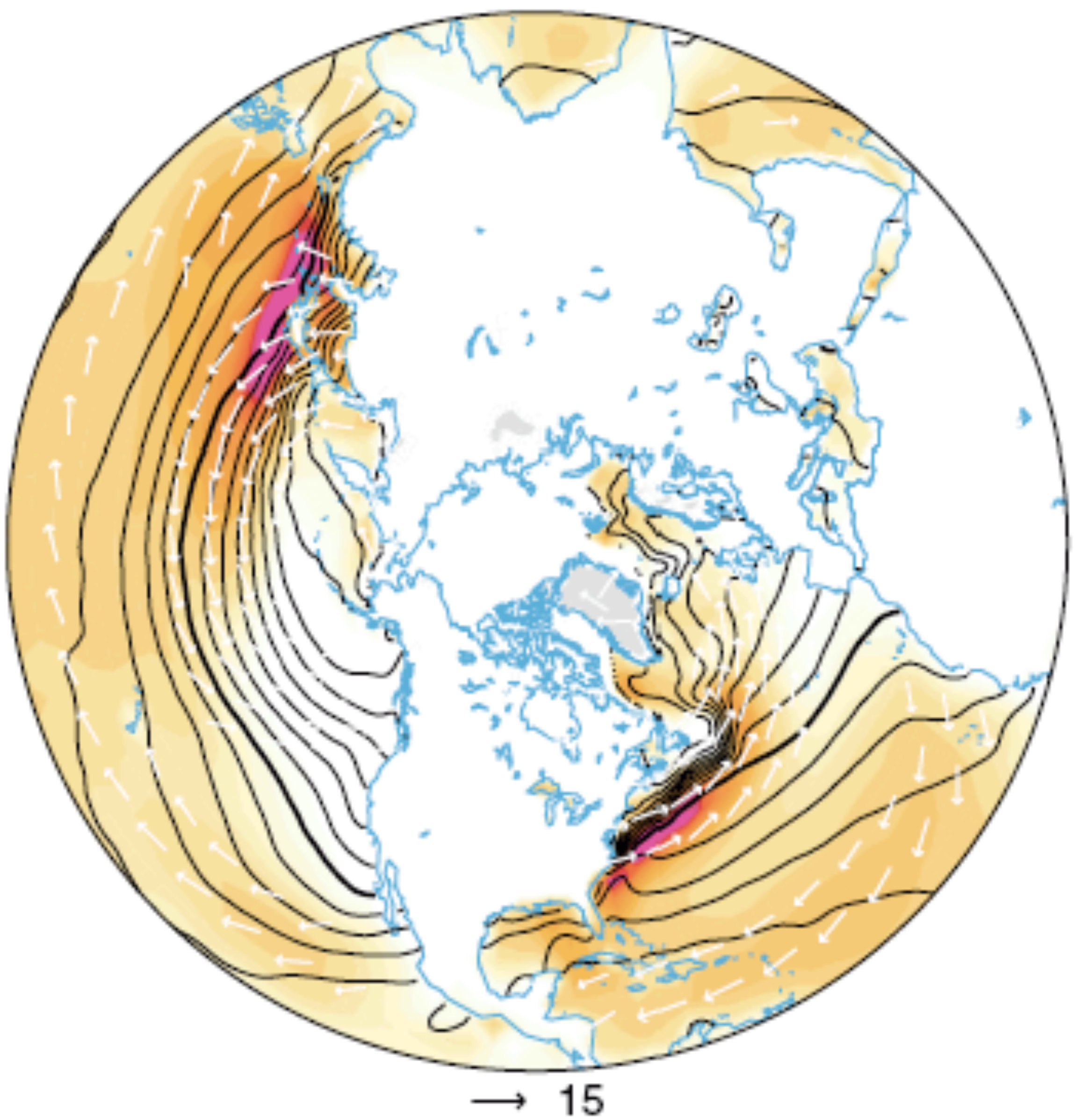


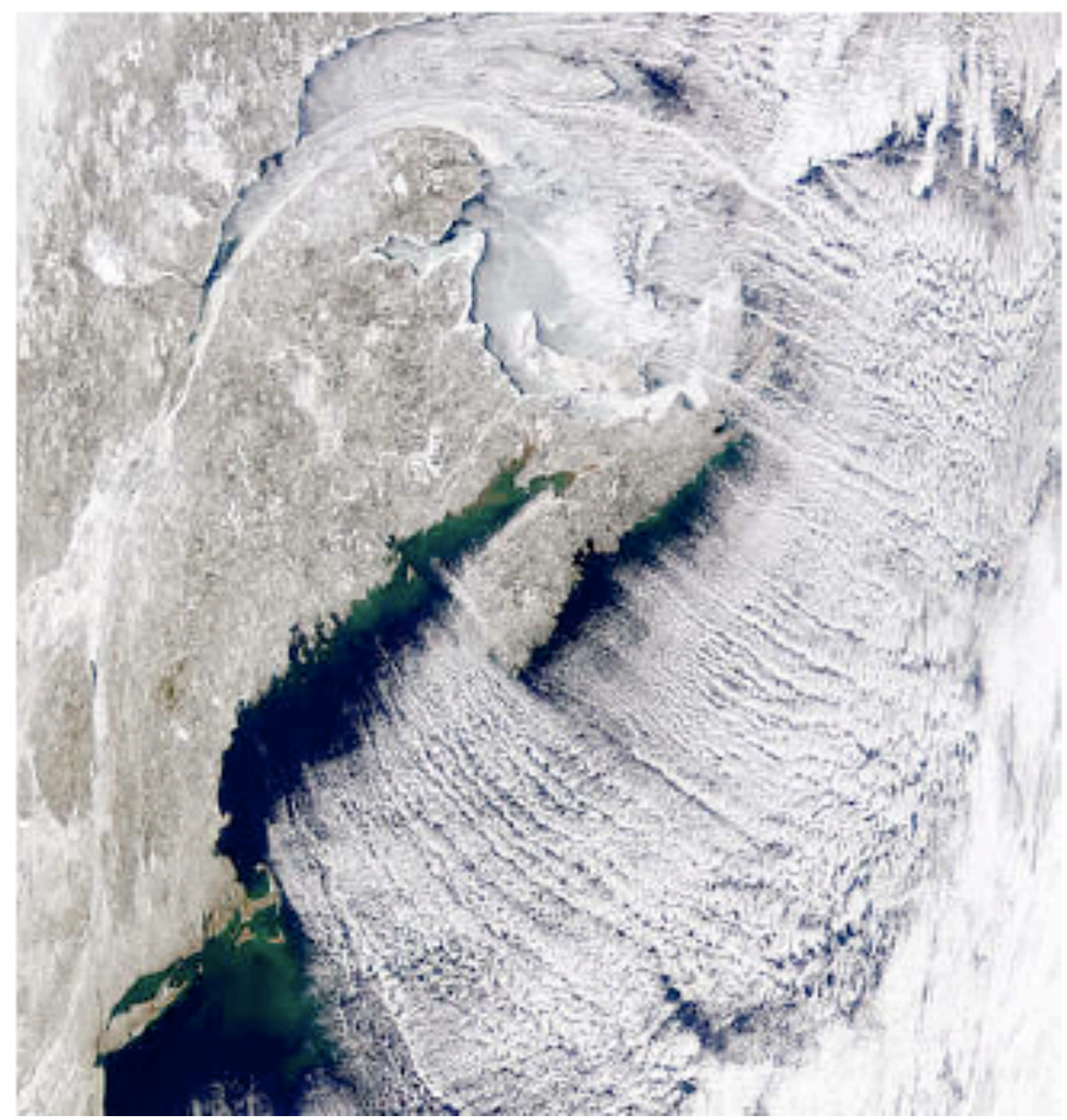


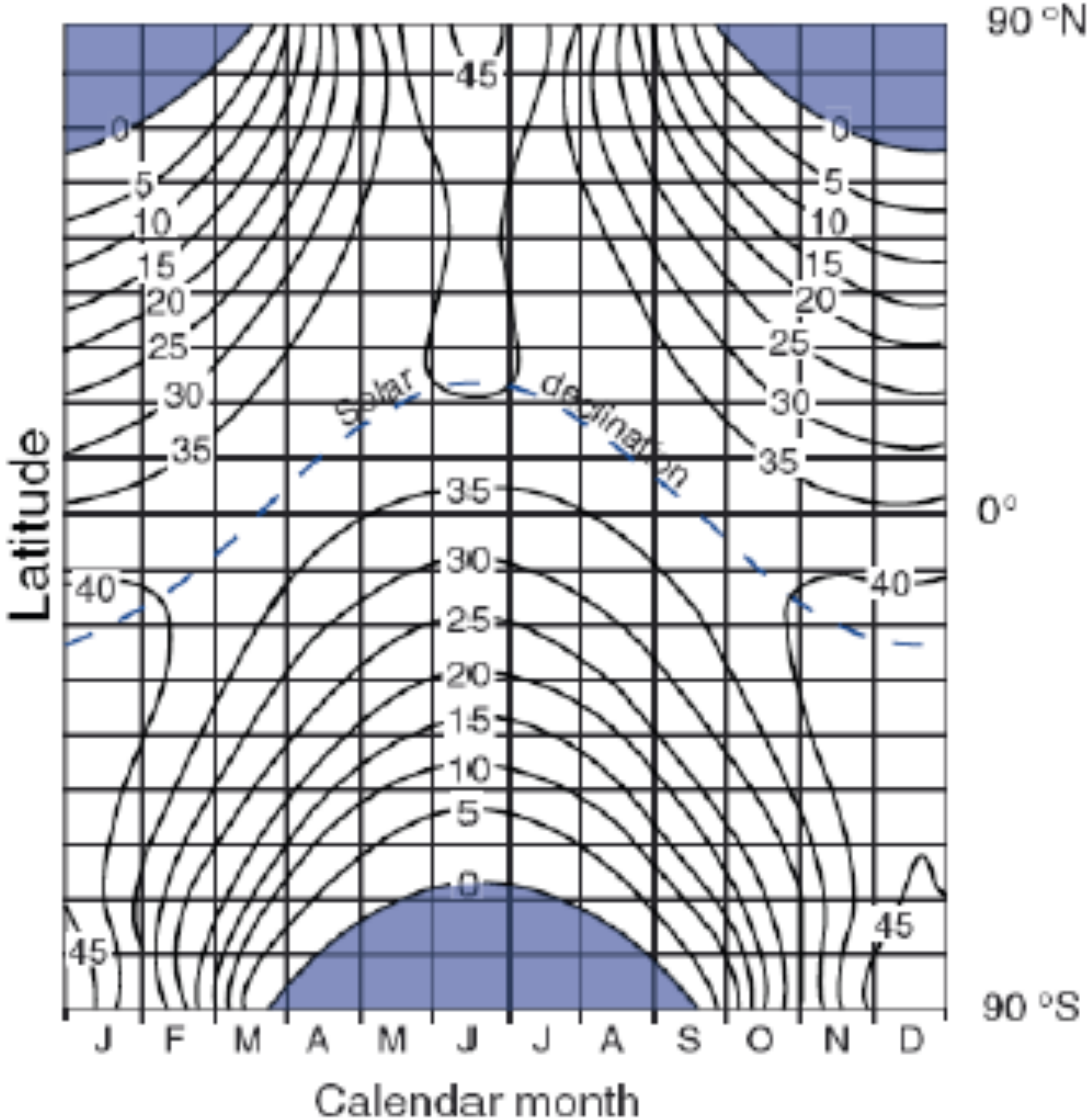
upward flux at
Earth's surface
(ERA-40)



upward flux at
Earth's surface
(ERA-40)

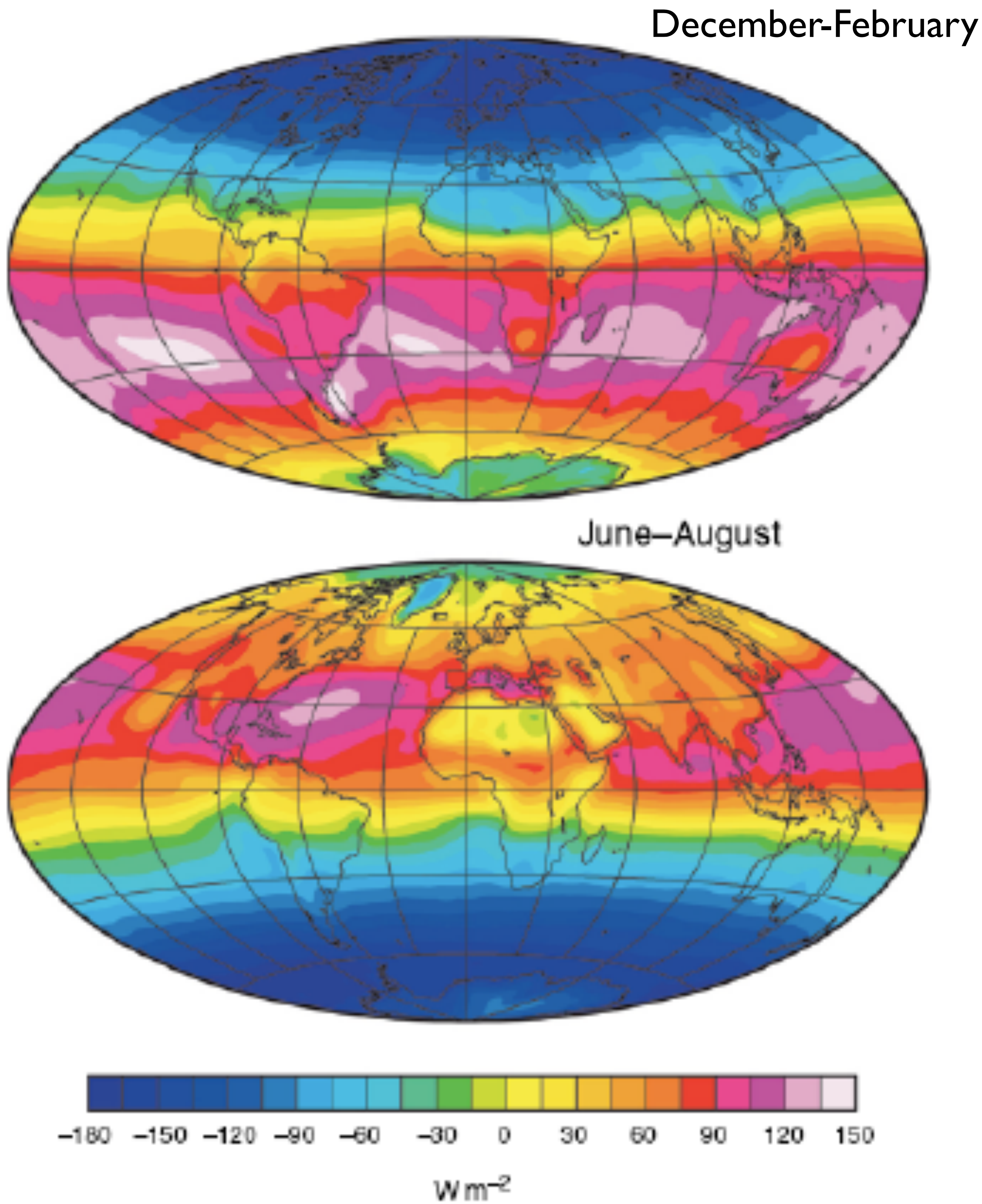


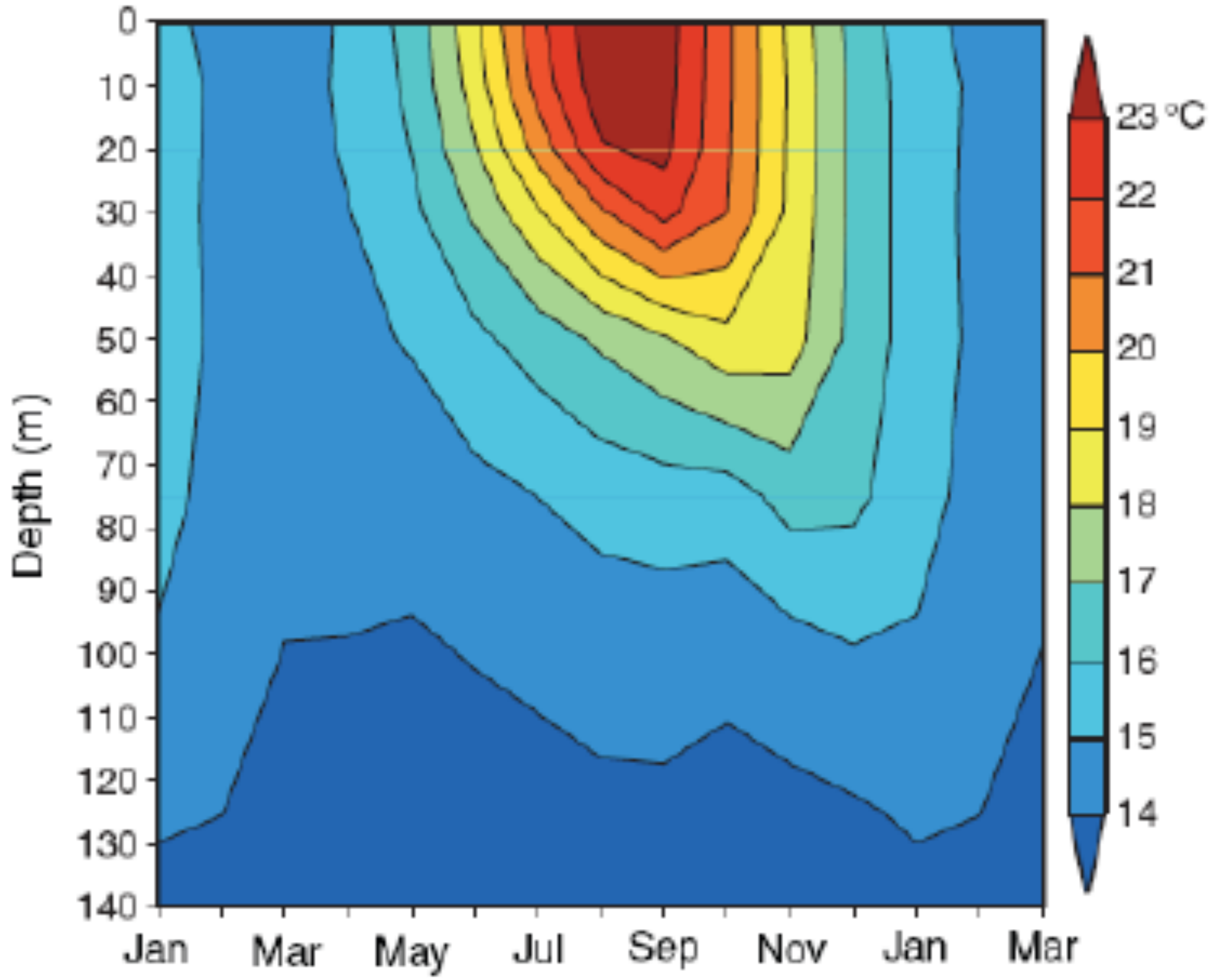




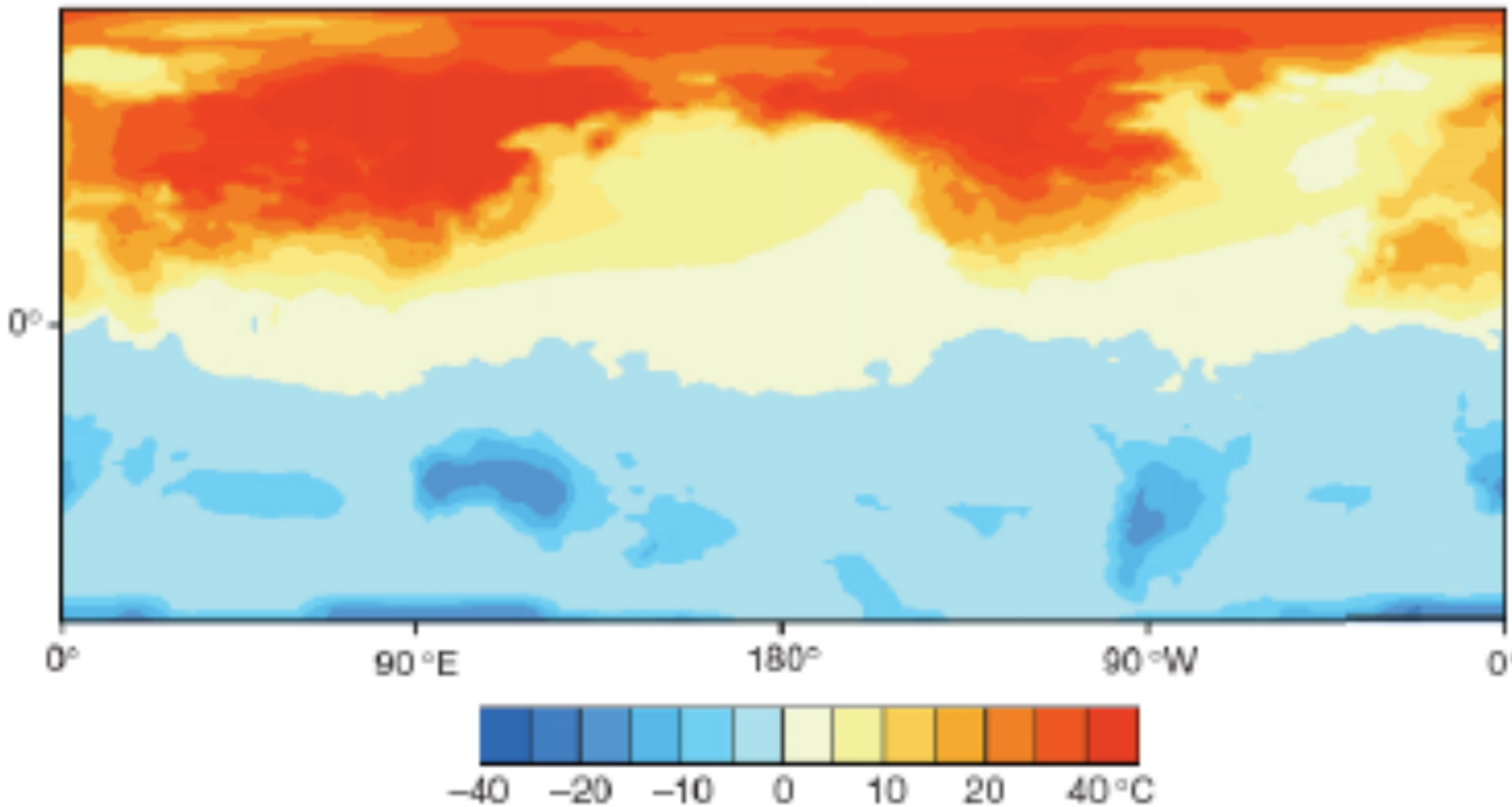
Annual cycle in insolation
(top of atmosphere)

TOA Net Radiation

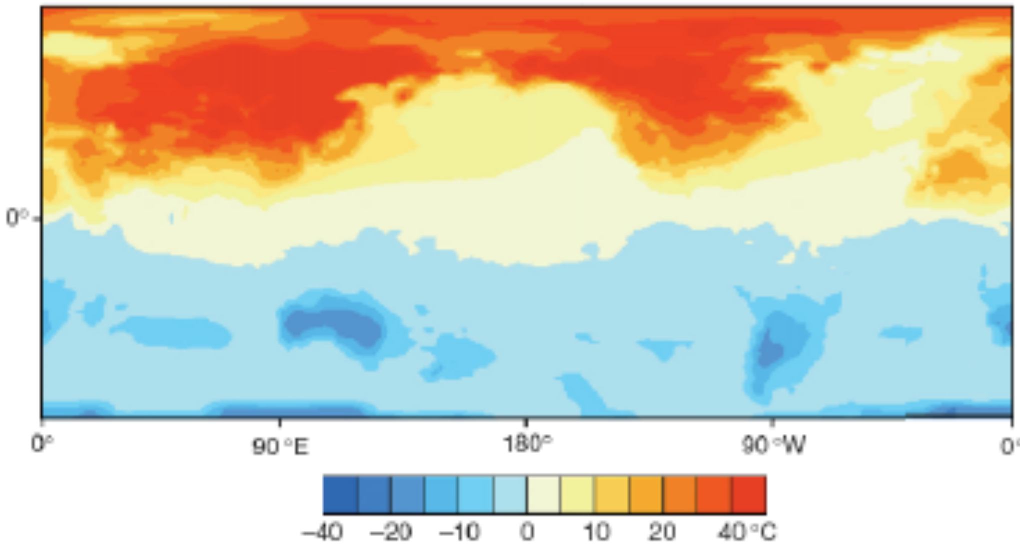




Climatological-mean subsurface T at Weather Ship N



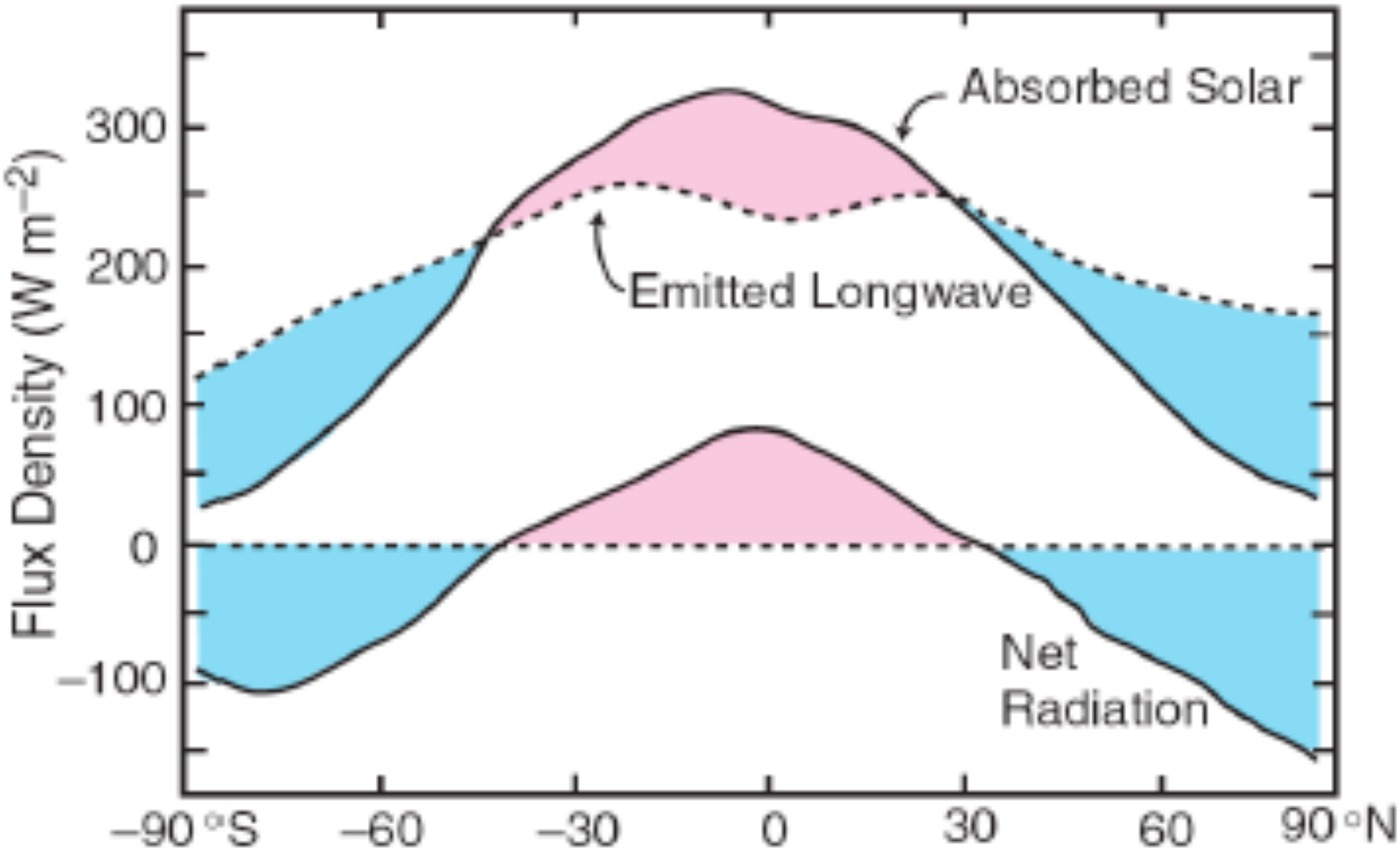
July minus January surface air T



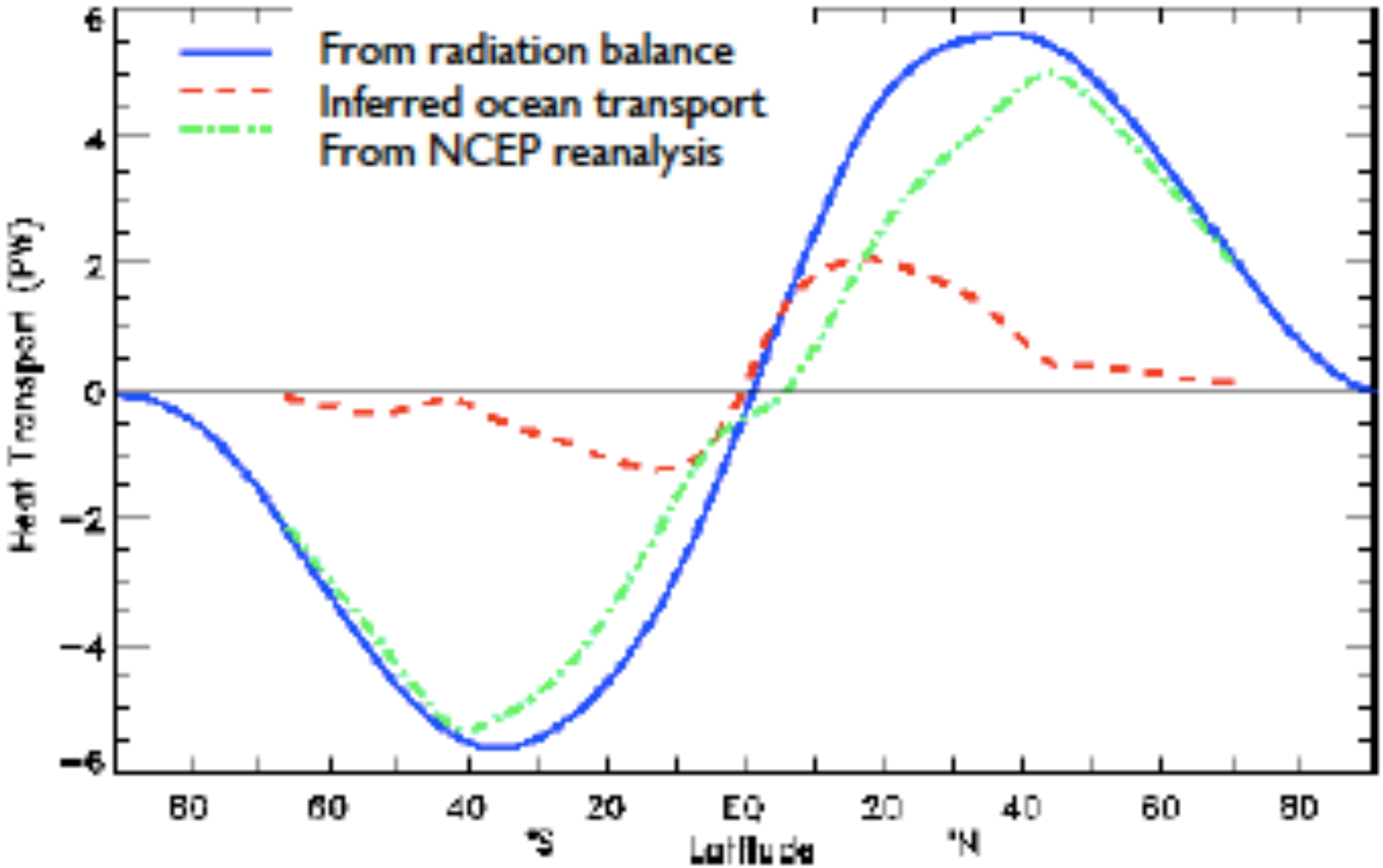
Conclusions concerning the annual cycle

- the Earth system is out of balance in seasonal means
- heat is stored in ocean mixed layer in summer
- oceans reject heat during winter
- oceanic heat storage damps the annual cycle in T
- the response is lagged relative to the forcing
- reflects differing heat capacities of atm. and ocean ML

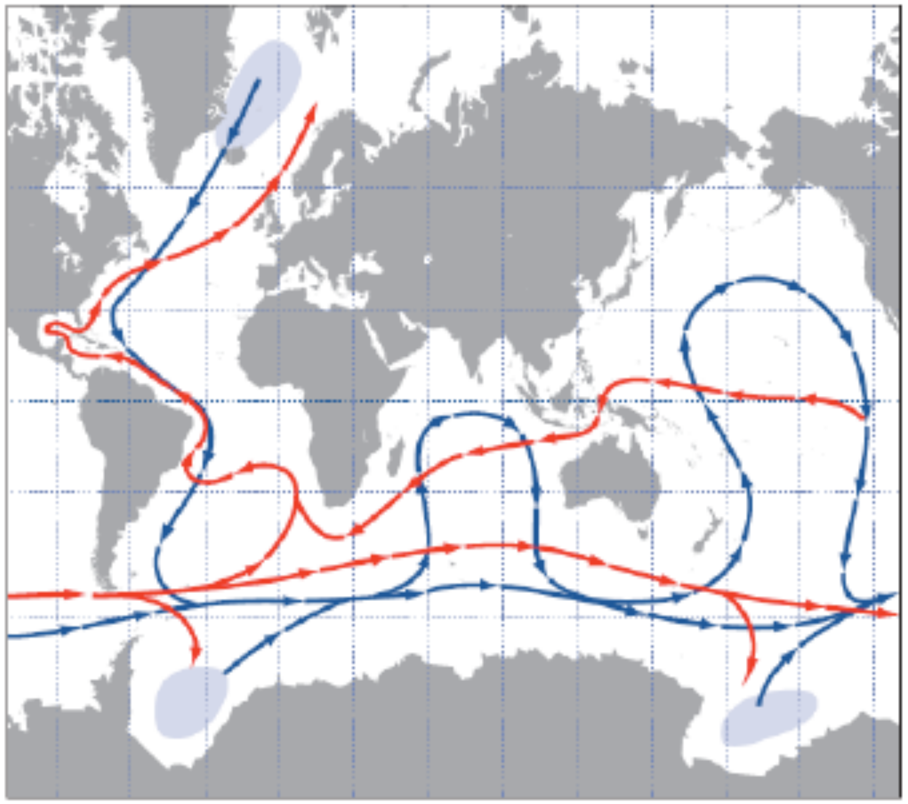
Balance requirement: Poleward transport of energy

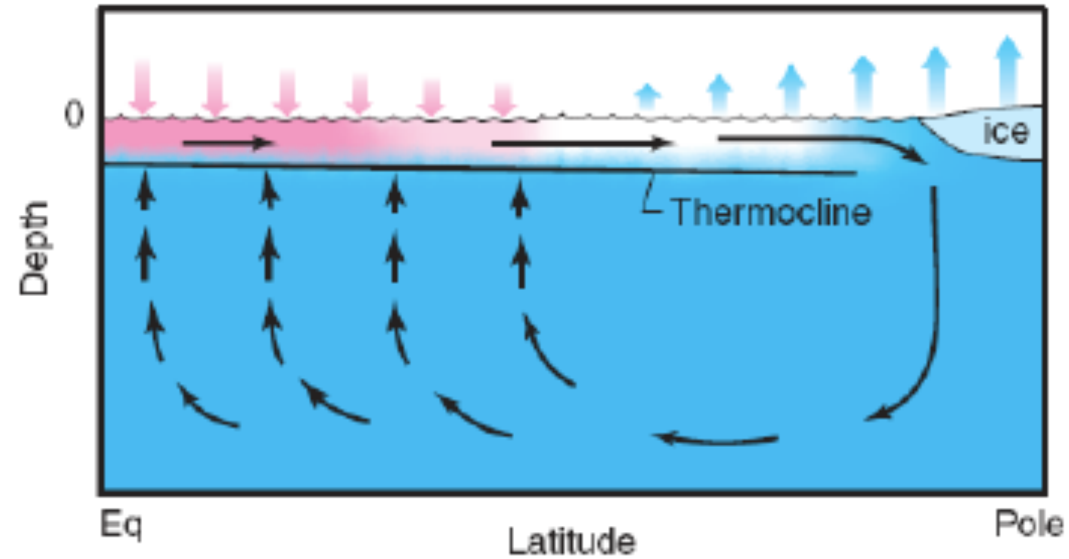


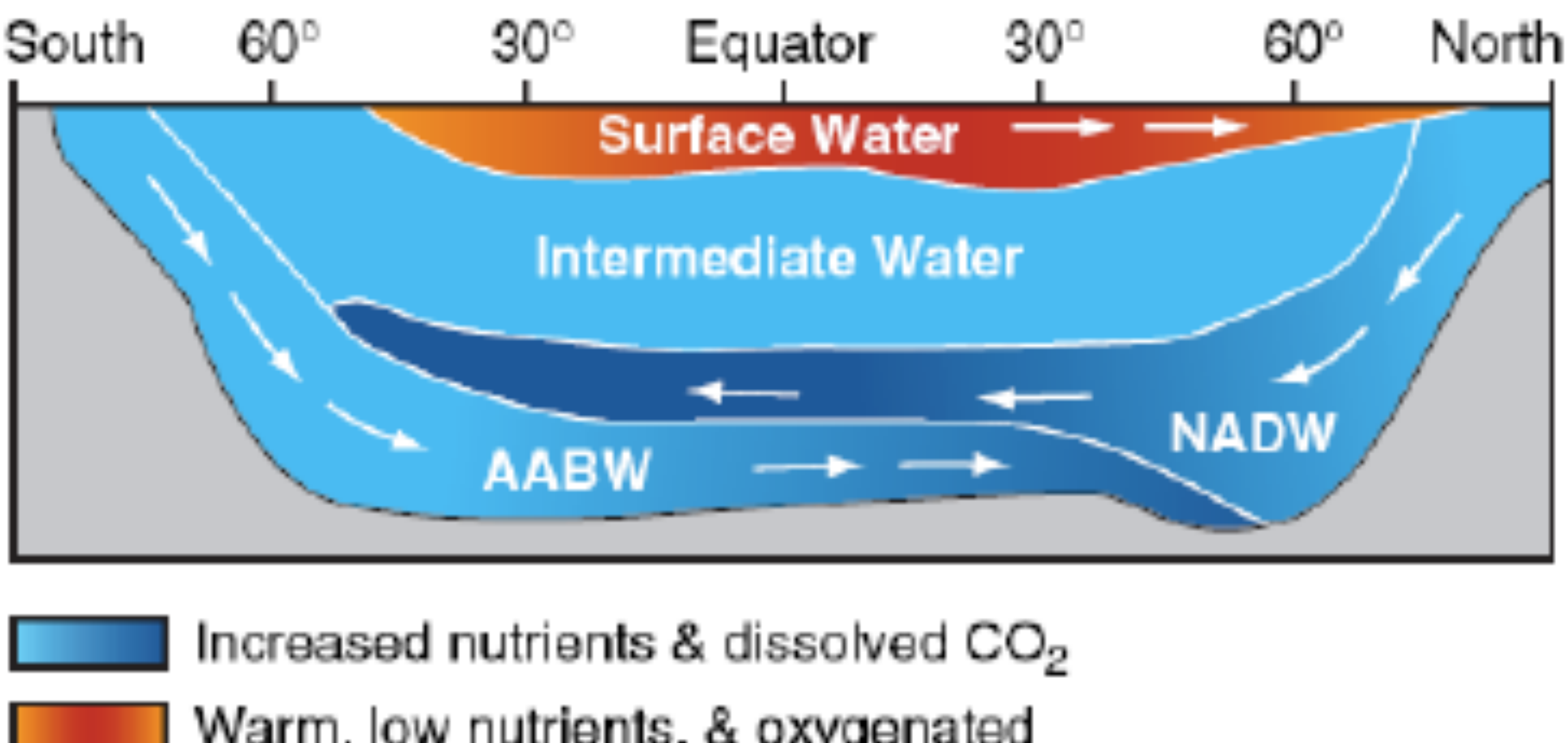
Atmosphere and ocean both contribute to the transport



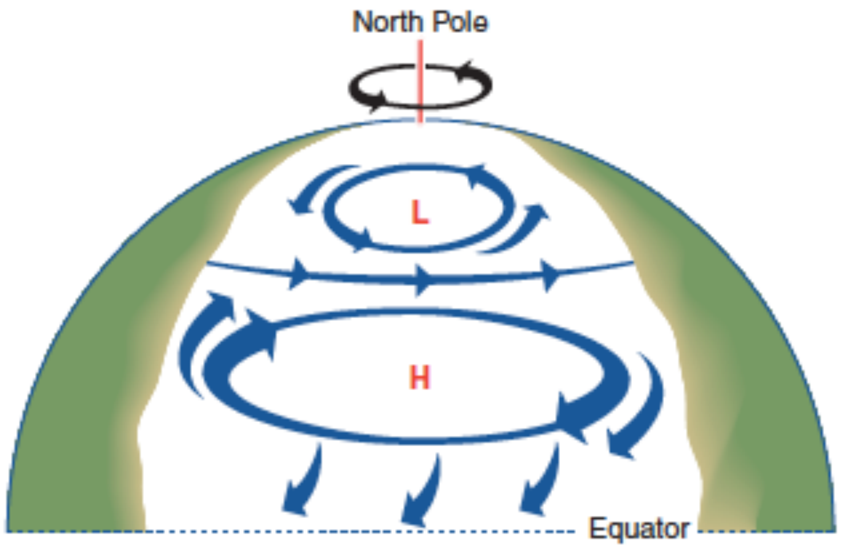
Contribution from the Oceanic THC

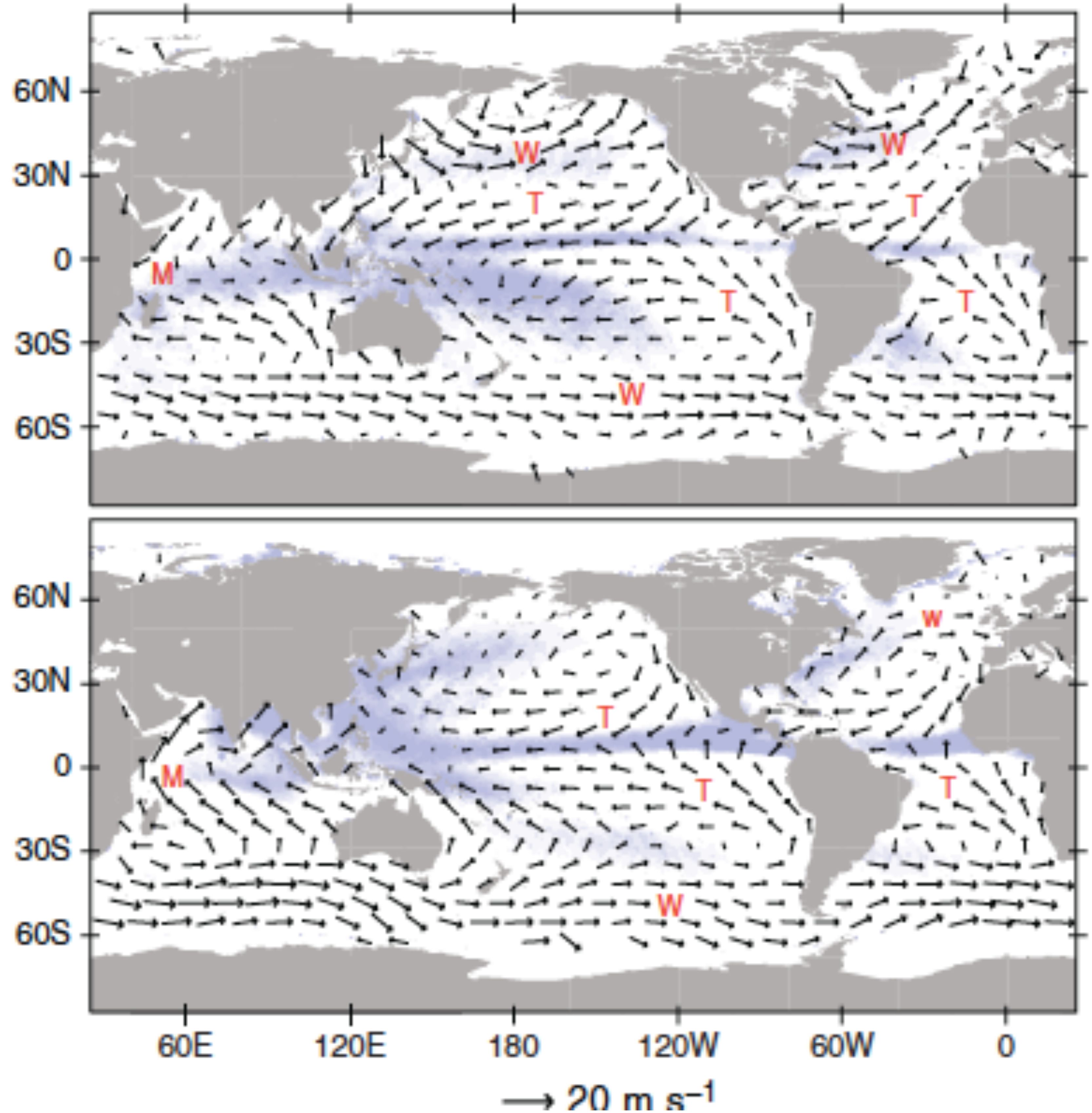




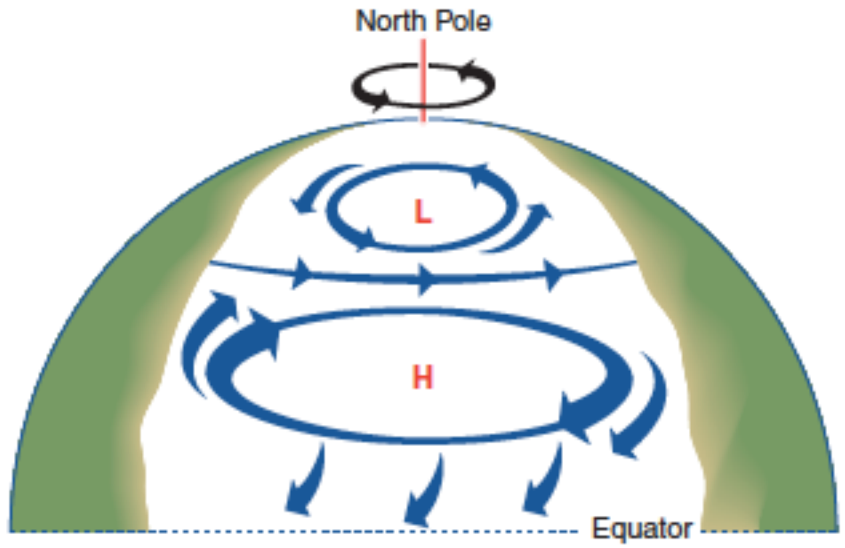


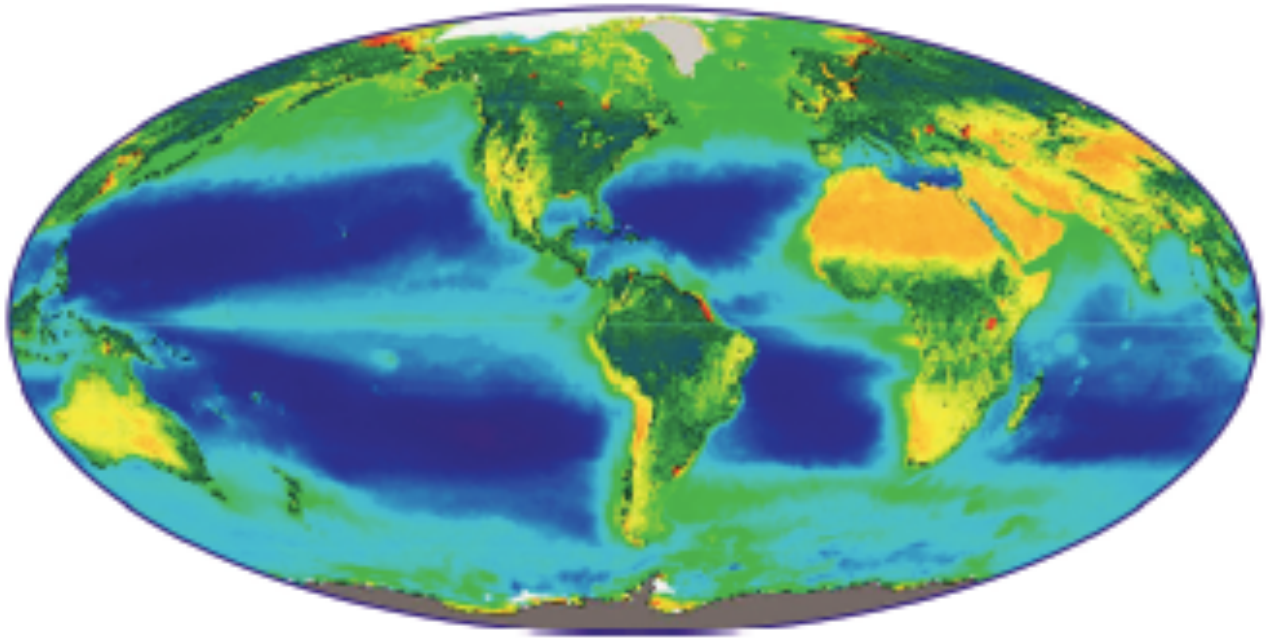
Contribution from the wind-driven currents

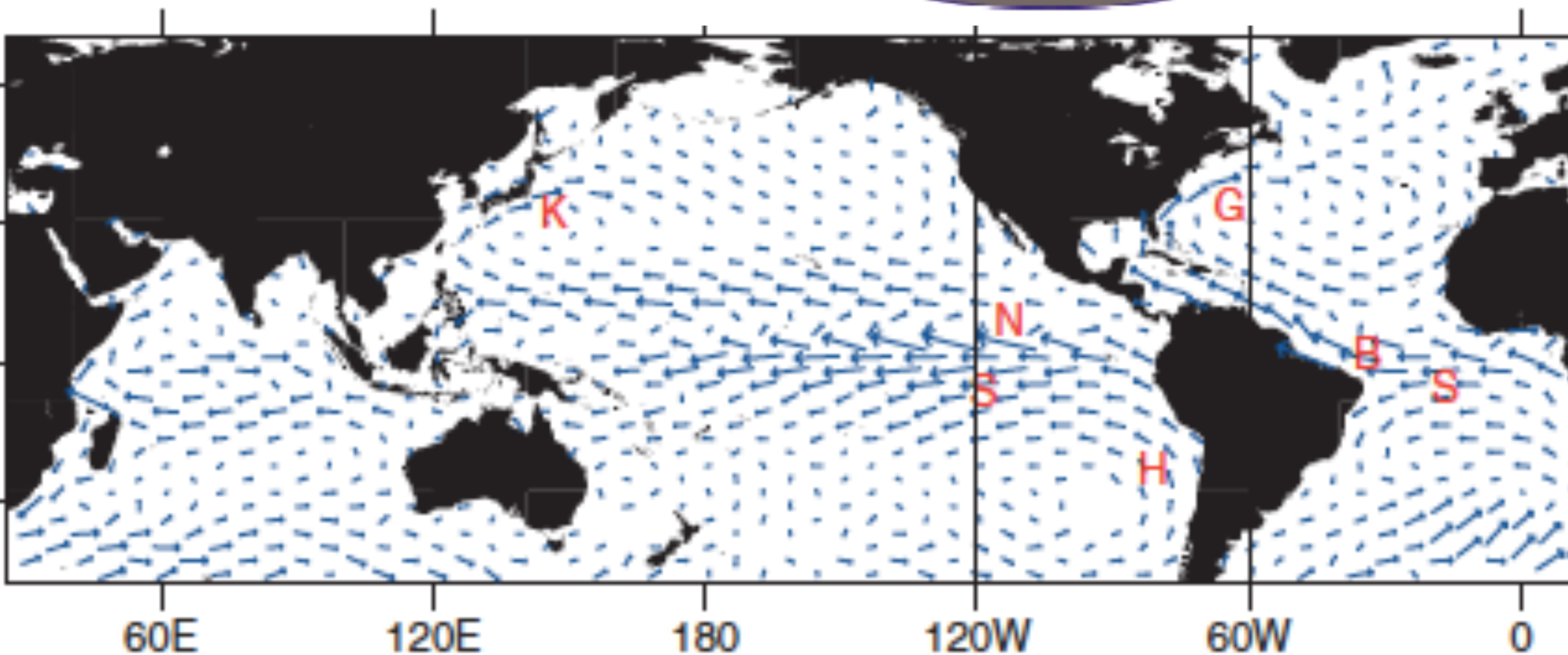




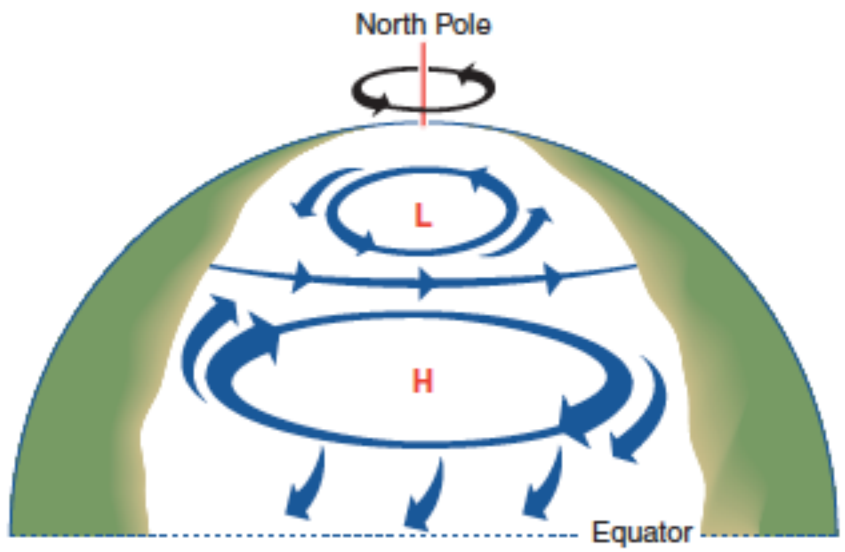
Contribution from the wind-driven currents



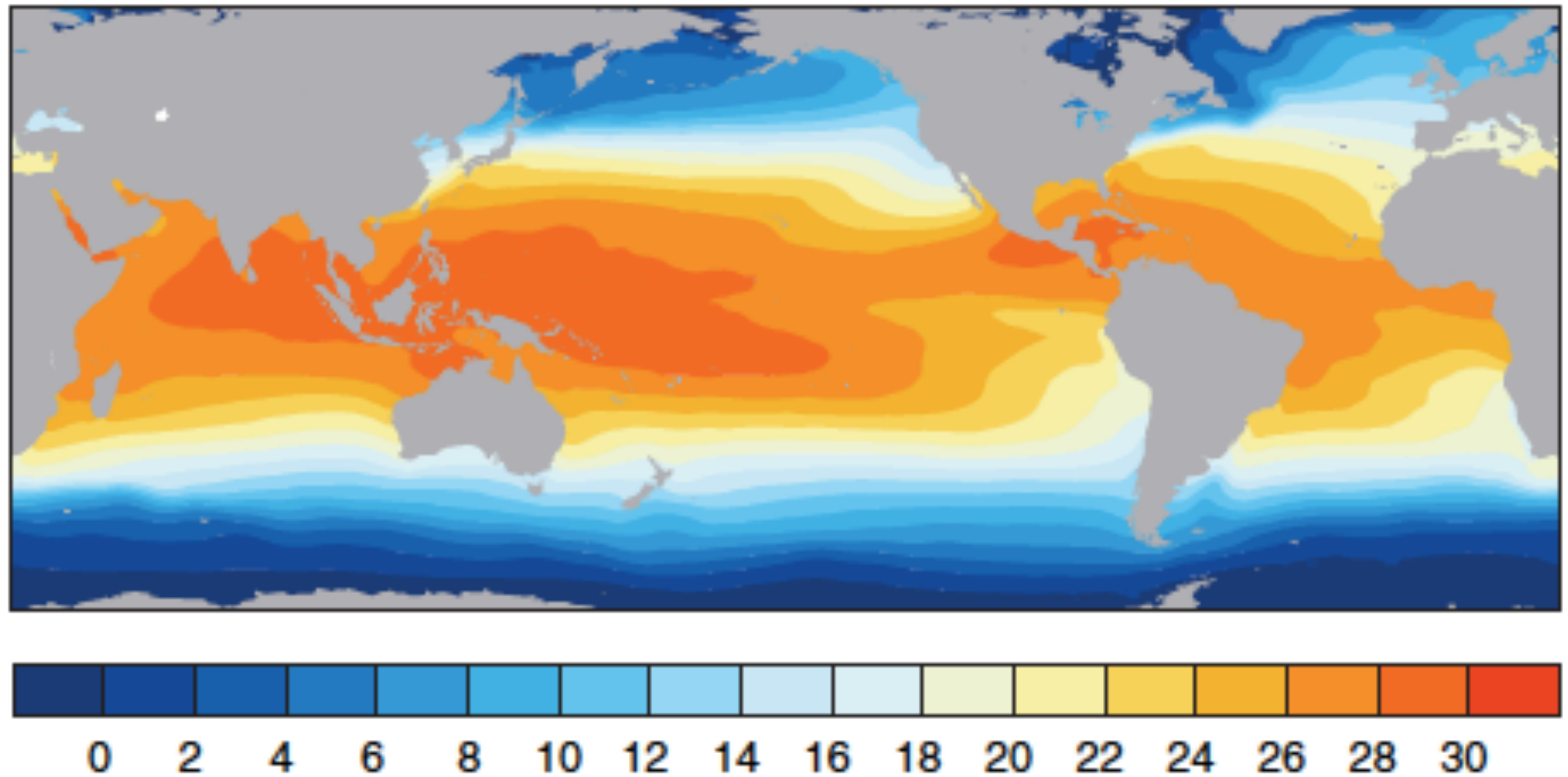




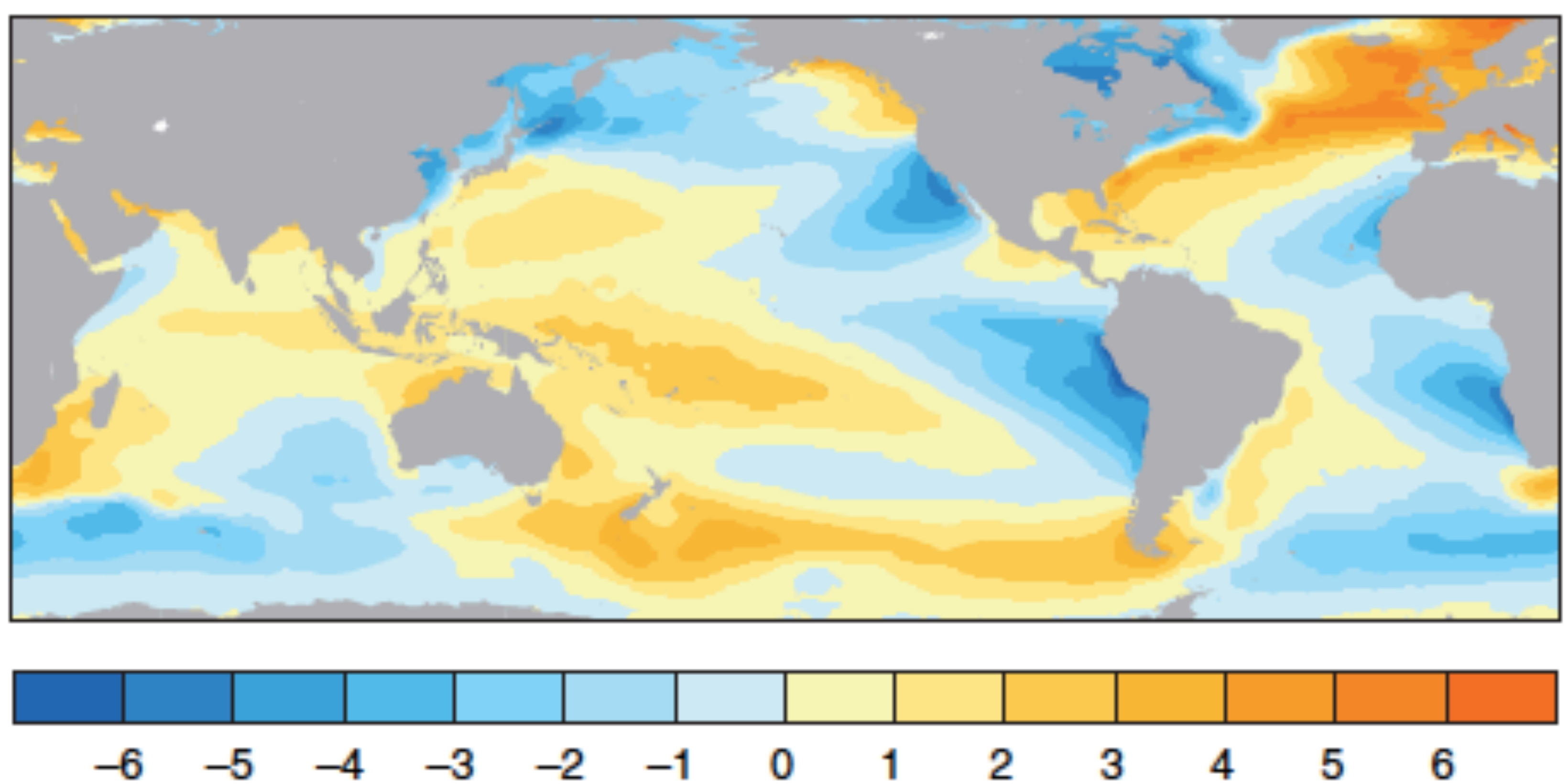
Contribution from the wind-driven currents



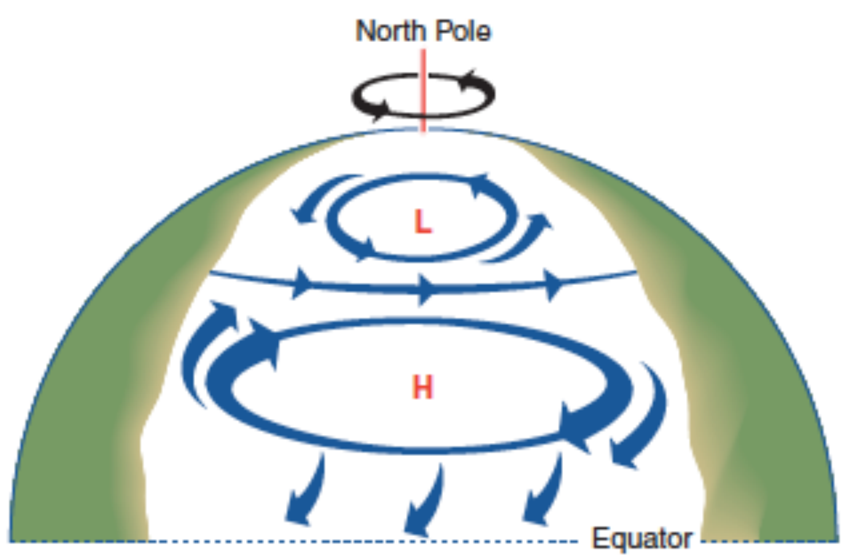
annual-mean SST

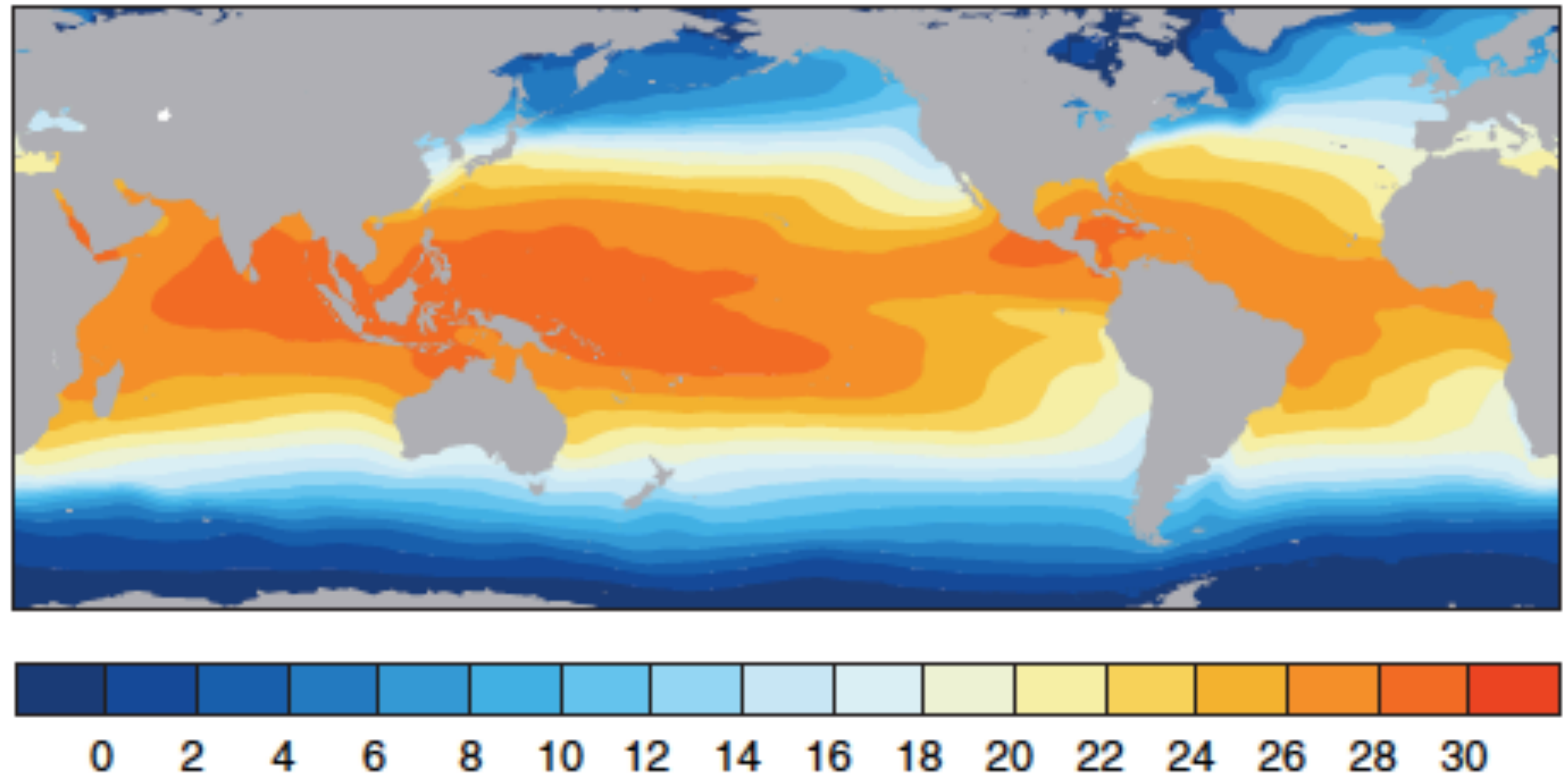


departure from zonal average

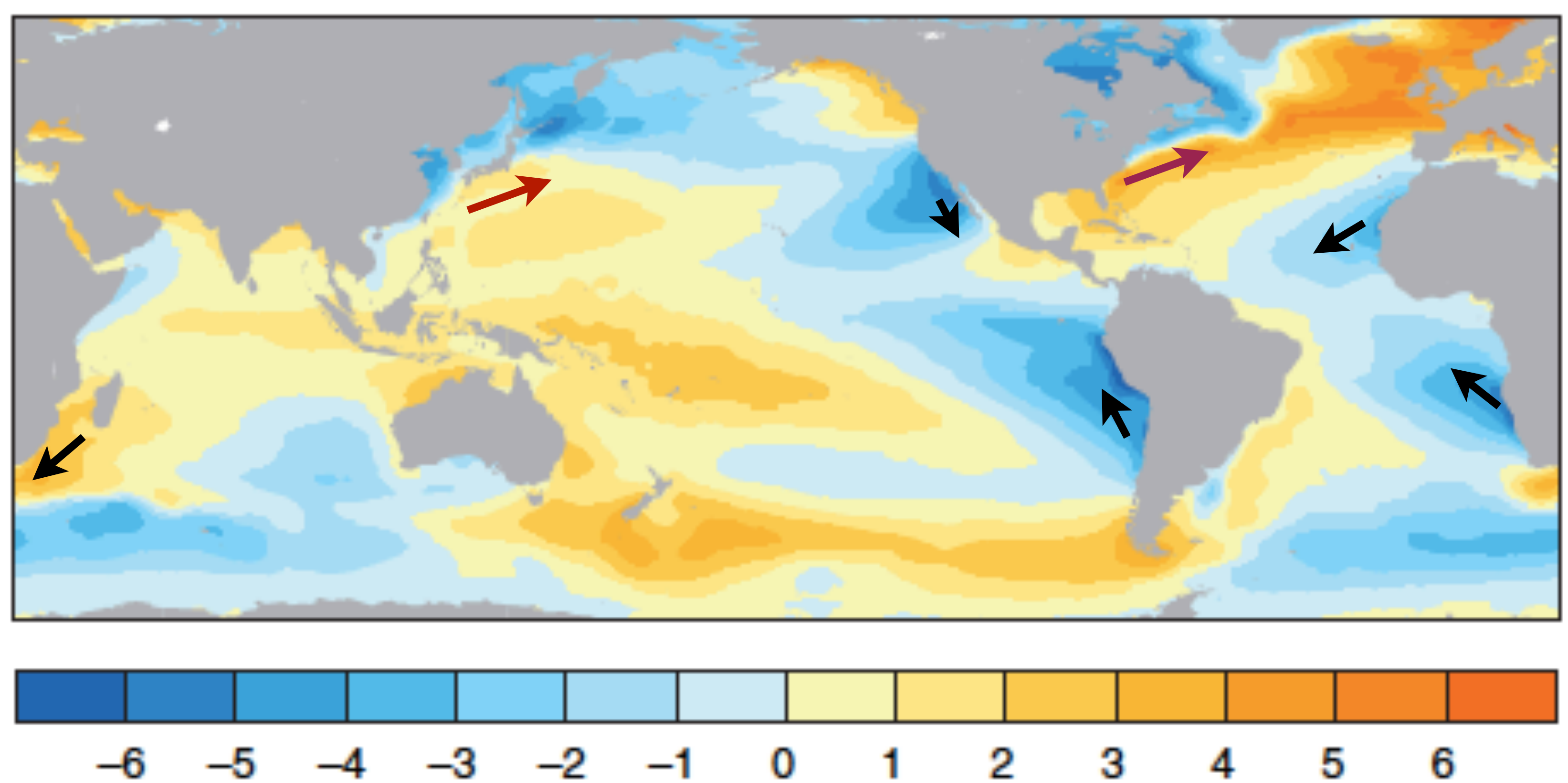


Contribution from the wind-driven currents

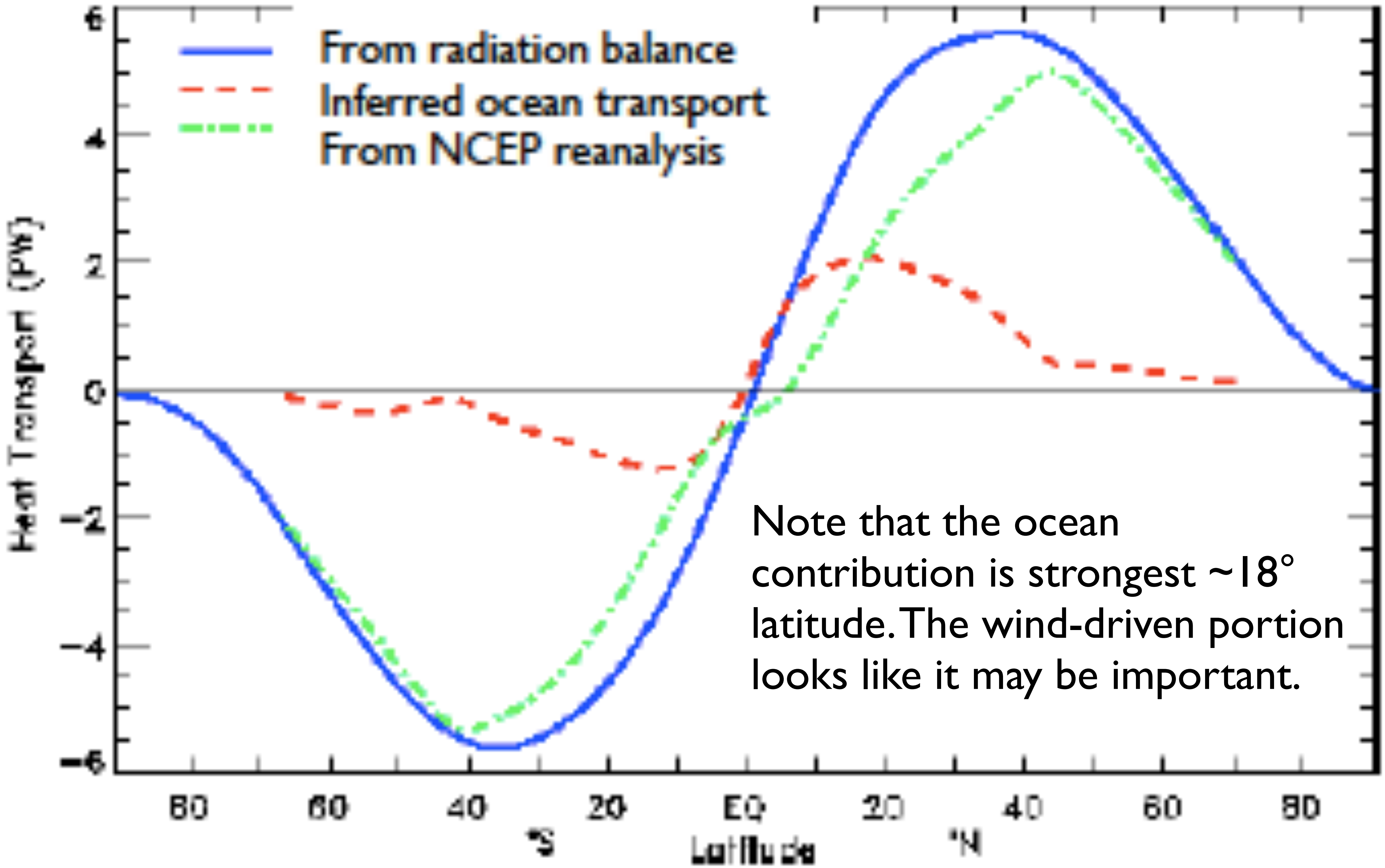




blue (red) arrows show cold (warm) surface currents

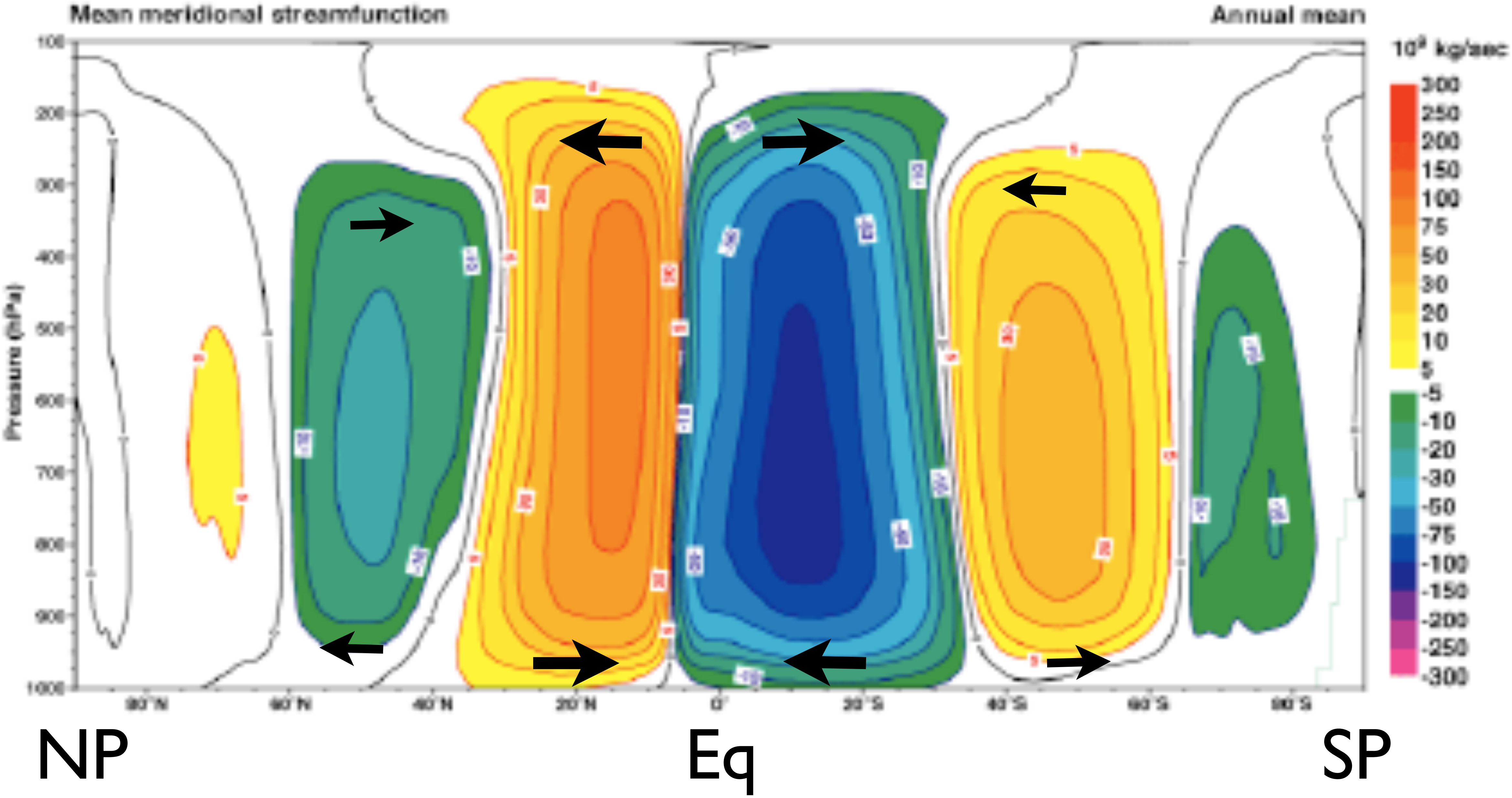


The total oceanic transport ...



Note that the ocean contribution is strongest ~18° latitude. The wind-driven portion looks like it may be important.

Role of the mean meridional circulations

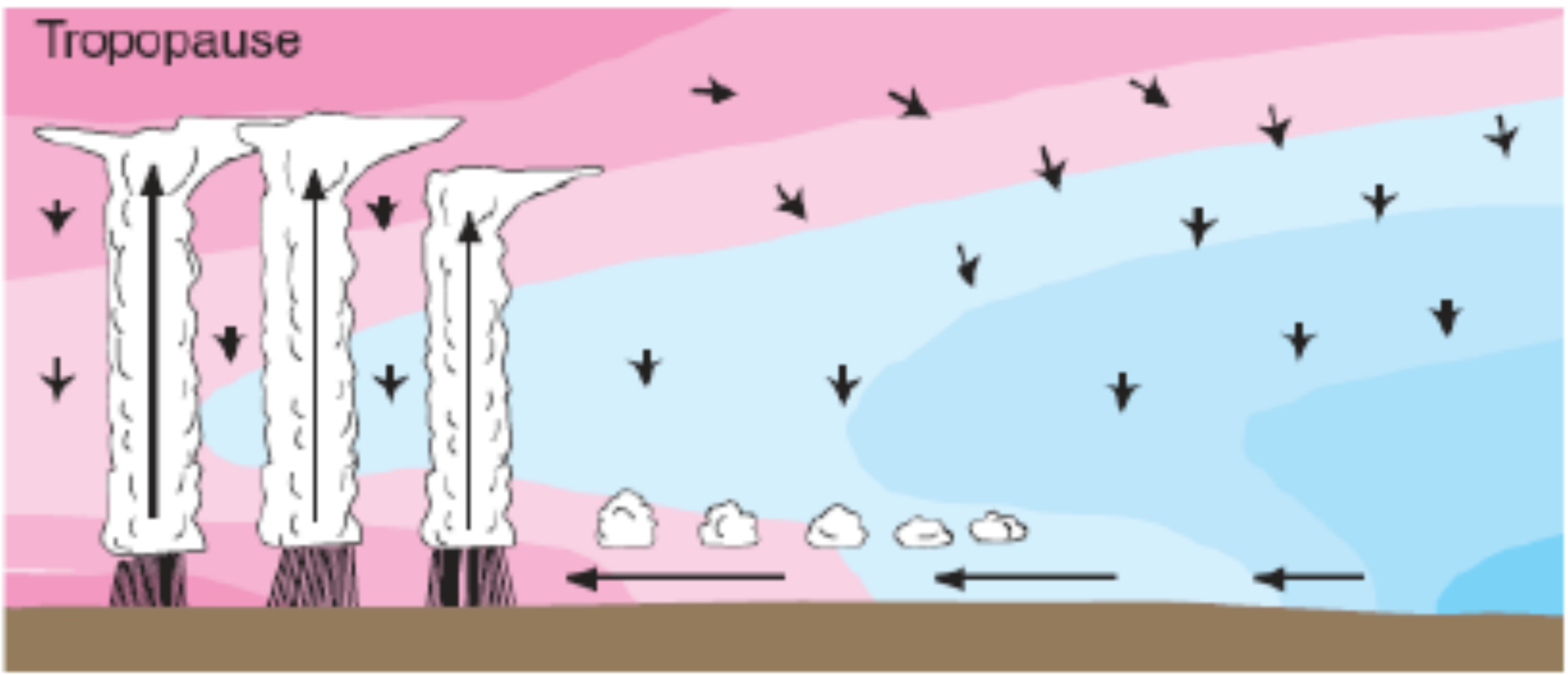


$$MSE = c_p T + Lq + \Phi$$ increases with height

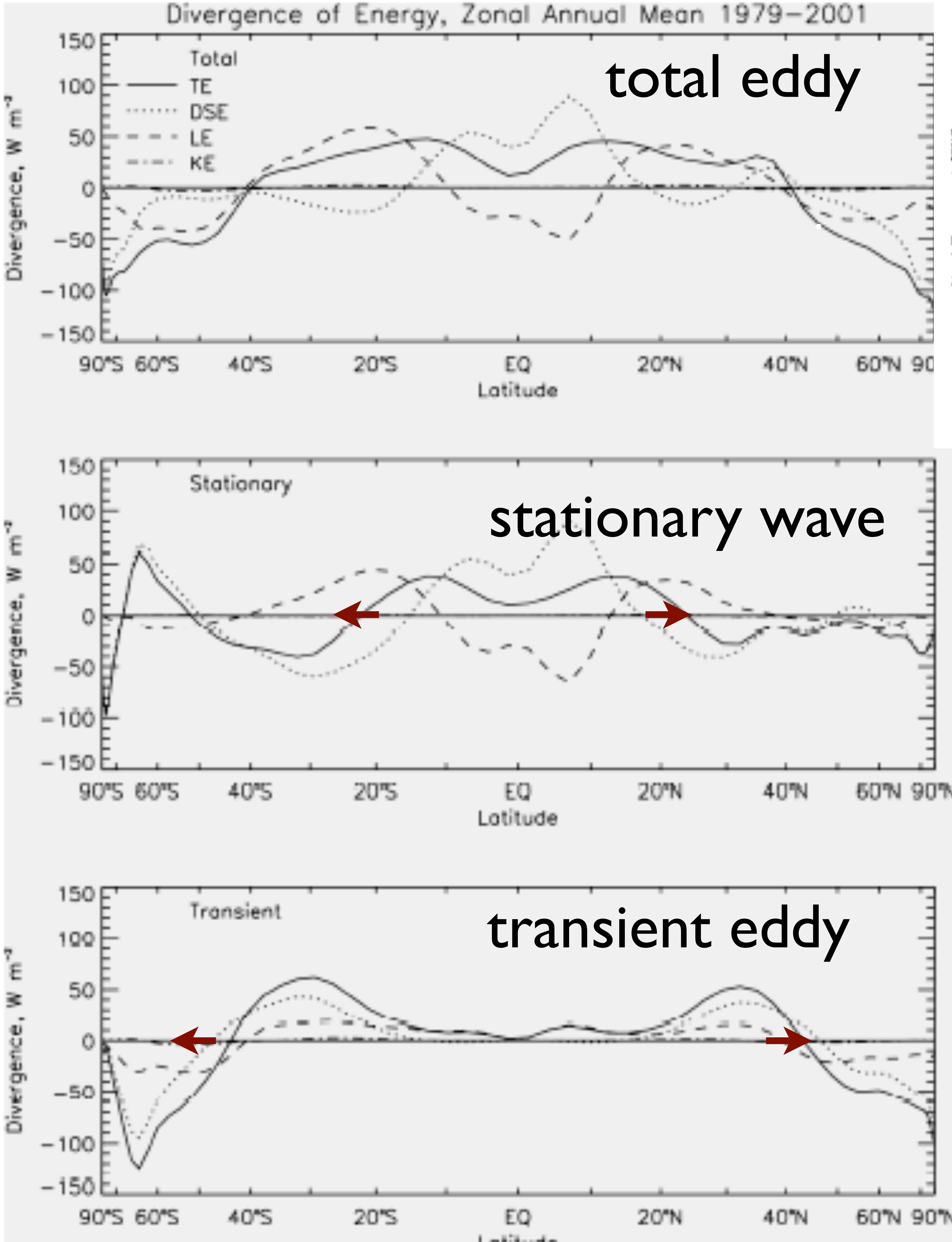
hence, the Hadley cells transport MSE poleward

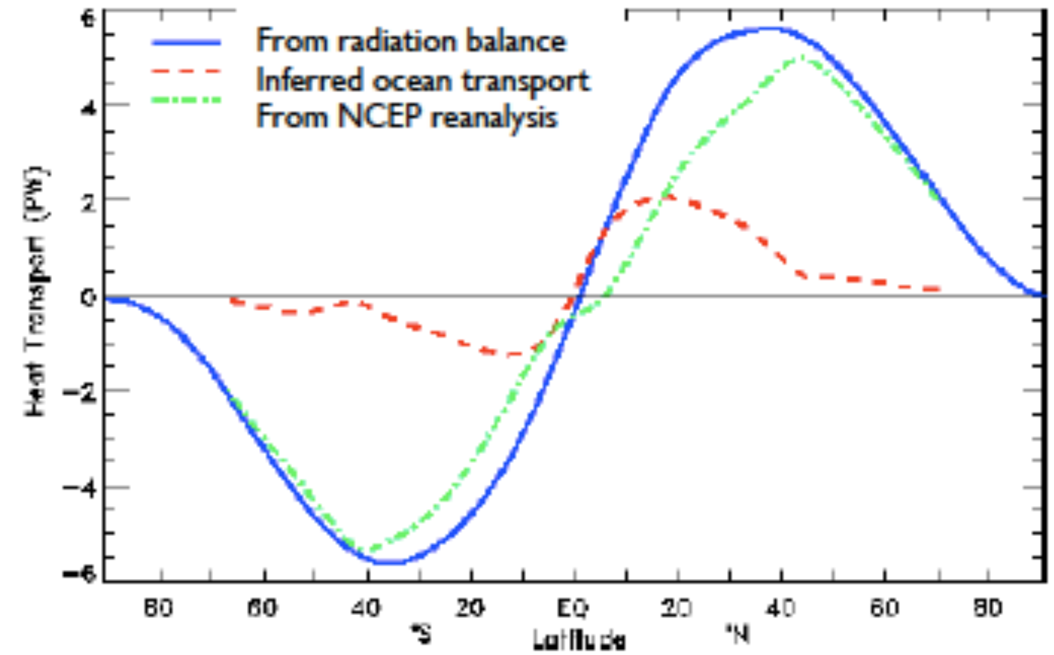
Ferrell cells transport MSE equatorward

Hadley cell analogous to this idealized schematic



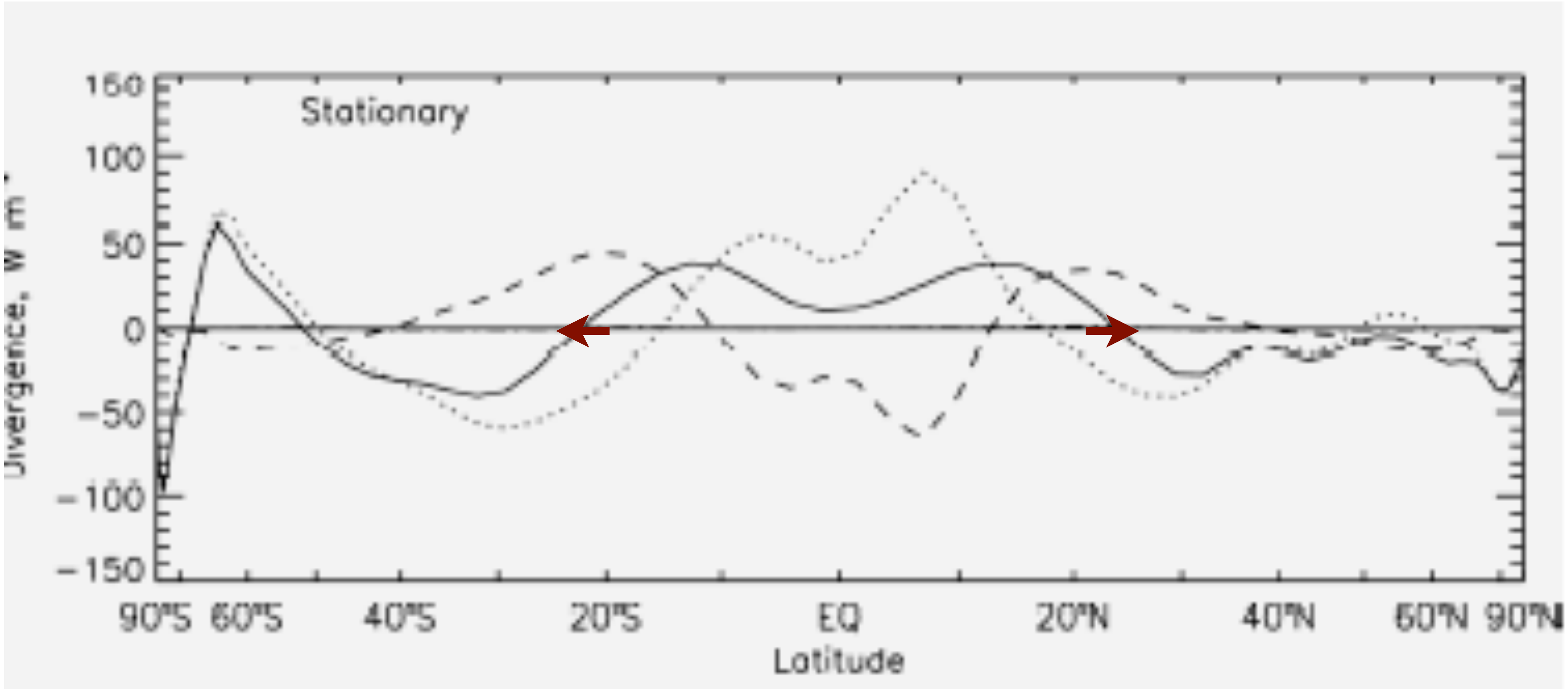
MSE transport is toward the right.



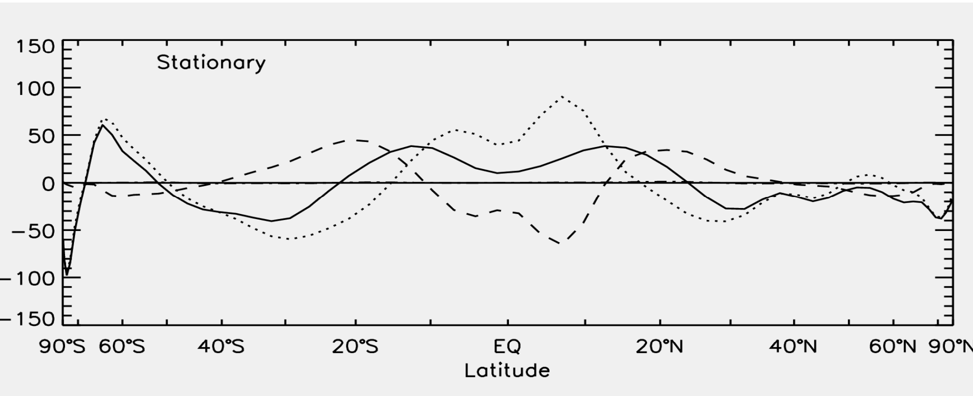


Role of the eddies

$$\frac{\partial}{\partial y}[v^* T^*]$$

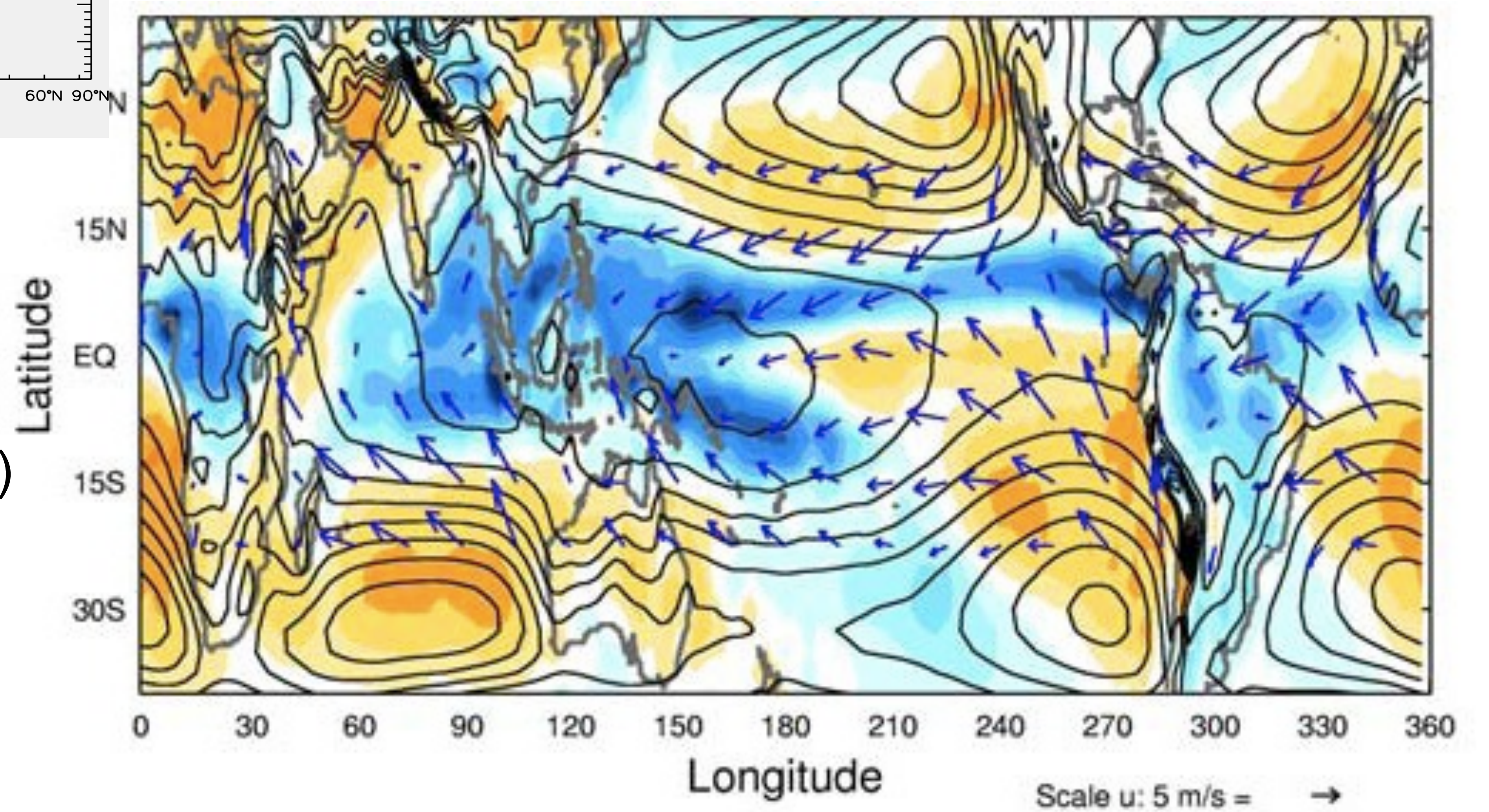


The stationary wave contribution
at low latitudes

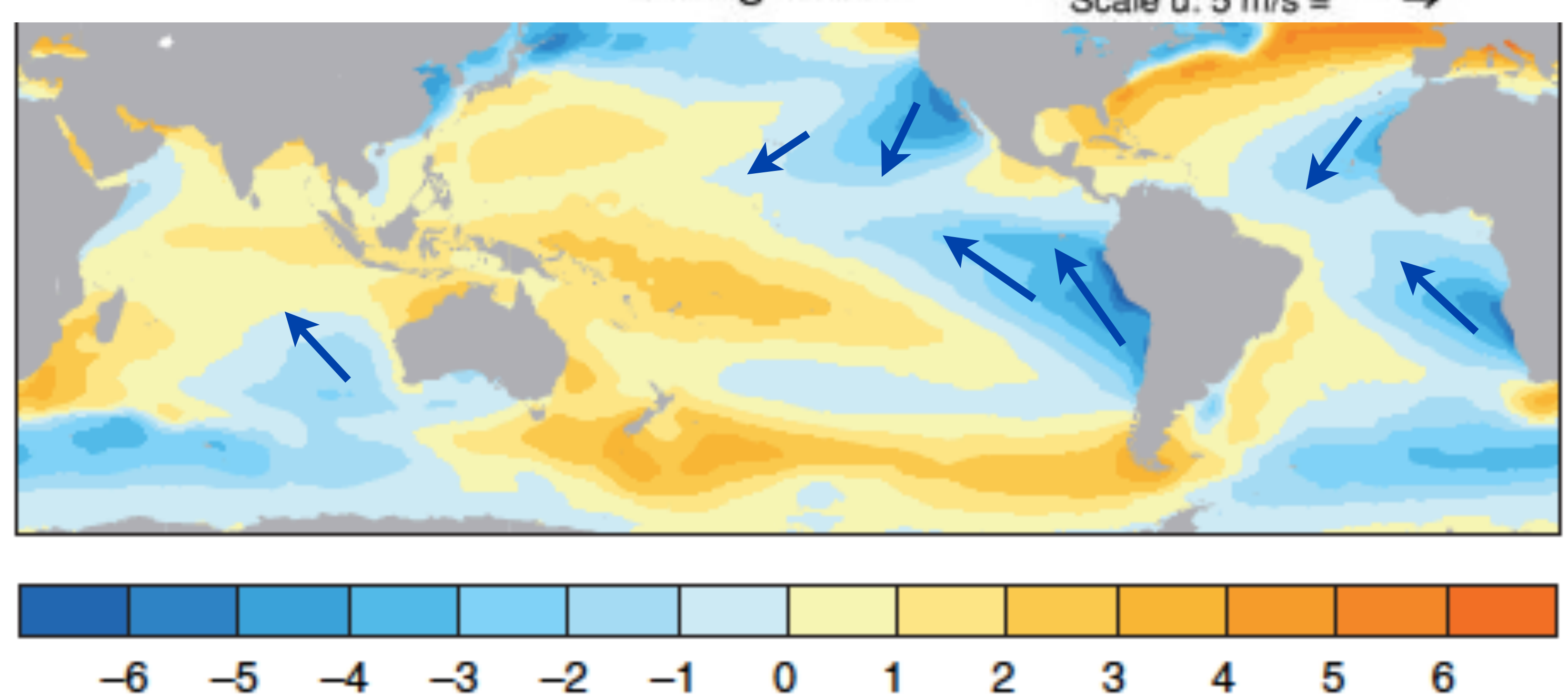


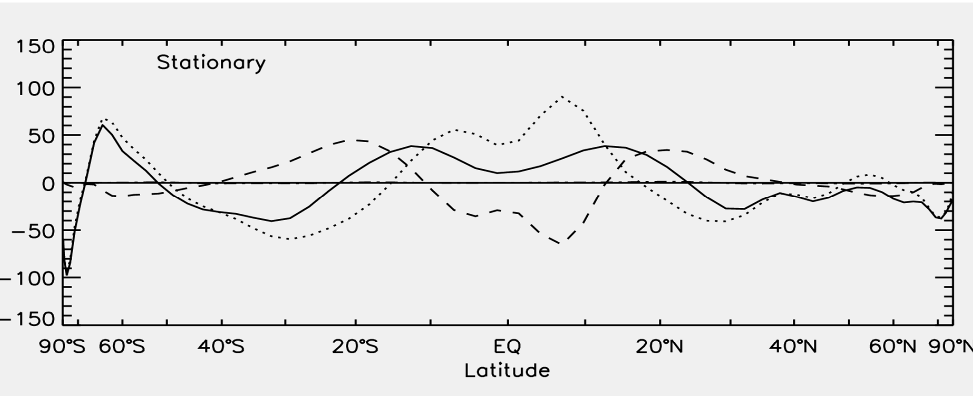
cold air injected into tropics

surface winds
SLP contours
vertical velocity (color)



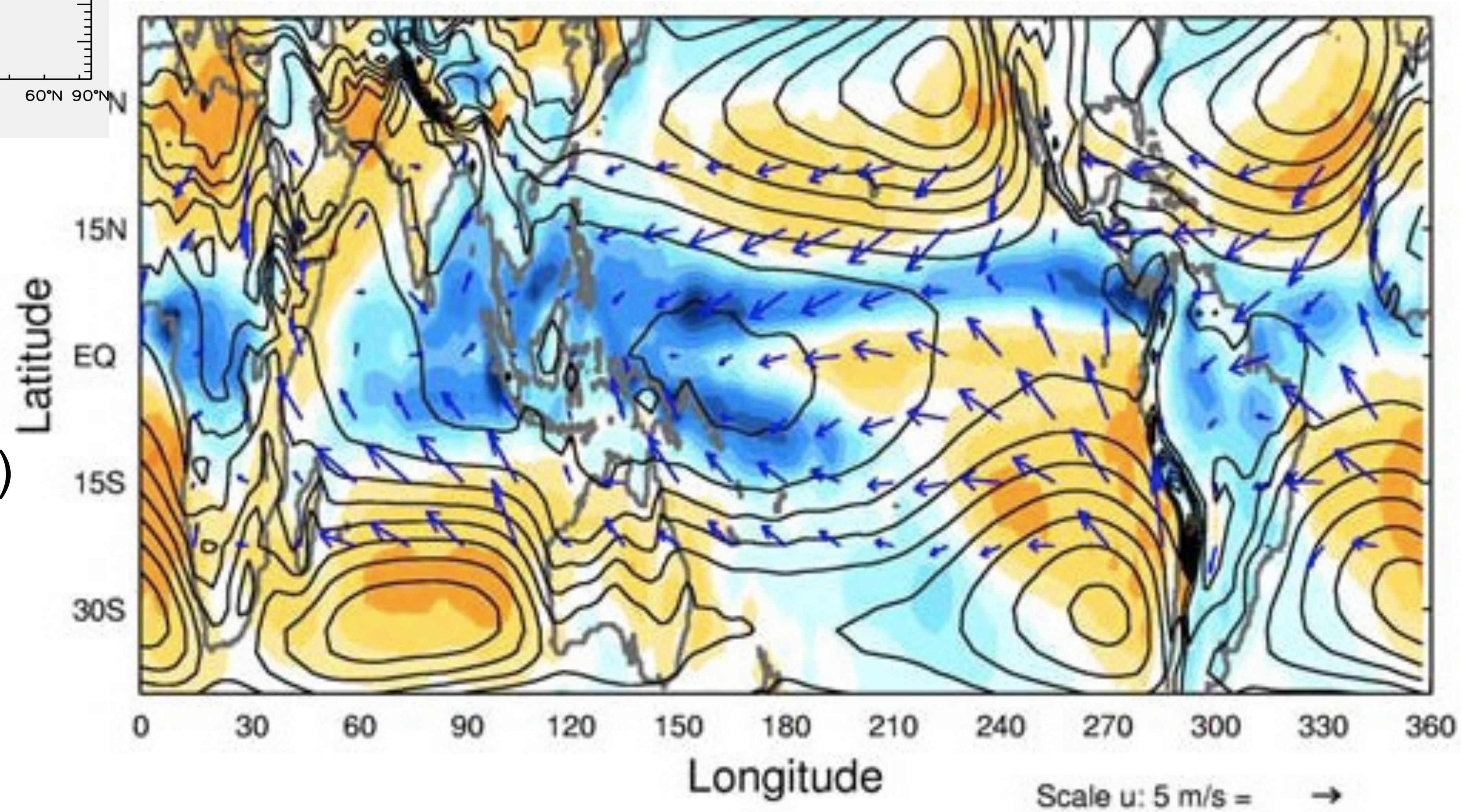
SST*





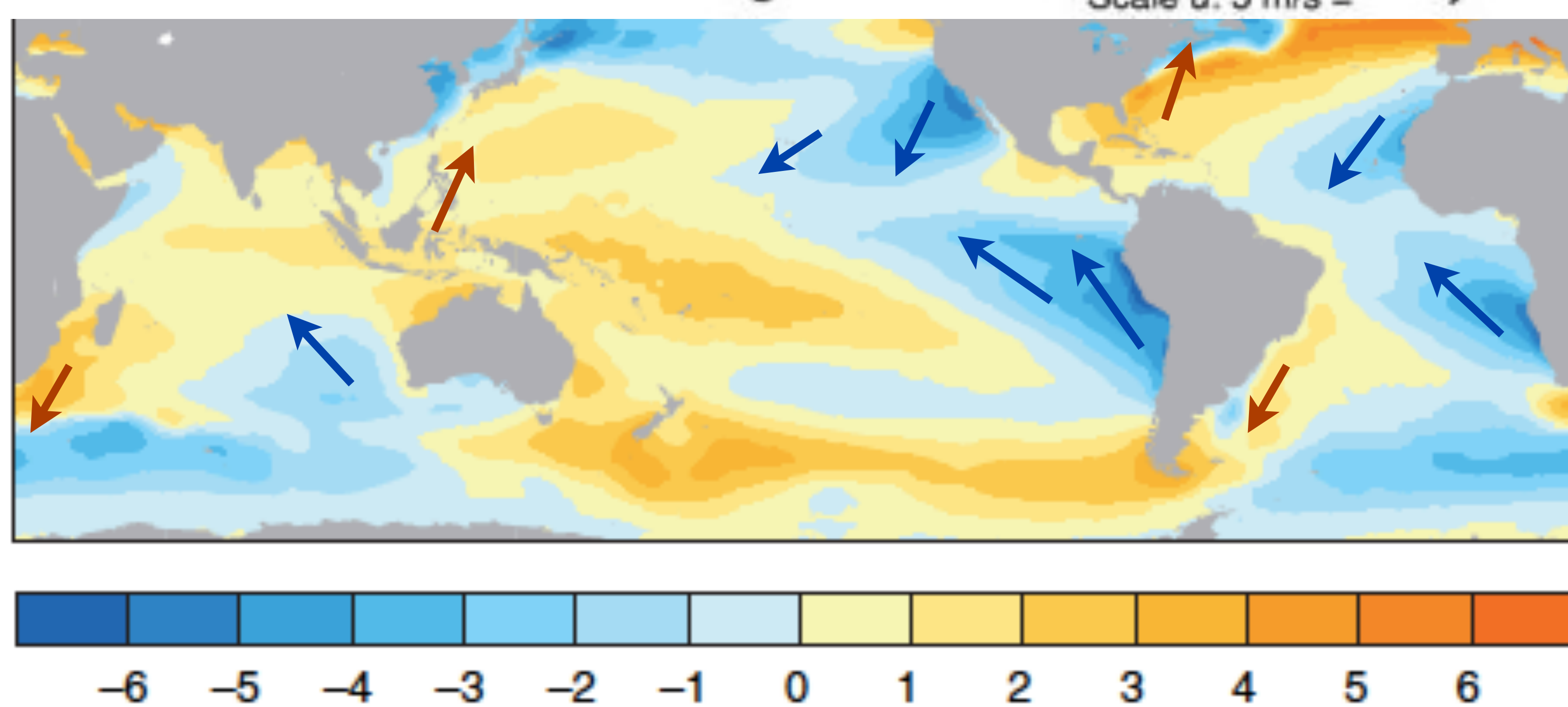
warm air exported in summer hemisphere

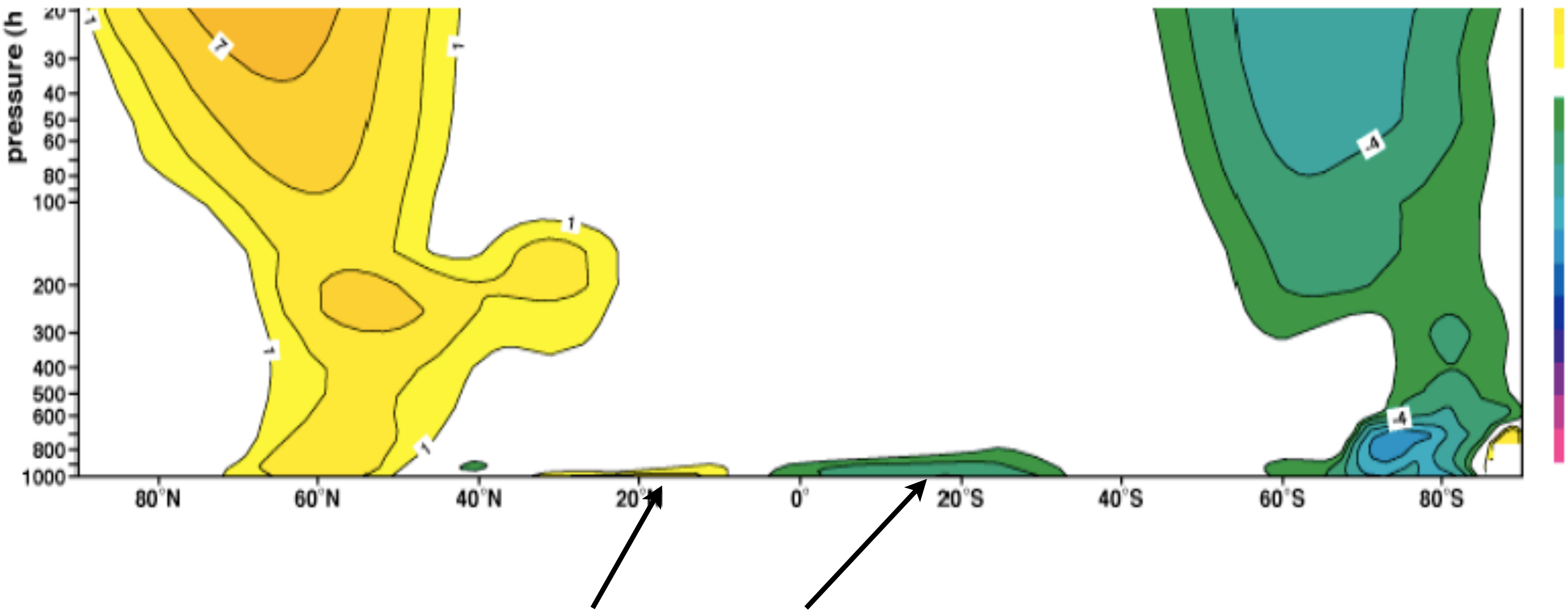
surface winds
SLP contours
vertical velocity (color)



SST departure from zonal averages

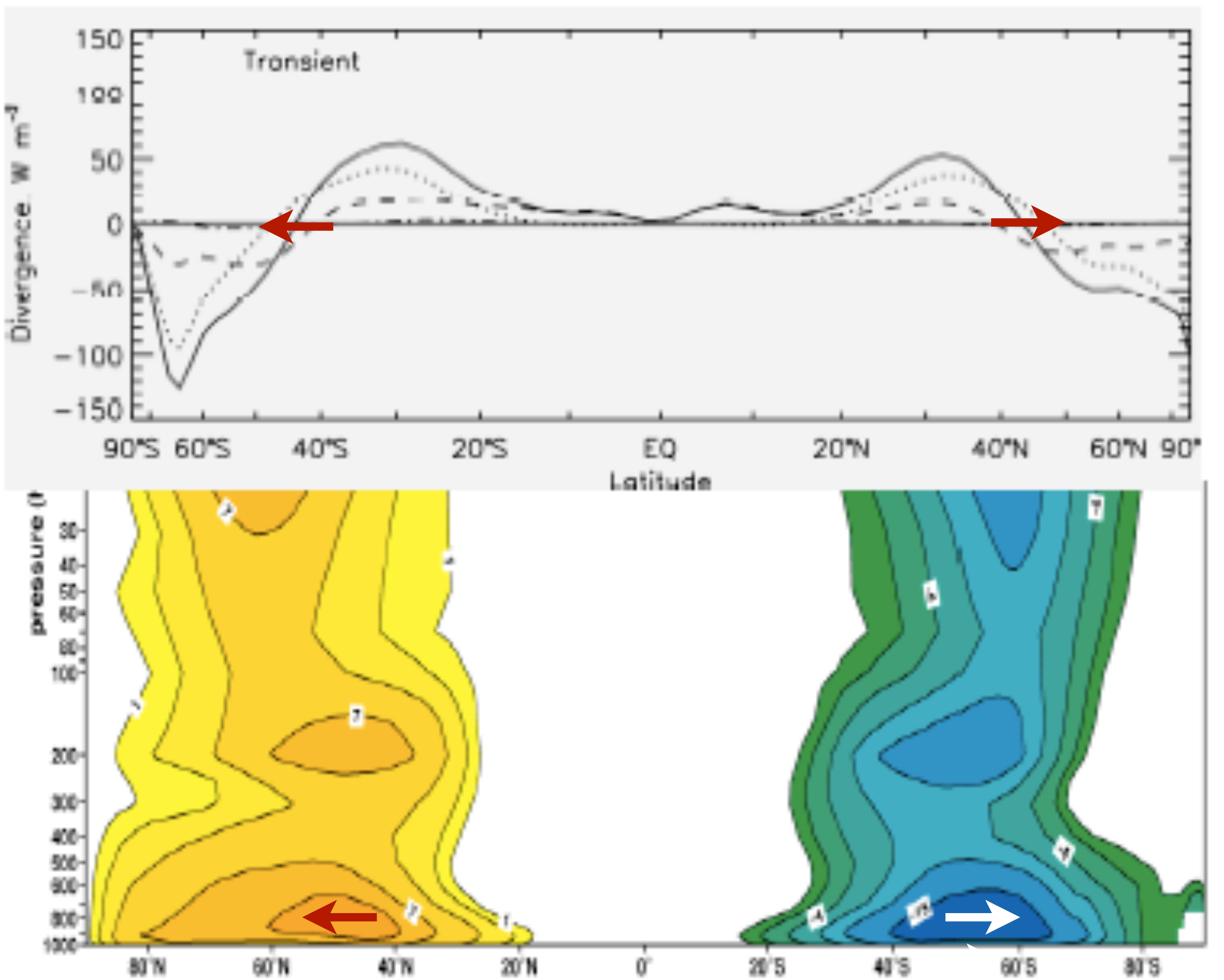
Wind arrows are transposed from upper panel



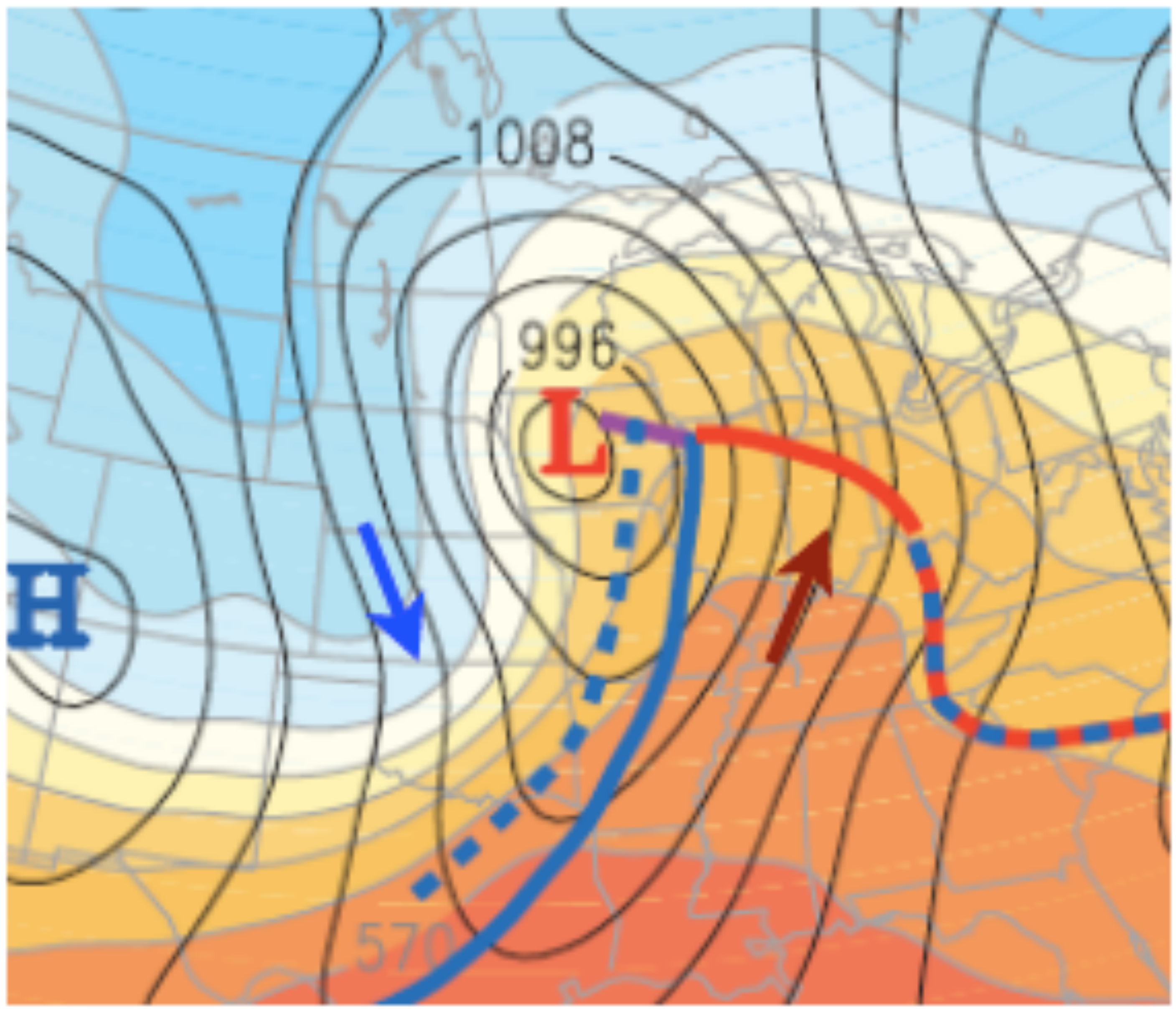


stationary wave contribution to poleward heat flux
ERA-40 Reanalysis

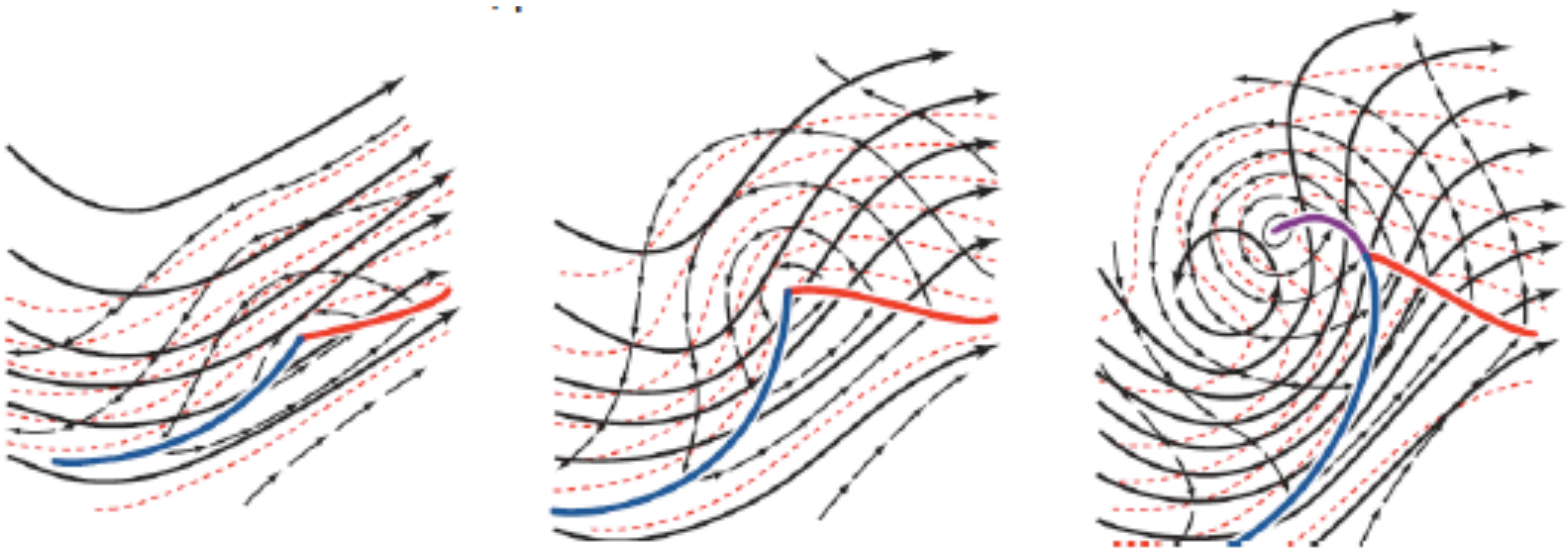
The transient eddy contribution



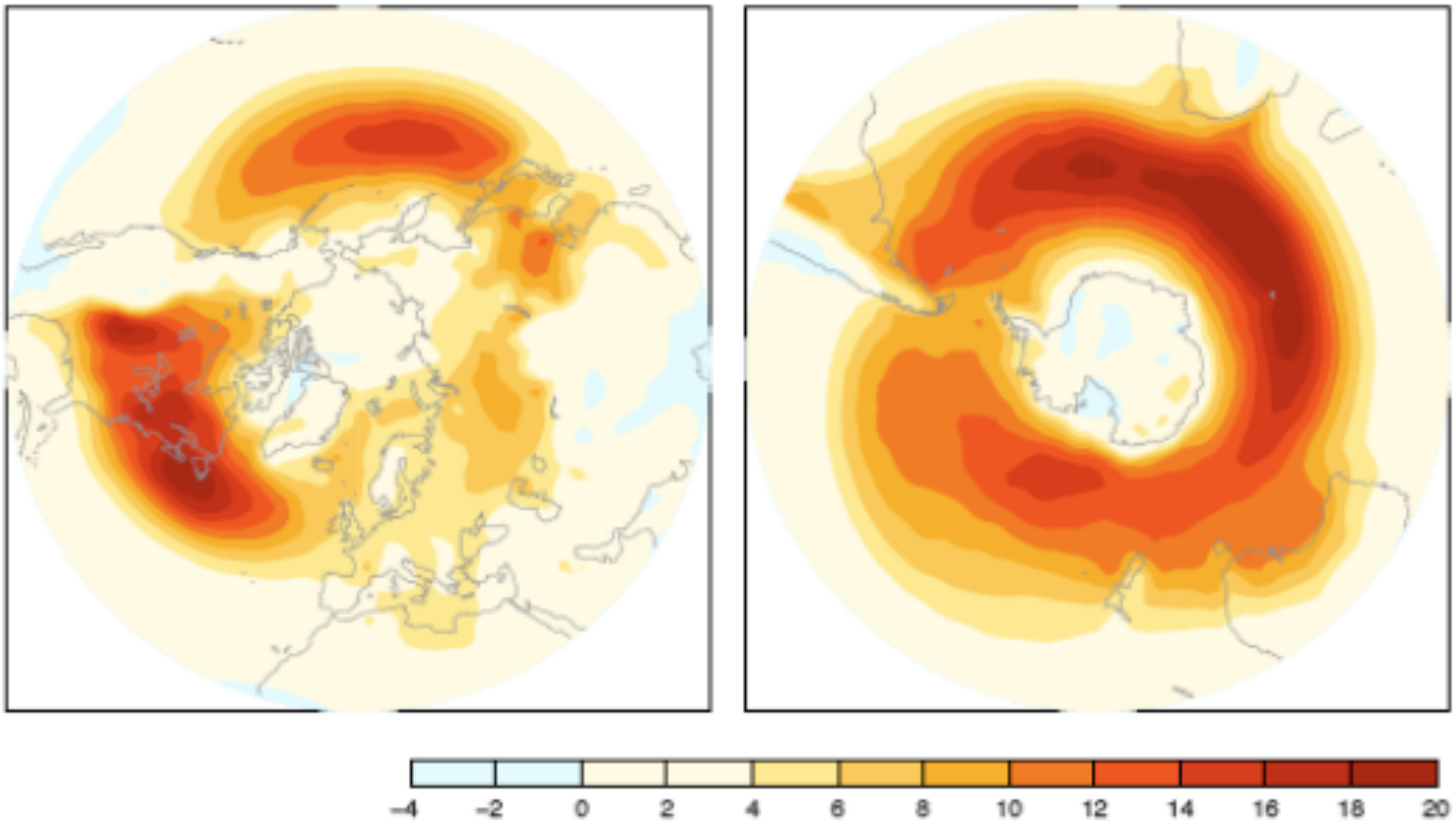
baroclinic waves dominate the transient eddy transport



baroclinic waves dominate the transient eddy transport



baroclinic waves are organized in “storm tracks”



$$\overline{v'T'}_{850\ \mathrm{hPa}}$$

spherical geometry

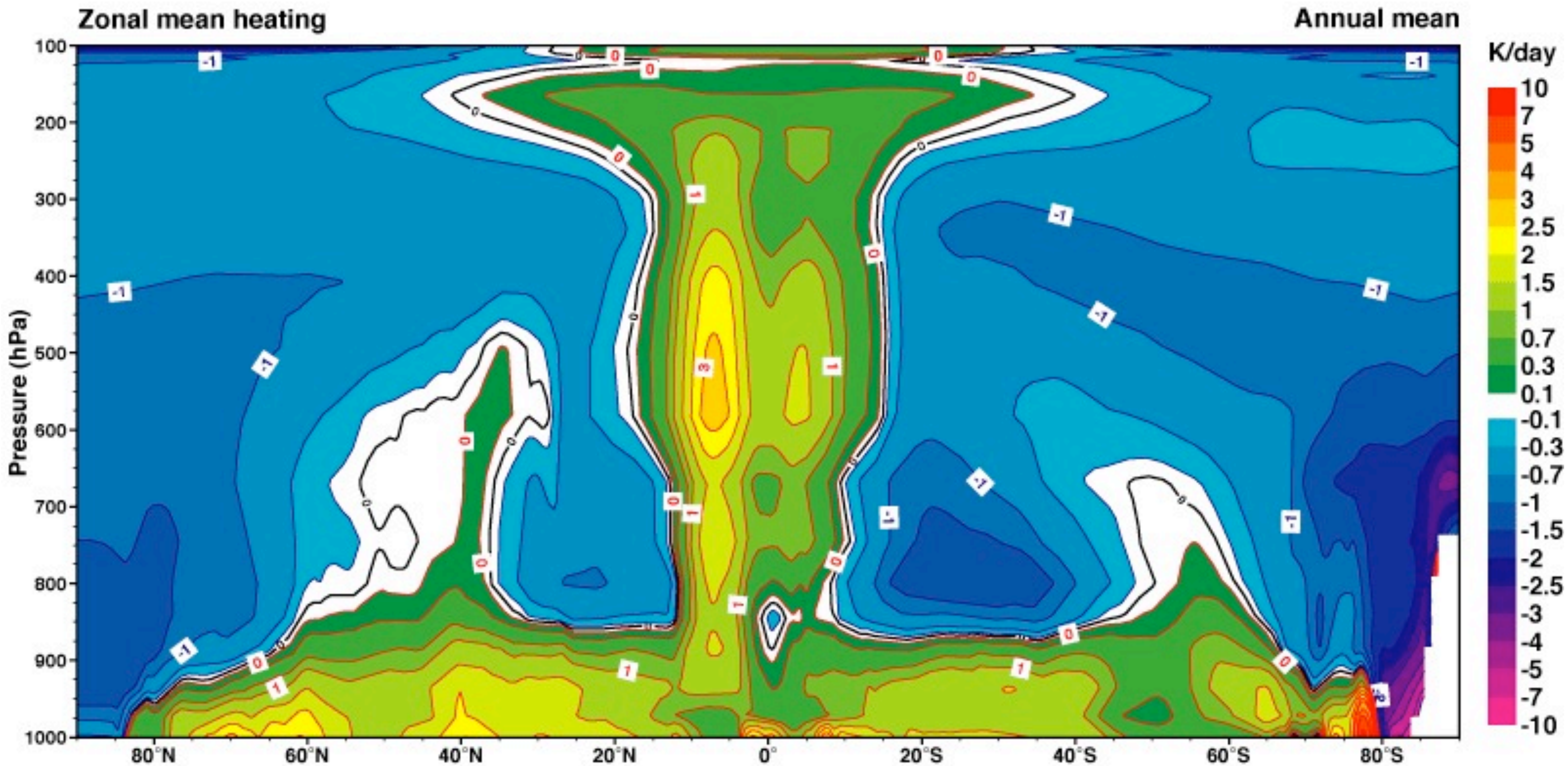
$$\frac{\partial[T]}{\partial t} = [\omega]\left(\frac{\kappa[T]}{p} - \frac{\partial[T]}{\partial p}\right) - \frac{[v]}{\cos\phi}\frac{\partial}{\partial y}[T]\cos\phi - \frac{1}{\cos\phi}\frac{\partial}{\partial y}[v^*T^*]\cos\phi - \frac{\partial}{\partial p}[\omega^*T^*] + [\mathrm{Q}]$$

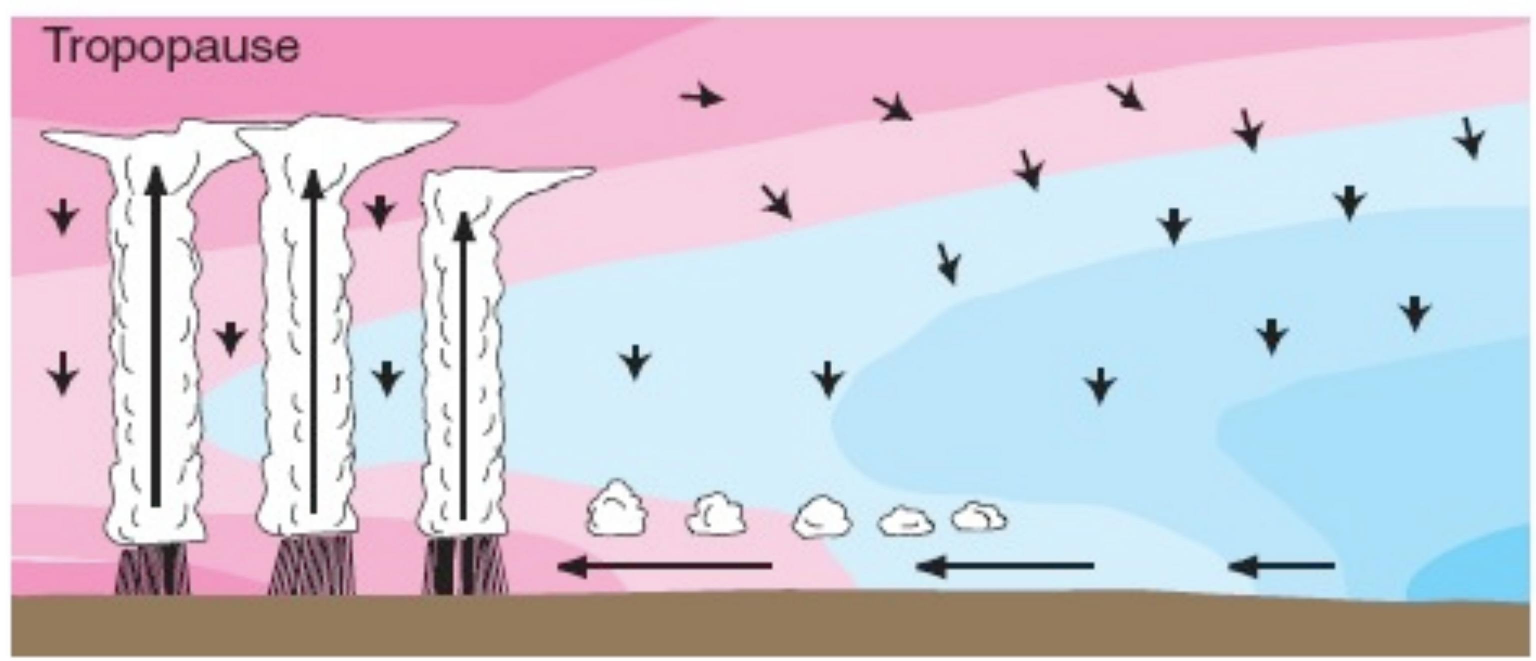
Cartesian geometry

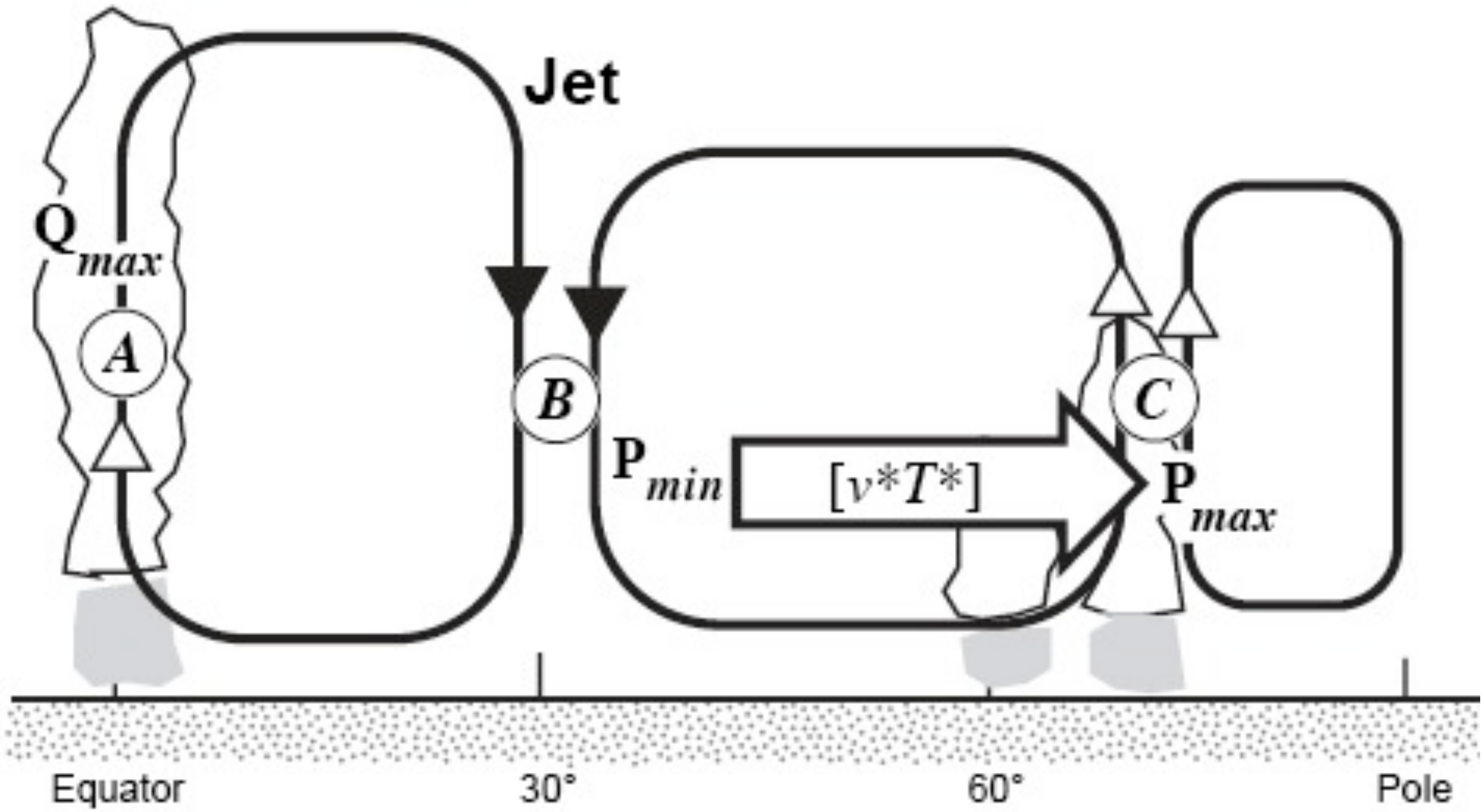
$$\frac{\partial[T]}{\partial t} = [\omega]\left(\frac{\kappa[T]}{p} - \frac{\partial[T]}{\partial p}\right) - [v]\frac{\partial[T]}{\partial y} - \frac{\partial}{\partial y}[v^*T^*] - \frac{\partial}{\partial p}[\omega^*T^*] + [\mathrm{Q}]$$

simplified

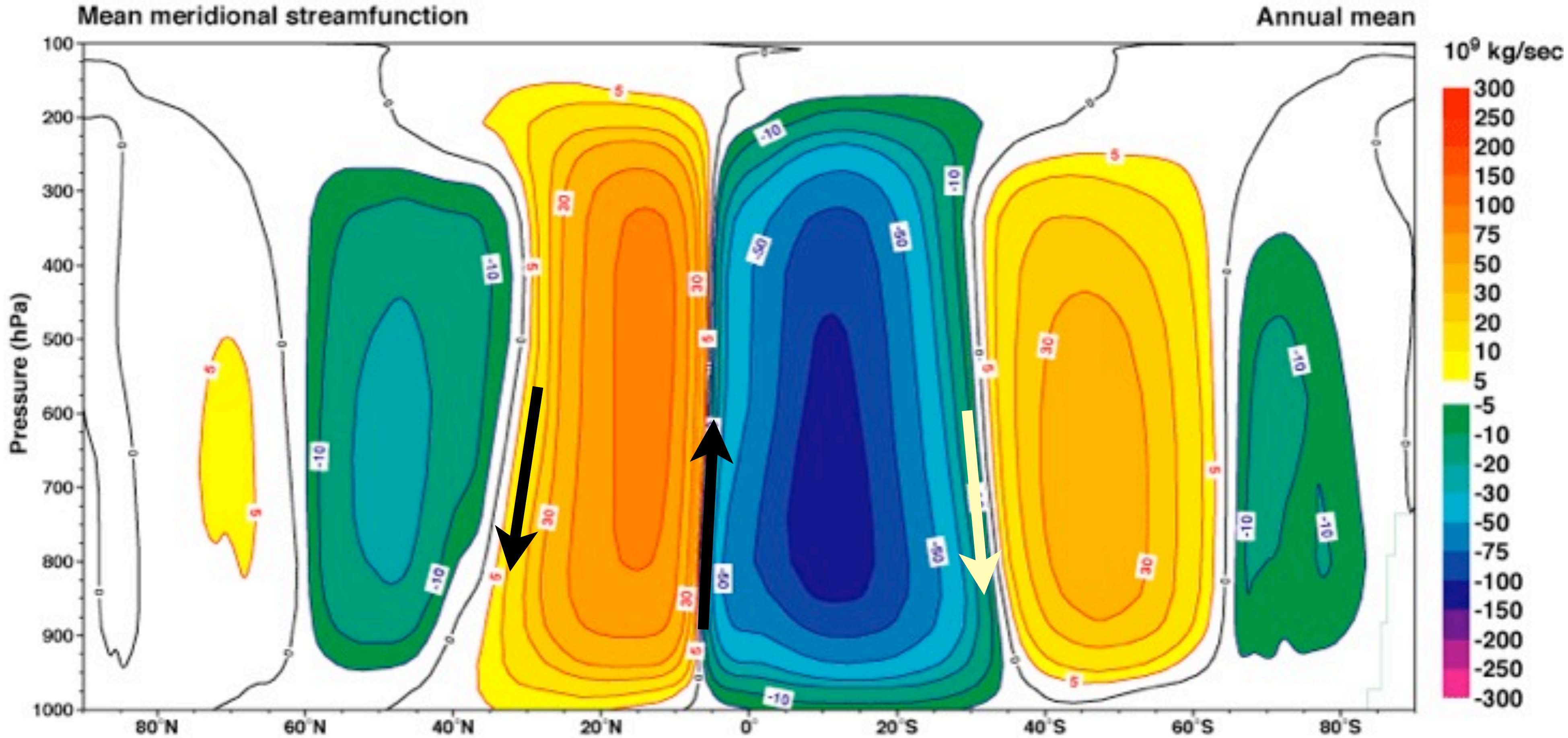
$$\frac{\partial[T]}{\partial t} = [\omega]\left(\frac{\kappa[T]}{p} - \frac{\partial[T]}{\partial p}\right) + \mathrm{P} + [\mathrm{Q}]$$


Zonal mean heating
Annual mean
K/day
Pressure (hPa)
100
200
300
400
500
600
700
800
900
1000
80°N
60°N
40°N
20°N
0°
20°S
40°S
60°S
80°S
10
7
5
4
3
2.5
2
1.5
1
0.7
0.3
0.1
-0.1
-0.3
-0.7
-1
-1.5
-2
-2.5
-3
-4
-5
-7
-10

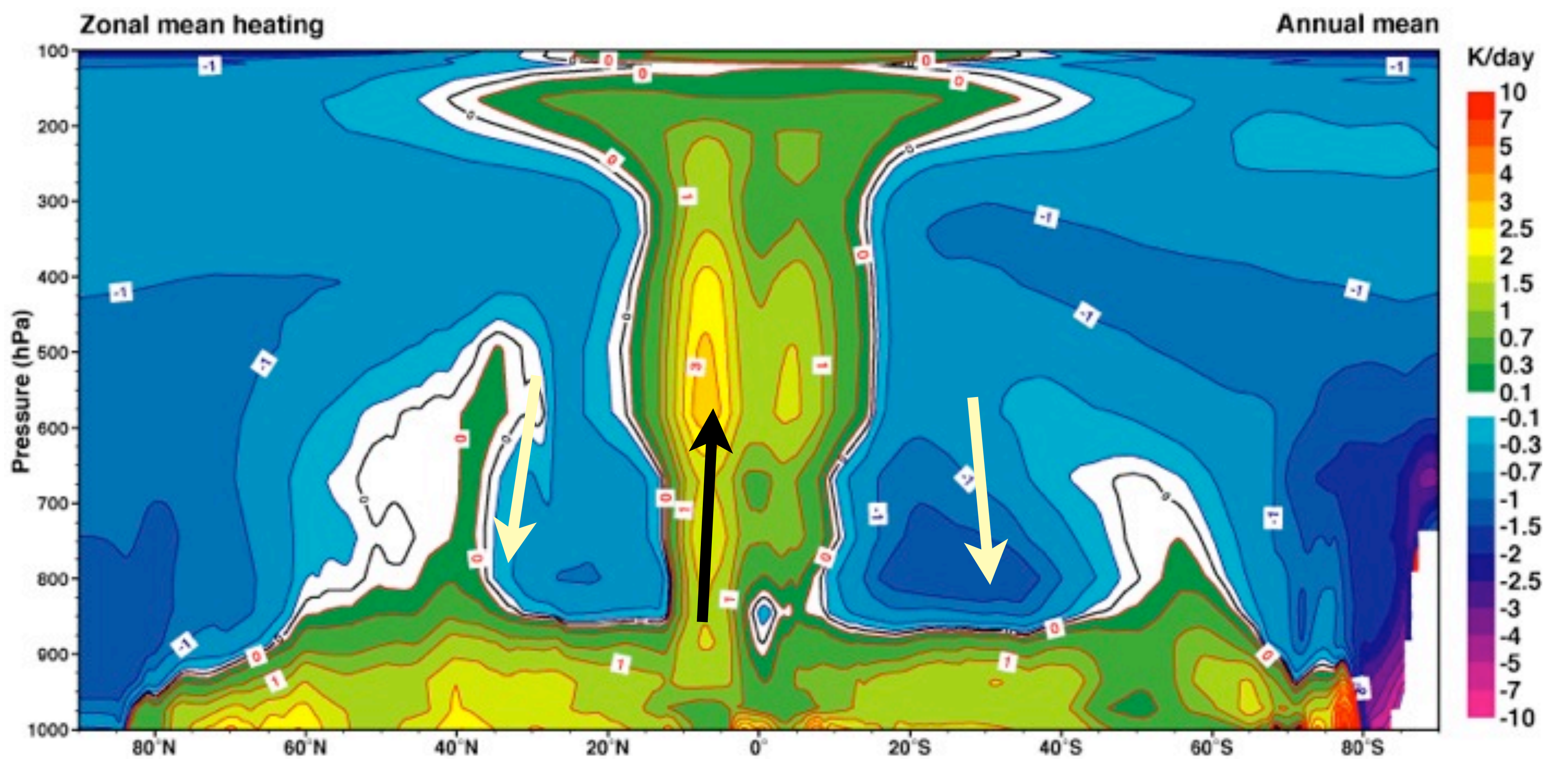

Tropopause



$$\frac{\partial[T]}{\partial t} = [\omega]\left(\frac{\kappa[T]}{p} - \frac{\partial[T]}{\partial p}\right) + \mathrm{P} + [\mathrm{Q}]$$

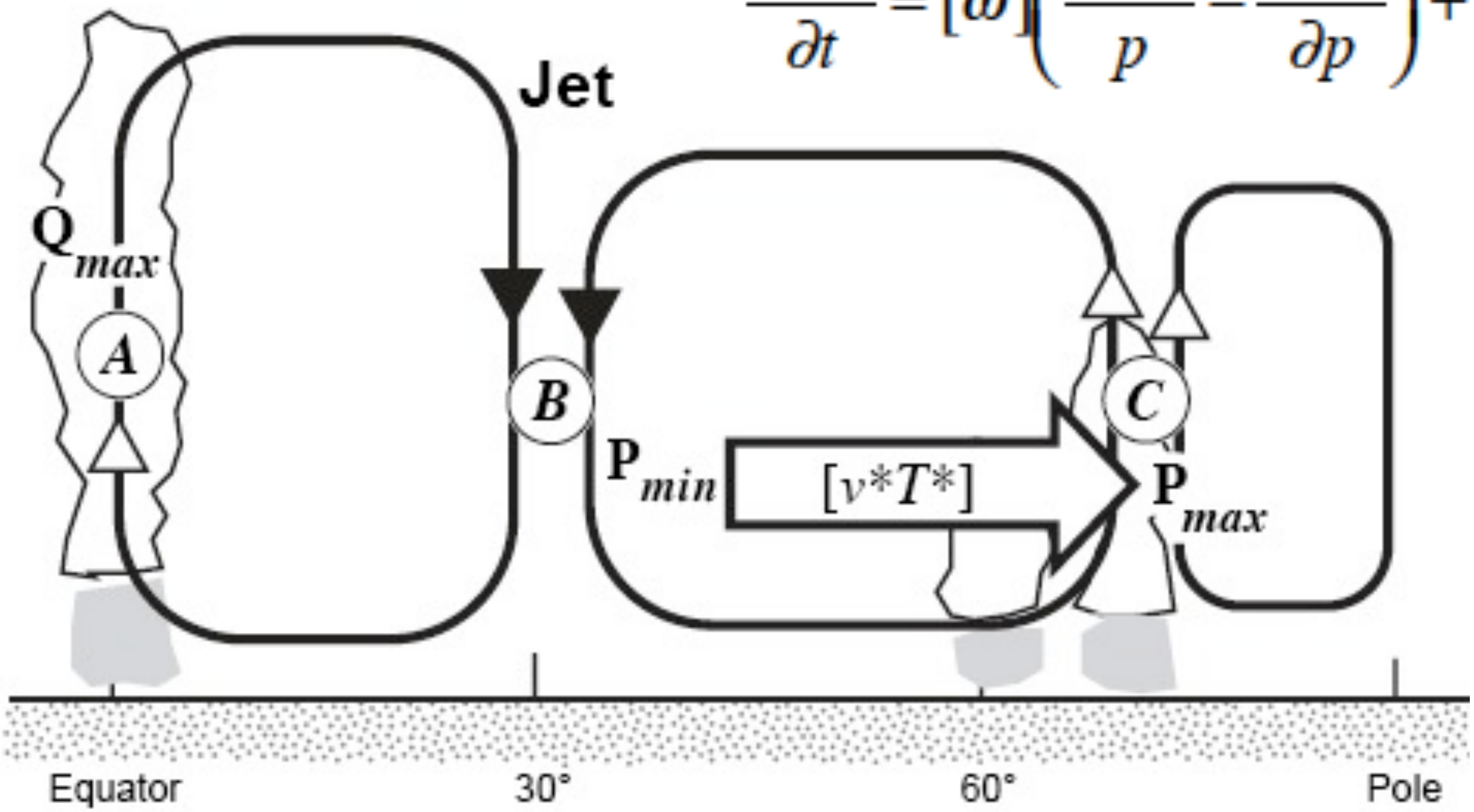


Mean meridional streamfunction
Annual mean
10^9 kg/sec
Pressure (hPa)



Zonal mean heating
Annual mean
K/day
Pressure (hPa)

$$\frac{\partial[T]}{\partial t} = [\omega]\left(\frac{\kappa[T]}{p} - \frac{\partial[T]}{\partial p}\right) + \mathrm{P} + [\mathrm{Q}]$$



$$A \quad Q_L = -\sigma\omega$$

$$B \quad Q_R + P_{min} = -\sigma\omega$$

$$C \quad Q_L + P_{max} = -\sigma\omega$$

MMC effective
at horizontal transport

$$\frac{\partial [u]}{\partial t} = [v]\left(f - \frac{\partial [u]}{\partial y}\right) + G + F_x$$

dynamic stability

$$\frac{\partial [T]}{\partial t} = [\omega]\left(\frac{\kappa [T]}{p} - \frac{\partial [T]}{\partial p}\right) + \mathrm{P} + [\mathrm{Q}]$$

static stability

Note the similarity in structure of the two equations

Role of the hydrologic cycle

$$MSE = c_p T + Lq + \Phi$$

c_p is the specific heat of dry air at constant p, 1004 J / kg

L is the latent heat of vaporization, 2.5×10^6 J / kg

q is the specific humidity, dimensionless

Φ is geopotential height

The mass balance for water vapor

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathbf{Q} = E - P$$

$$\mathbf{Q} = \frac{1}{g} \int_0^{p_0} q\mathbf{V}\, dp$$

the vertically-integrated moisture transport

$$\overline{\mathbf{Q}} = \mathbf{Q}_M + \mathbf{Q}_T$$

$$\mathbf{Q}_M = \frac{1}{g} \int_0^{p_0} \overline{q}\,\overline{\mathbf{V}}\, dp$$

$$\mathbf{Q}_T = \frac{1}{g} \int_0^{p_0} \overline{q'\mathbf{V}'}\, dp$$

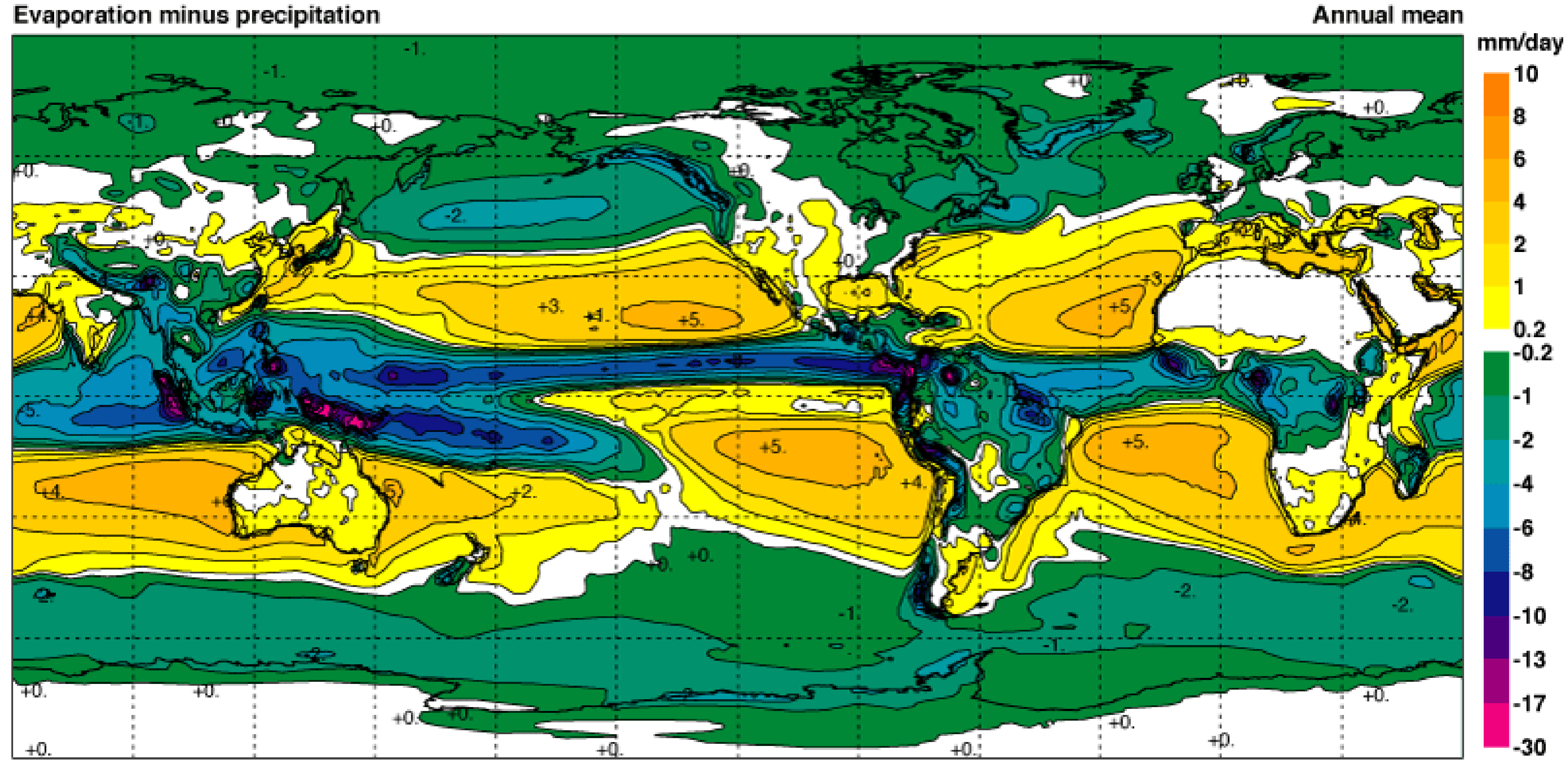
$$\frac{\partial W}{\partial t} + \nabla \cdot \mathbf{Q} = E - P$$

Averaged over a season

$$\frac{\partial W}{\partial t} = 0$$

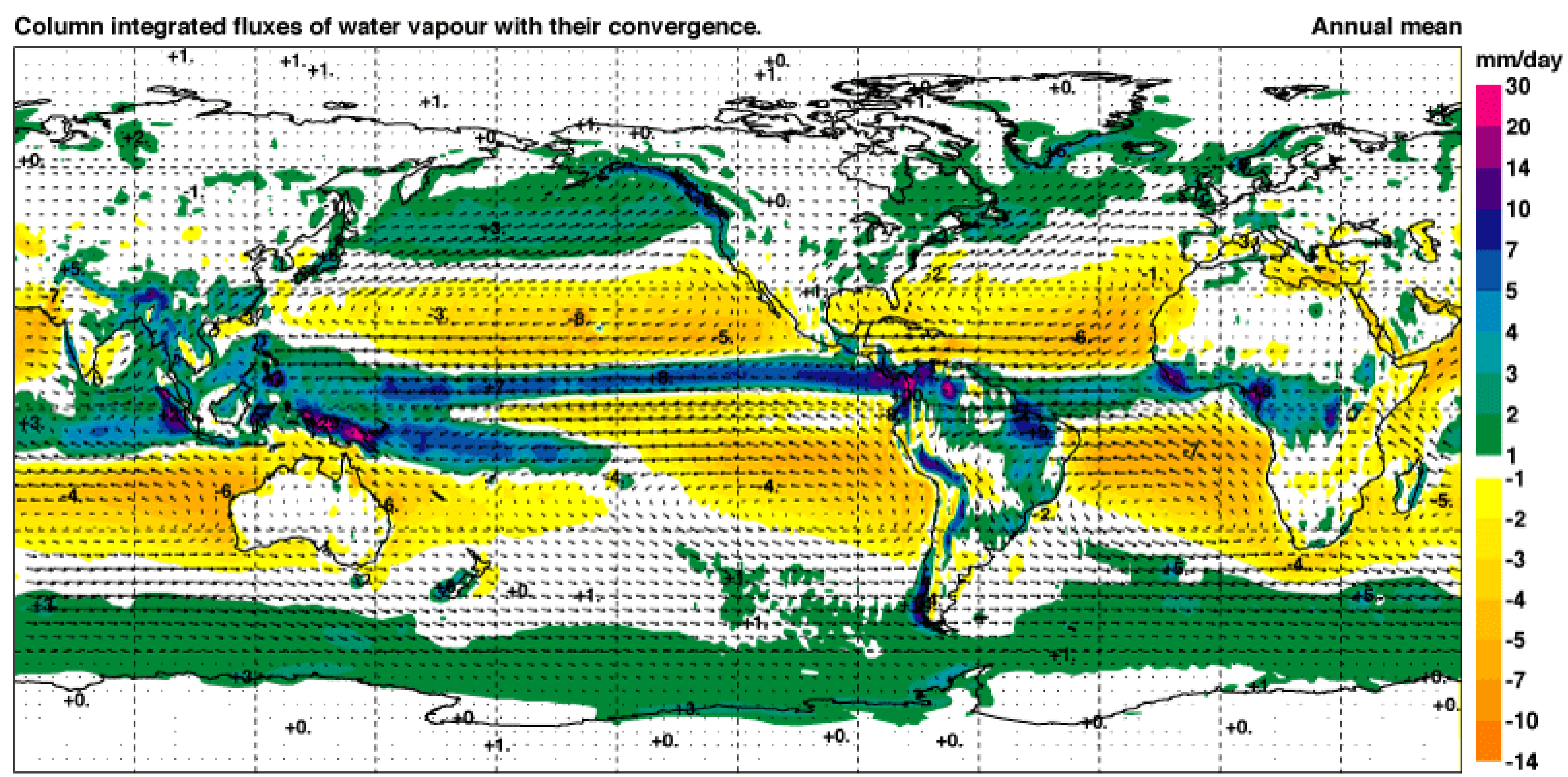
$$\nabla \cdot \mathbf{Q} = E - P$$

$$E - P$$



$$\frac{\partial W}{\partial t} + \nabla \cdot \mathbf{Q} = E - P$$

$$\nabla \cdot \mathbf{Q}$$



ERA-40 data

$$\nabla \cdot \mathbf{Q} = E - P$$

Approximations
(voluntary)

$$\mathbf{Q} = \frac{1}{g}\int_0^{p_0} q\mathbf{V}\,dp \quad \sim \widehat{q}\widehat{\mathbf{V}}\frac{\delta p}{g}$$

"slab"

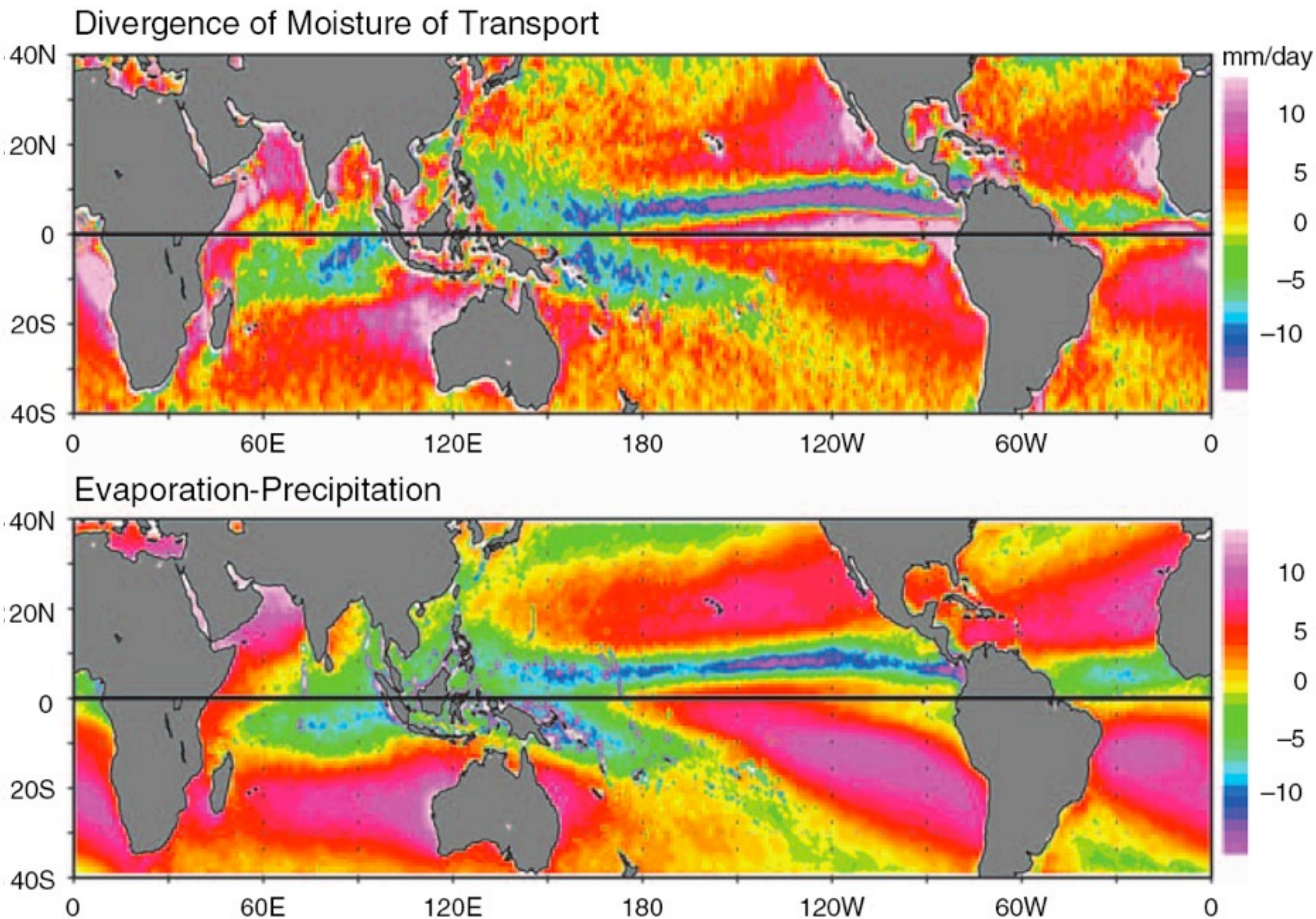
$$\nabla \cdot q\mathbf{V} = q\nabla \cdot \mathbf{V} + \mathbf{V} \cdot \nabla q$$

ignore advection

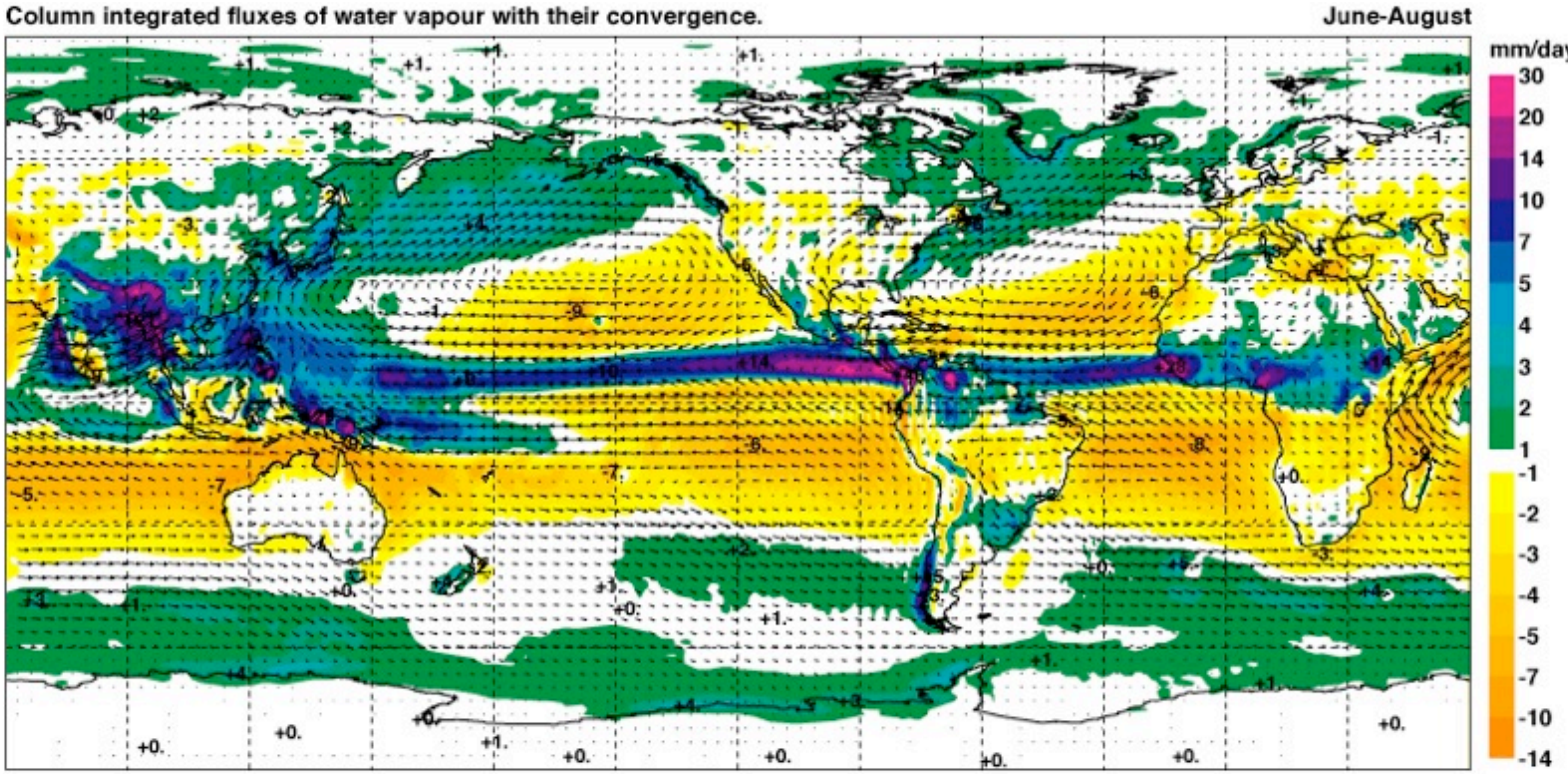
$$\nabla \cdot \mathbf{Q} \sim \nabla \cdot \mathbf{V}\left(\frac{q_0 \delta p}{g}\right)$$

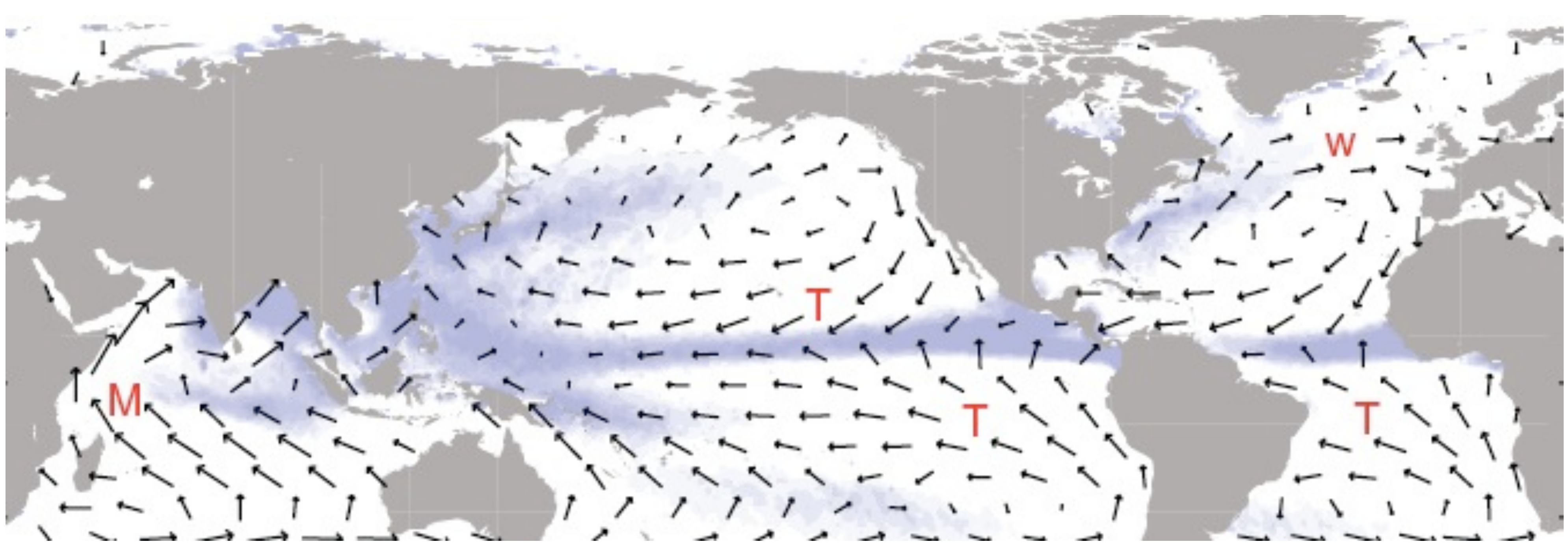
approximate slab wind by surface wind

Water vapor   Annual-mean conditions   (satellite data)



Boreal summer

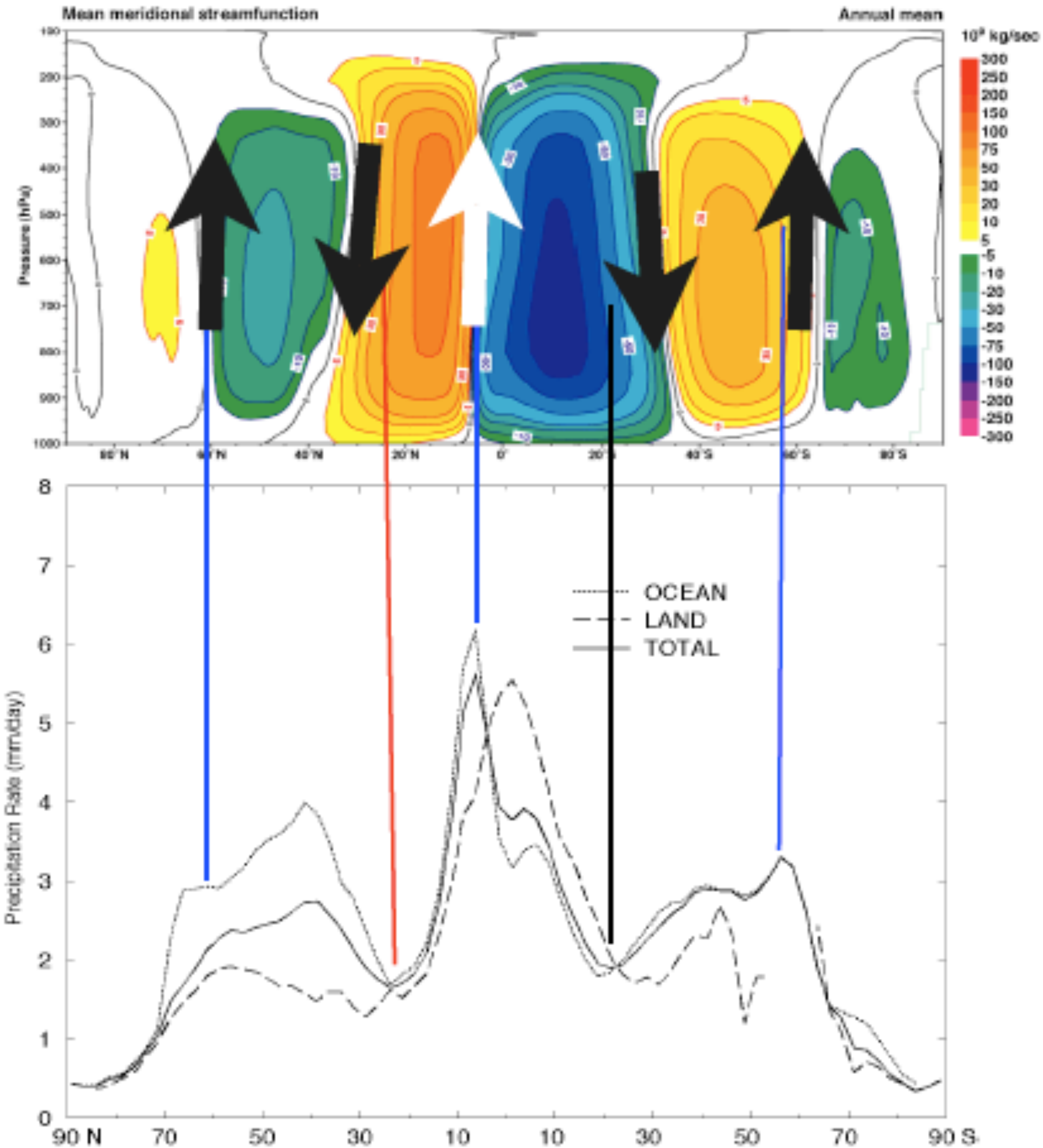




in the zonal average

$$\frac{1}{\cos\phi}\frac{\partial}{\partial y}\left[\mathbf{Q}\right]\cos\phi = \left[E\right] - \left[P\right]$$

Role of the MMC in water vapor transport



On land surface

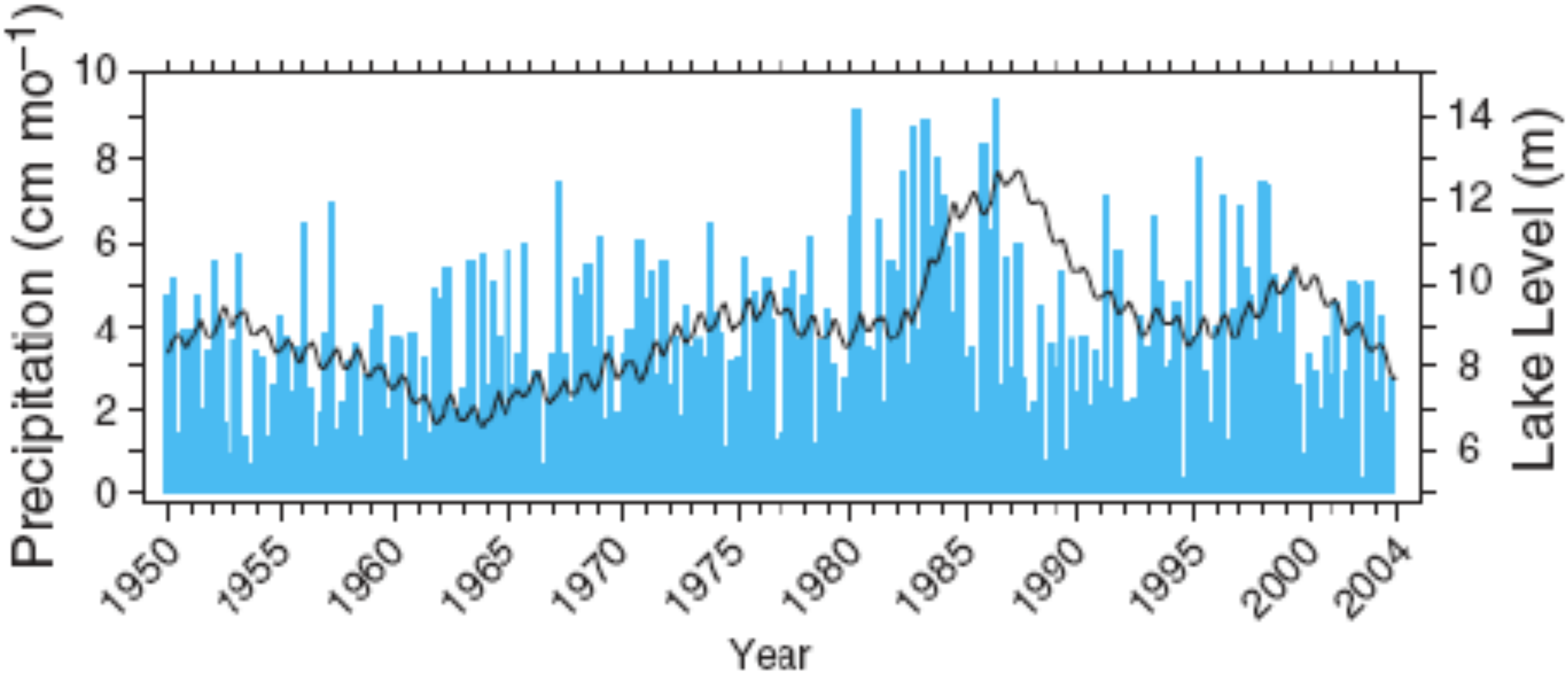
$$P - E = \frac{\partial}{\partial t} Storage + R$$

Land + Atmosphere

$$\frac{\partial}{\partial t} Storage + R = -\nabla \cdot \mathbf{Q}$$

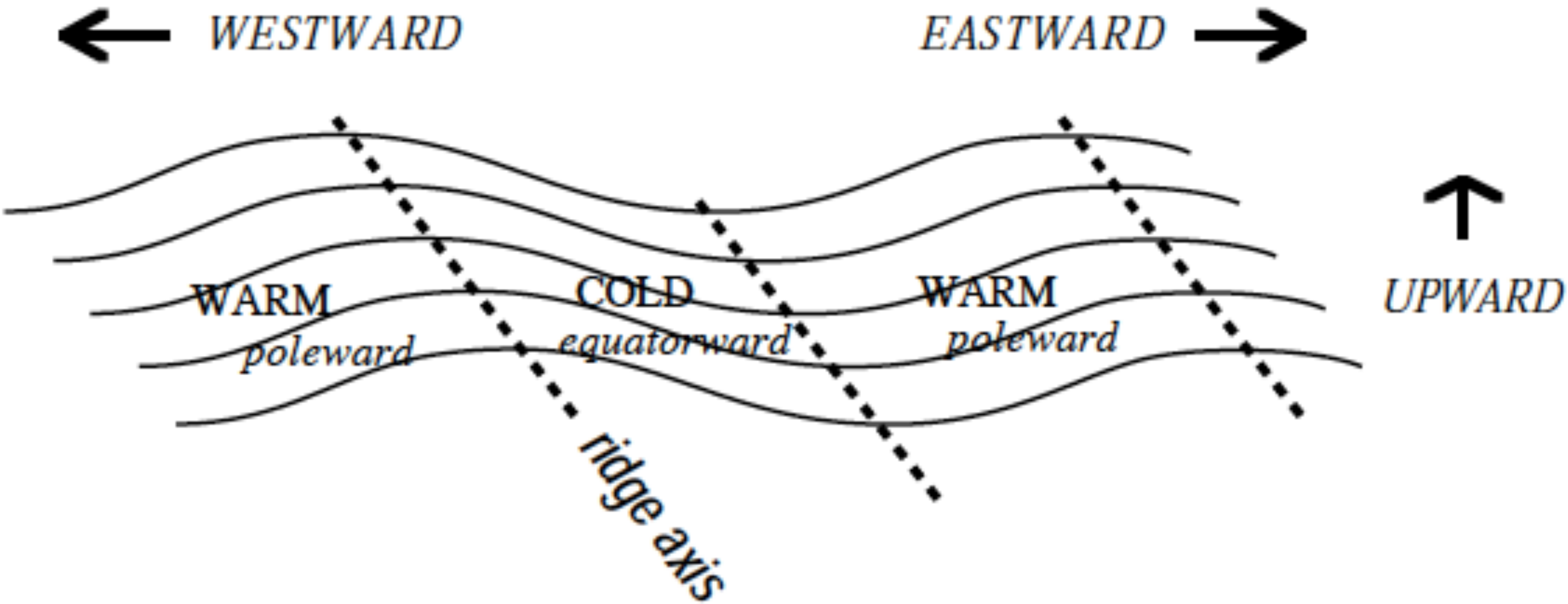
Land-locked basin

$$\frac{\partial}{\partial t} Storage = -\nabla \cdot \mathbf{Q} = P - E$$

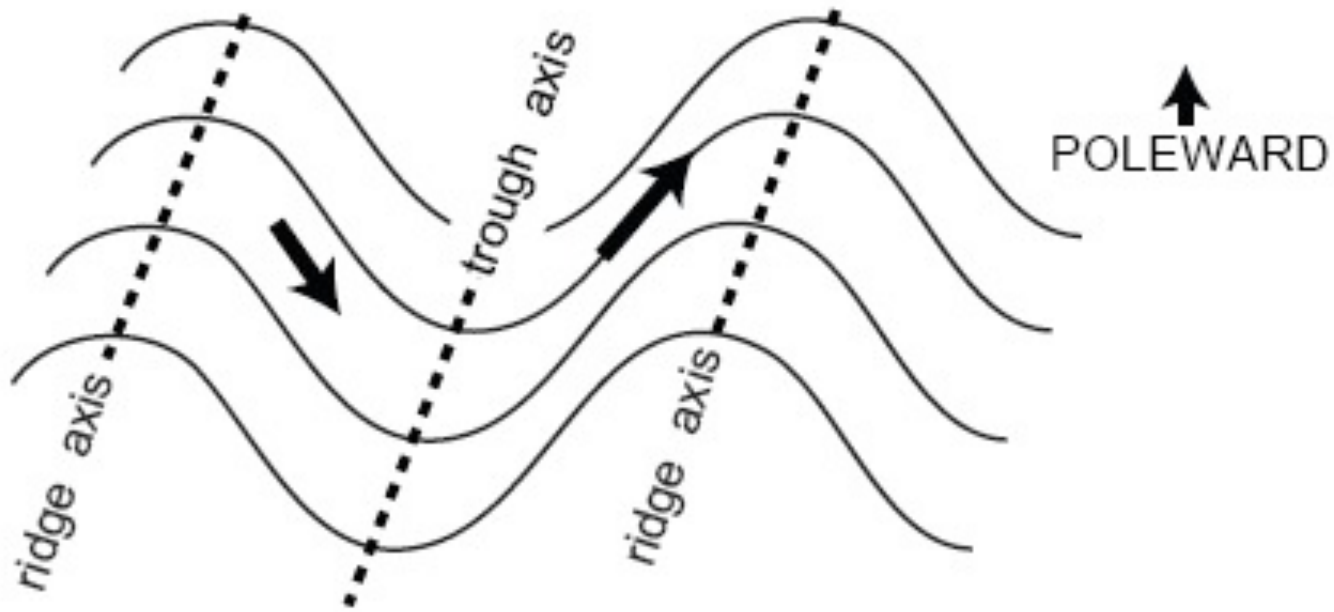


$$\frac{\partial}{\partial t} Storage + R = -\nabla \cdot \mathbf{Q}$$

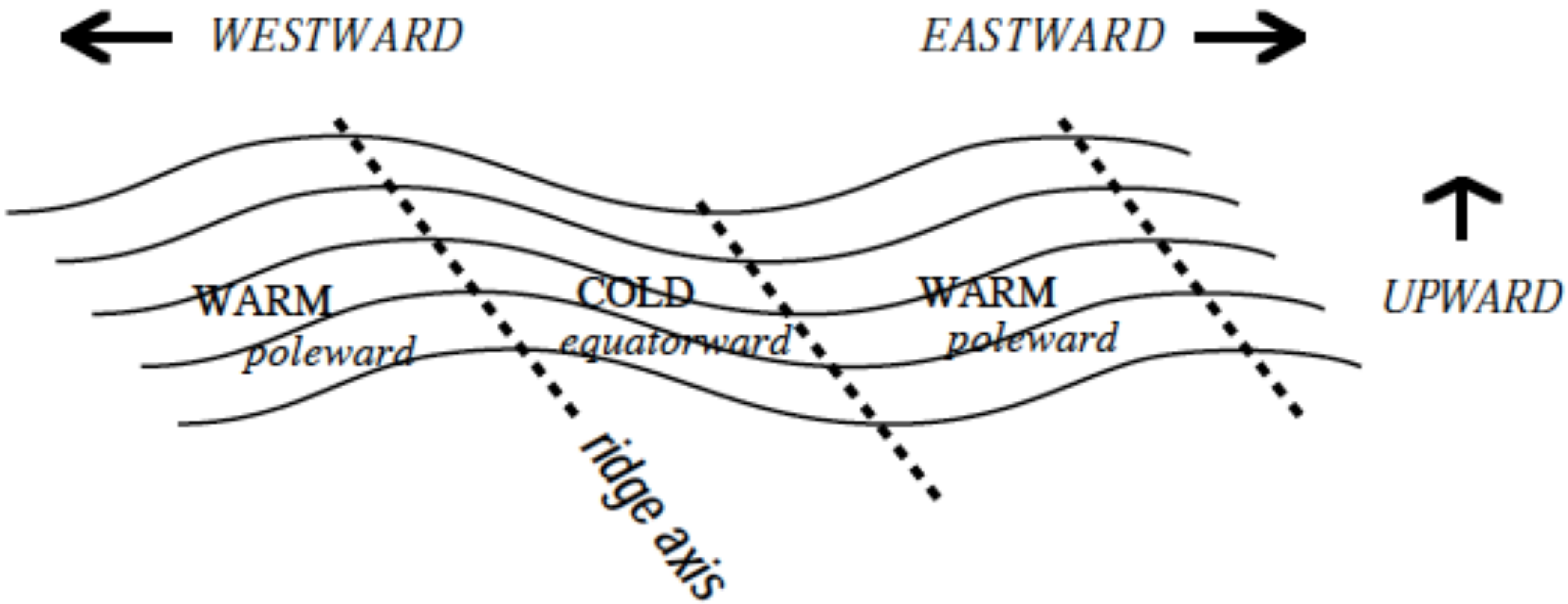
Closed drainage basin: $R = 0$



In waves that transport heat poleward
the ridges and troughs tilt westward with height



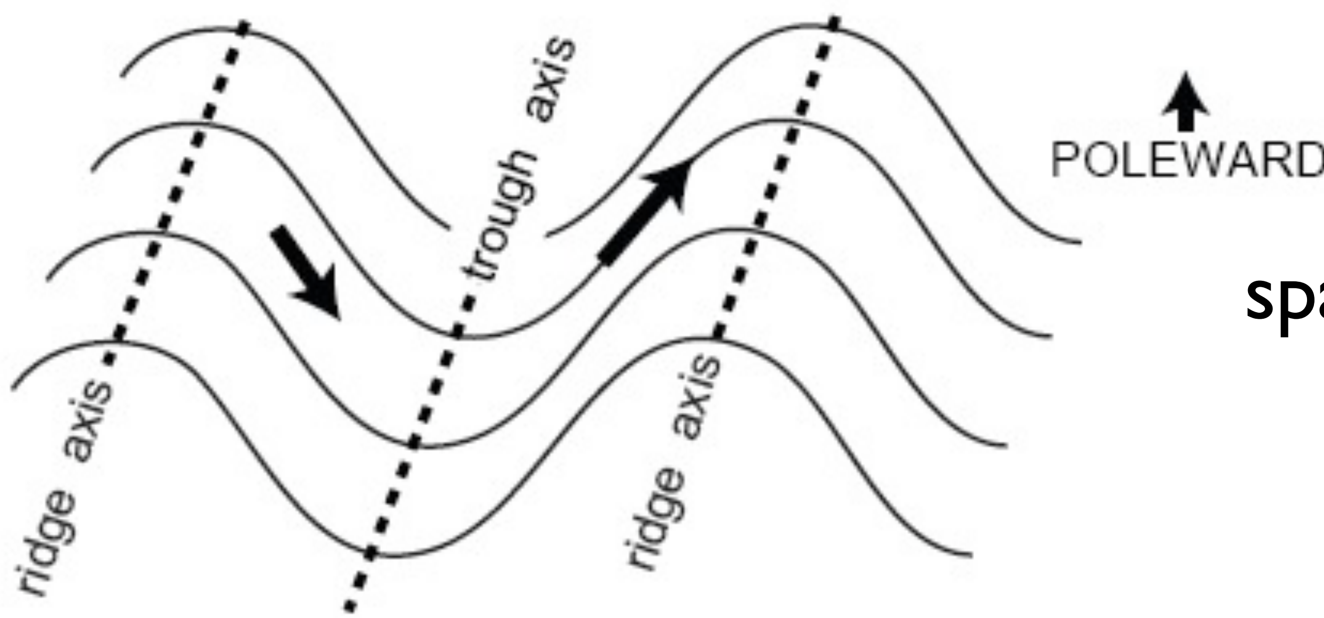
$$[u^* v^*]$$



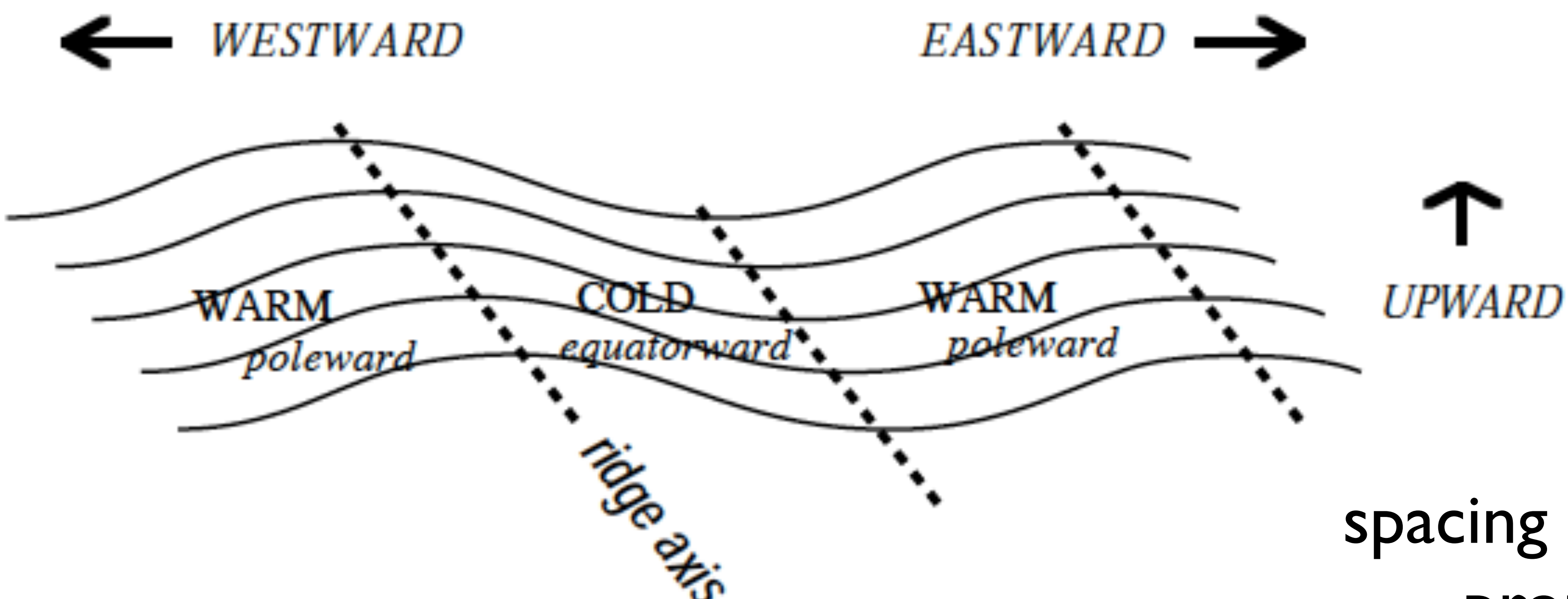
$$[v^* T^*]$$

An interesting parallel

It's not chance coincidence



spacing between contours proportional to u



spacing between contours proportional to T

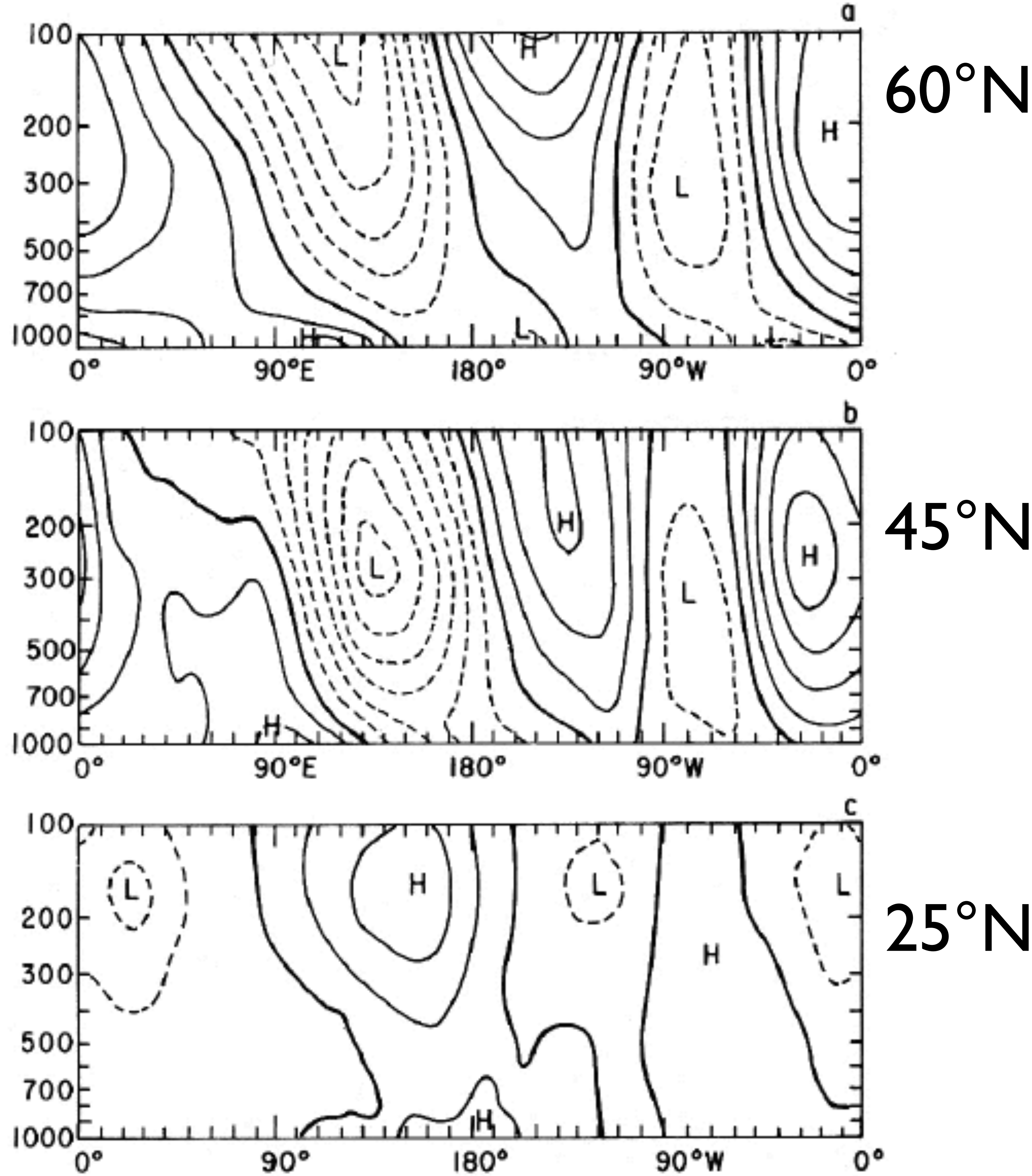
The math is identical

NH wintertime stationary waves

geopotential height

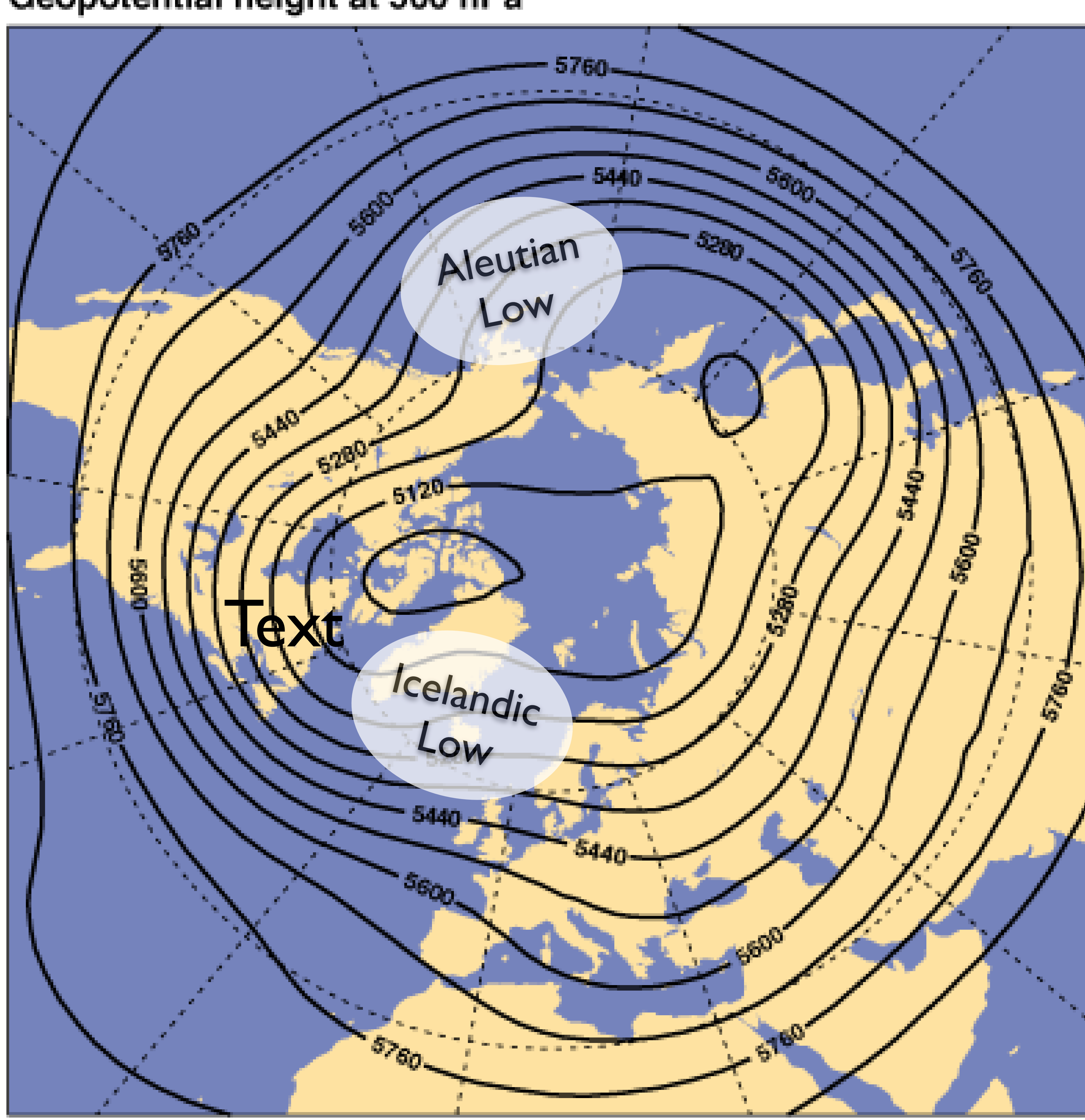
Z^*

Note westward tilt with height



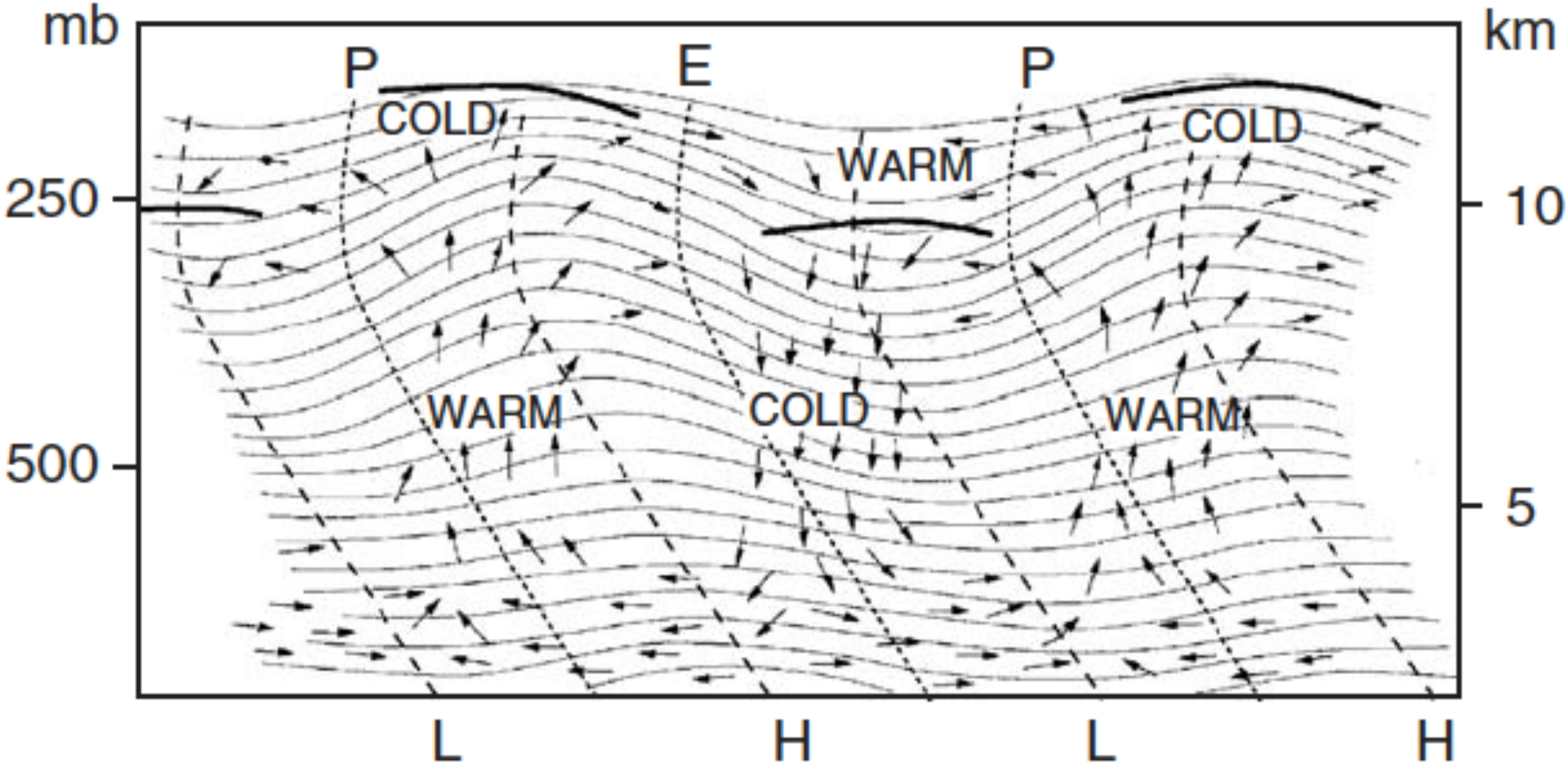
NH wintertime stationary waves

Z_{500}



Note westward tilt with height

Idealized baroclinic waves



tilt westward with height

The eddy flux of geopotential $[v^*\Phi^*]$

second order term in the meridional transport of moist static energy

directed opposite to transport of zonal momentum

work term in kinetic energy cycle

The eddy flux of geopotential $\left[v^*\Phi^*\right]$

second order term in the meridional transport of moist static energy

compare

$$c_p\, r\left(v,T\right)\sigma(v)\,\sigma(T) \quad \text{with} \quad r\left(v,\Phi\right)\sigma(v)\,\sigma(\Phi)$$

$$r\left(v,T\right) > r\left(v,\Phi\right) \quad \text{and} \quad c_p\sigma(T^*) > g\sigma(Z^*)$$

factor of 3 *factor of 3*

Hence, $c_p\left[v^*T^*\right] \gg \left[v^*\Phi^*\right]$

The eddy flux of geopotential $[v^*\Phi^*]$

is directed opposite to the transport of westerly momentum

First prove that $[v^*\Phi^*] = [v_a^*\Phi]$

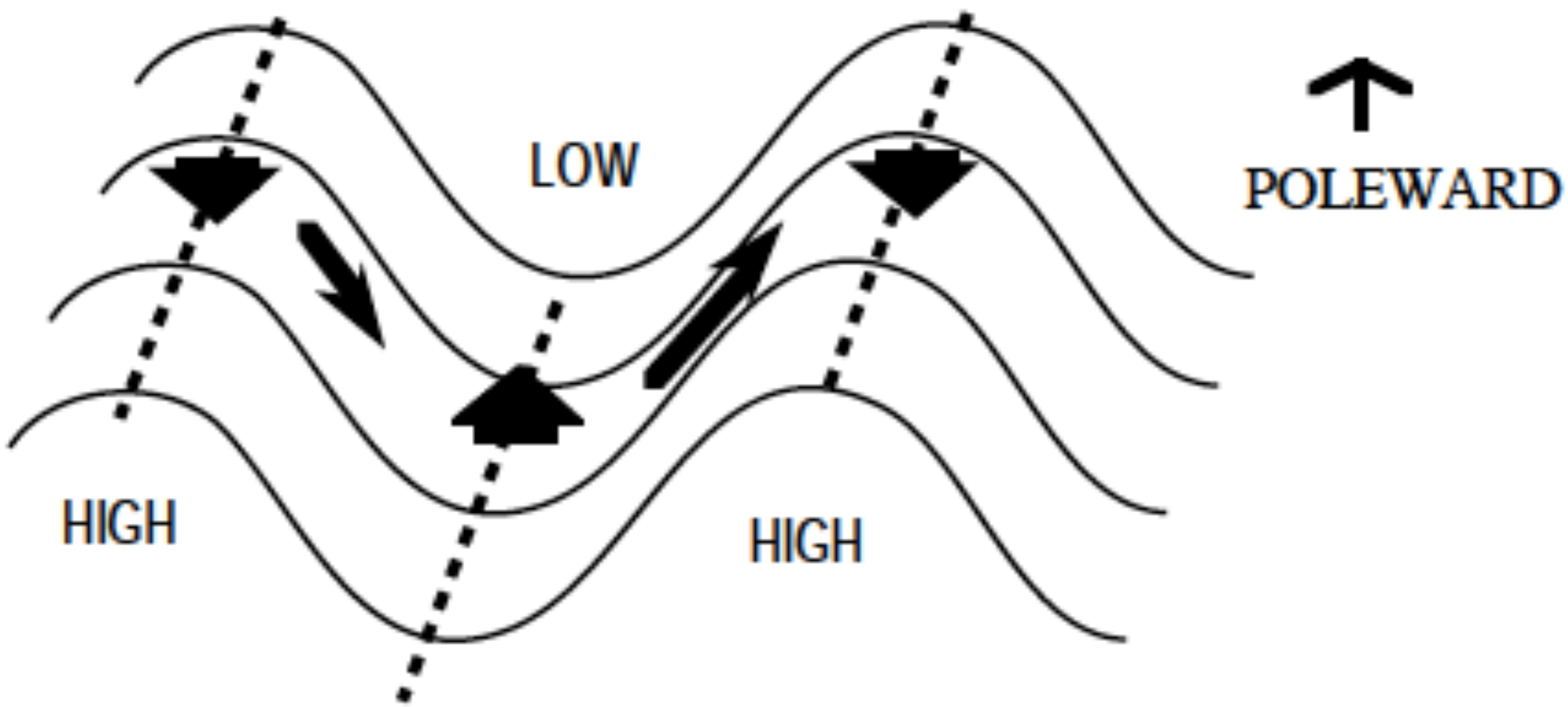
i.e., that the flux of geopotential is accomplished entirely by the ageostrophic component of *v*

$$[v_g^*\Phi^*] = [\Phi^*\partial\Phi^*/\partial x]/f$$

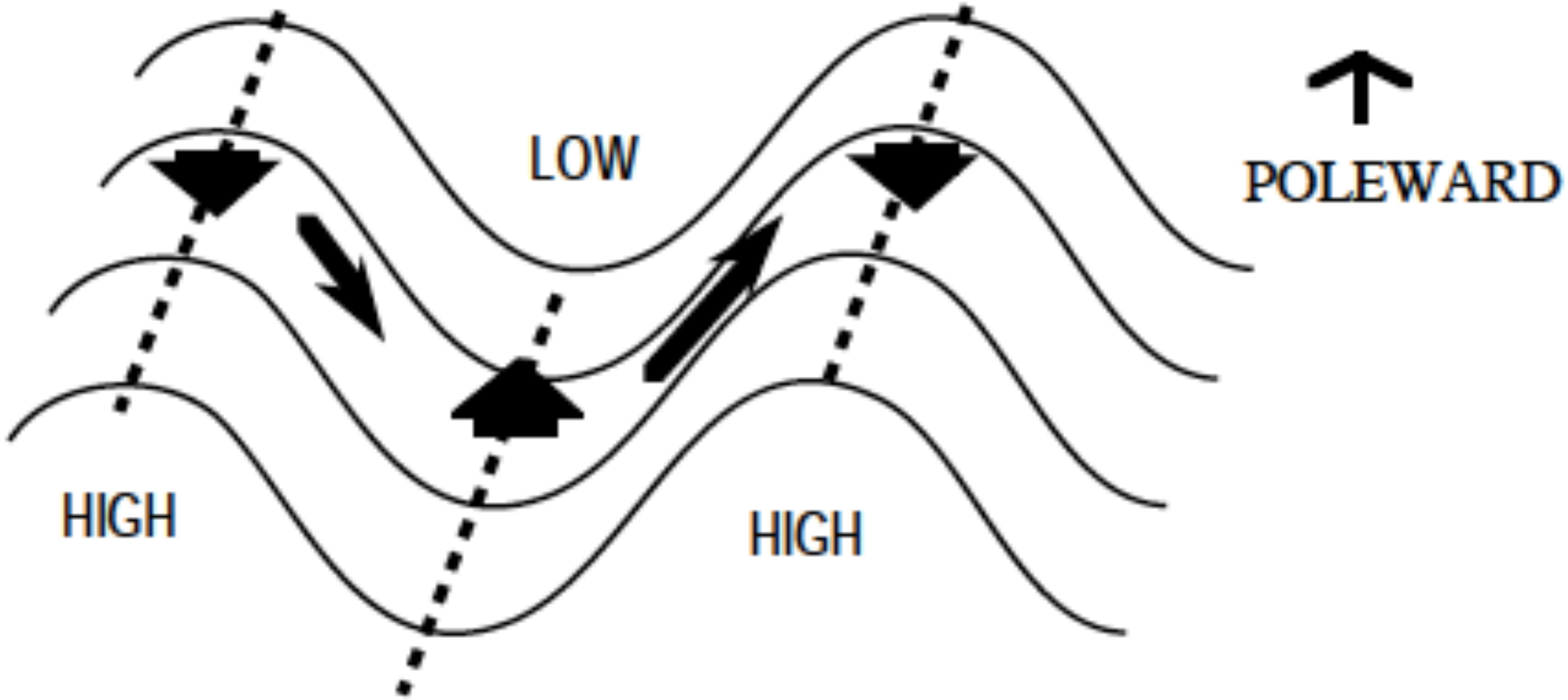
$$= \partial/\partial x[\Phi^{*2}/2]/f$$

$$= 0$$

The eddy flux of geopotential



and the flux of zonal momentum are in the opposite direction



for stationary waves

$$\frac{Du^*}{Dt} = [u]\frac{\partial u^*}{\partial x} = f v_a^*$$

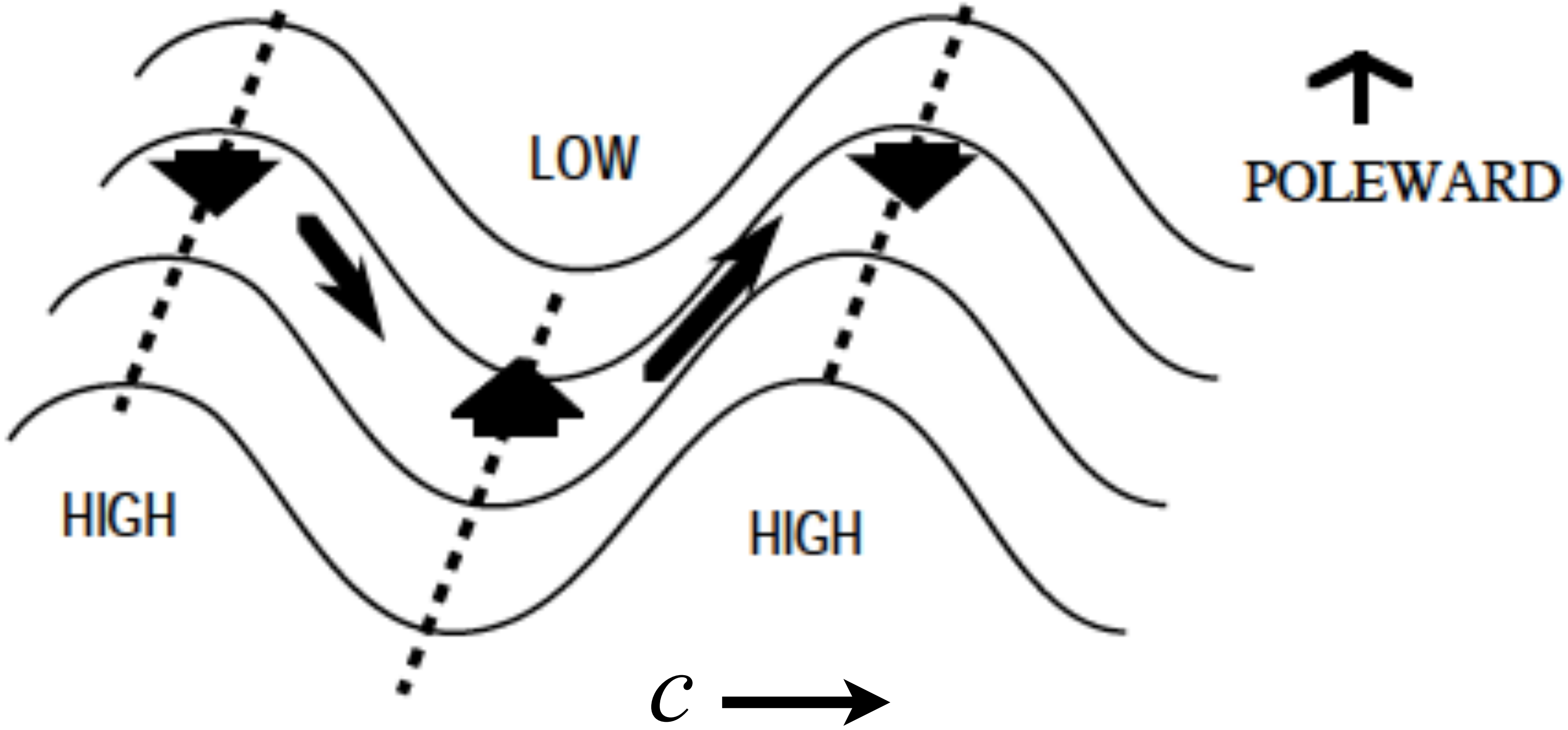
D/Dt Lagrangian time derivative

multiplying by Φ^*, zonally averaging, and using the identity

$$\left[\Phi^* \frac{\partial u^*}{\partial x}\right] = -\left[u^* \frac{\partial \Phi^*}{\partial x}\right] = -f[u^* v^*]$$

we obtain

$$[v_a^* \Phi^*] = -[u][u^* v^*]$$

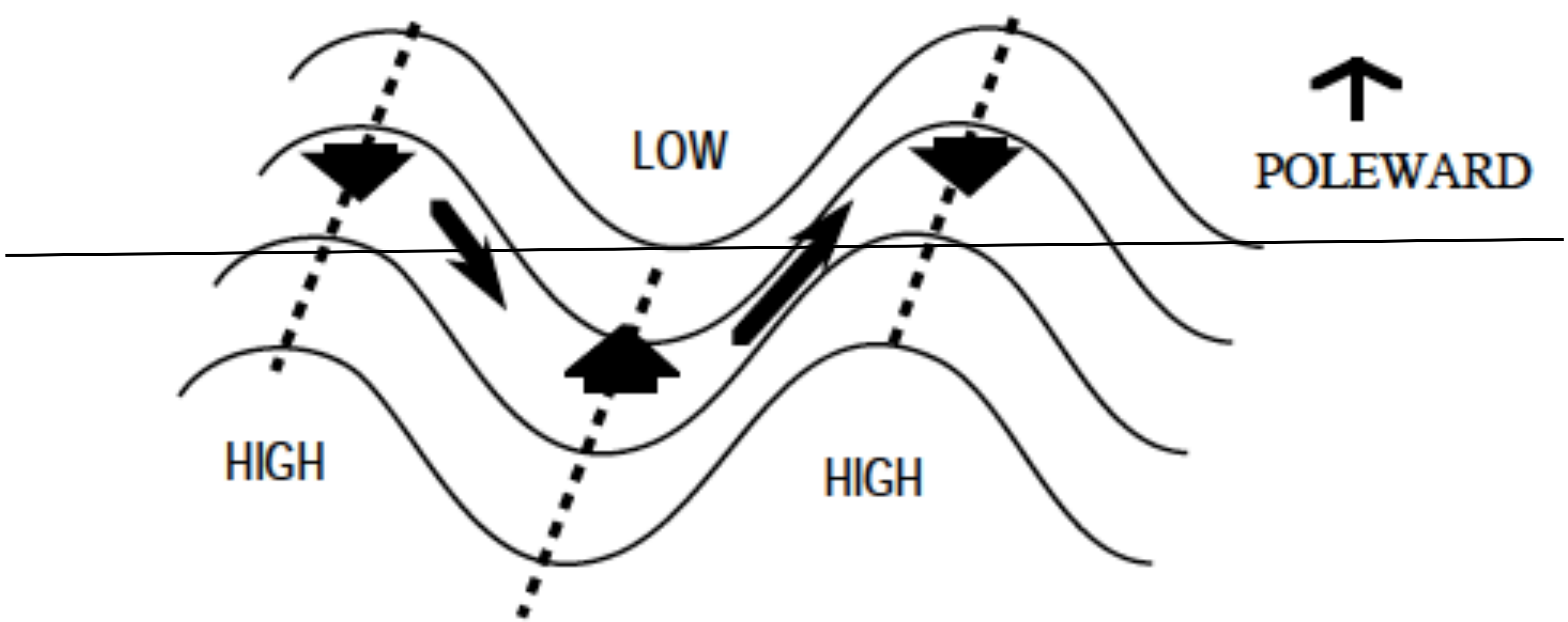


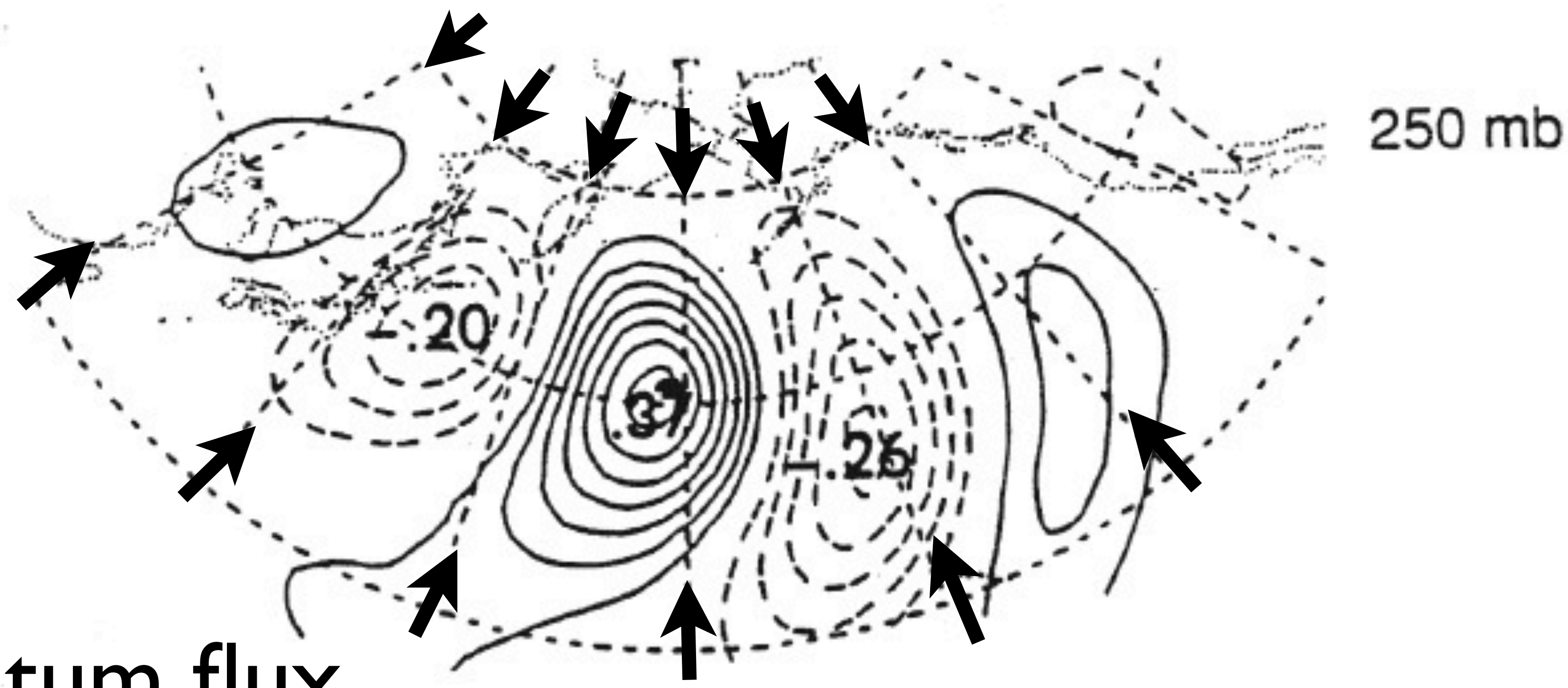
for waves propagating zonally with phase velocity c
replace $-u$ by $-(u-c)$

Above ~700 hPa $u > c$

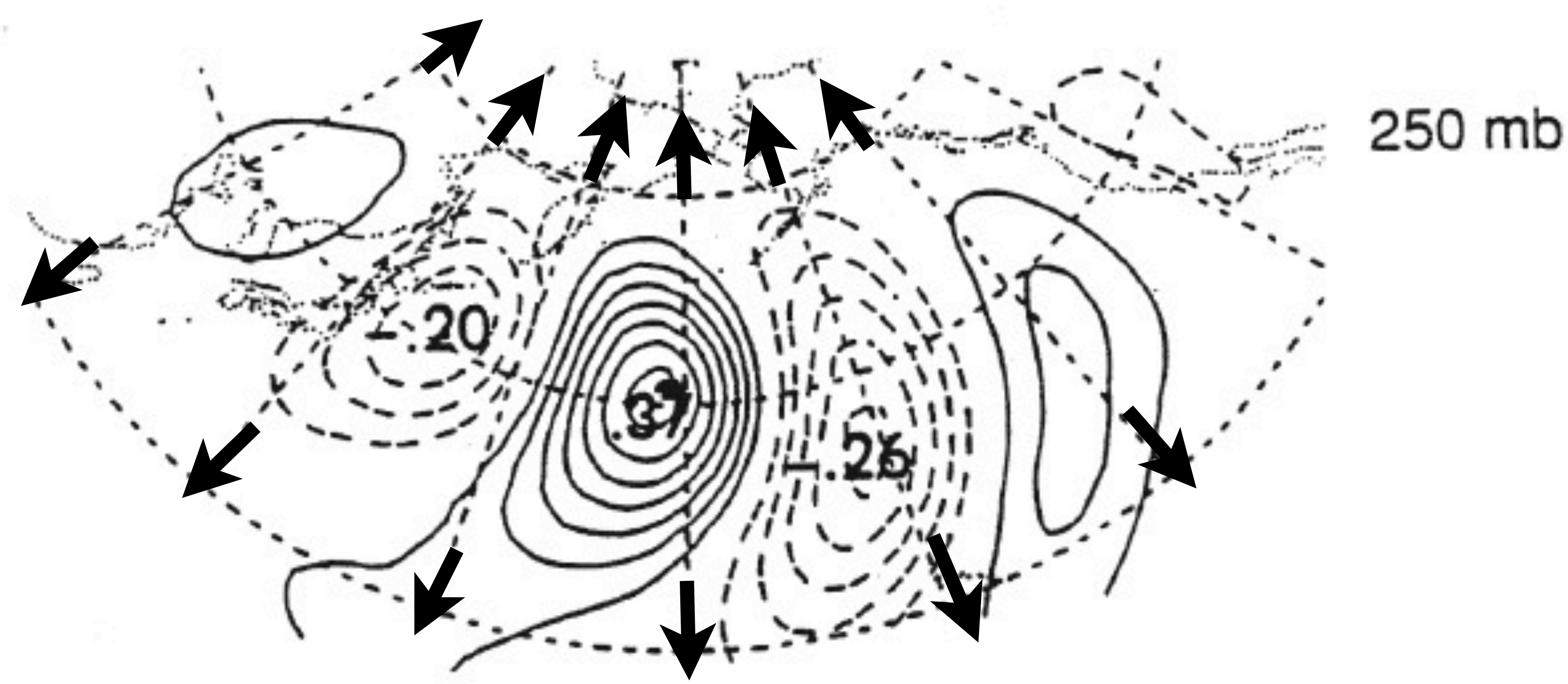
The eddy flux of geopotential $[v^*\Phi^*]$

work term in kinetic energy cycle by which the eddies equatorward of the latitude circle do work on the eddies poleward of it.

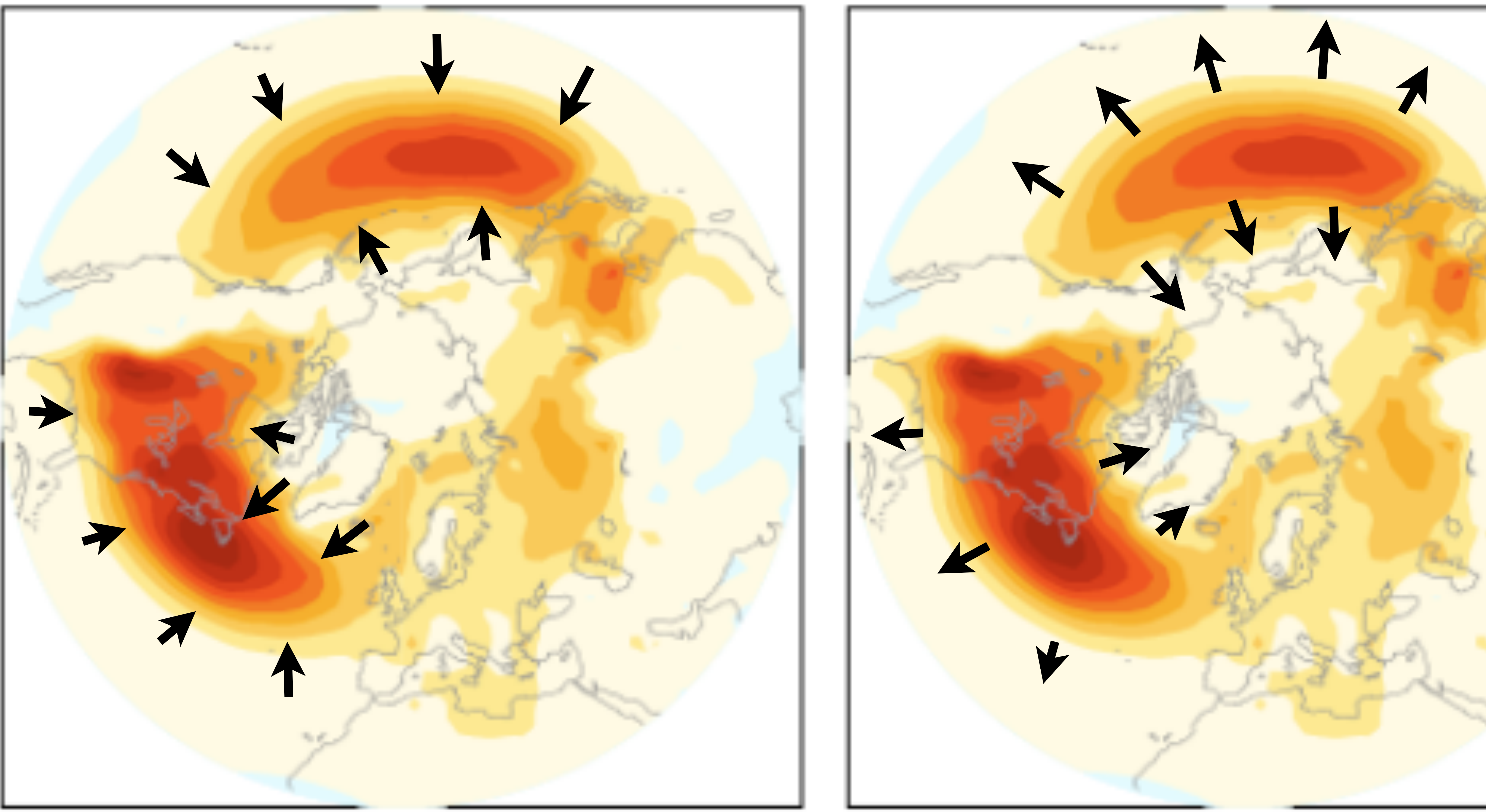




momentum flux



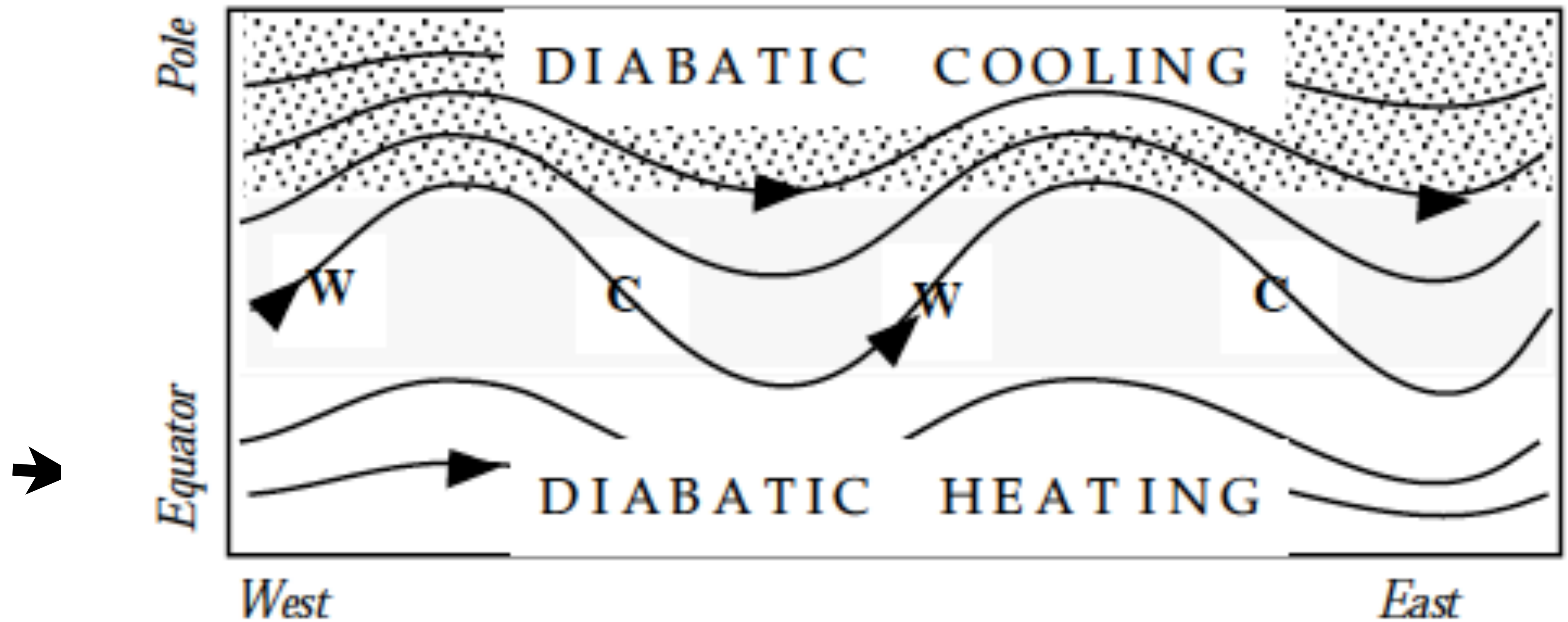
geopotential flux



momentum flux

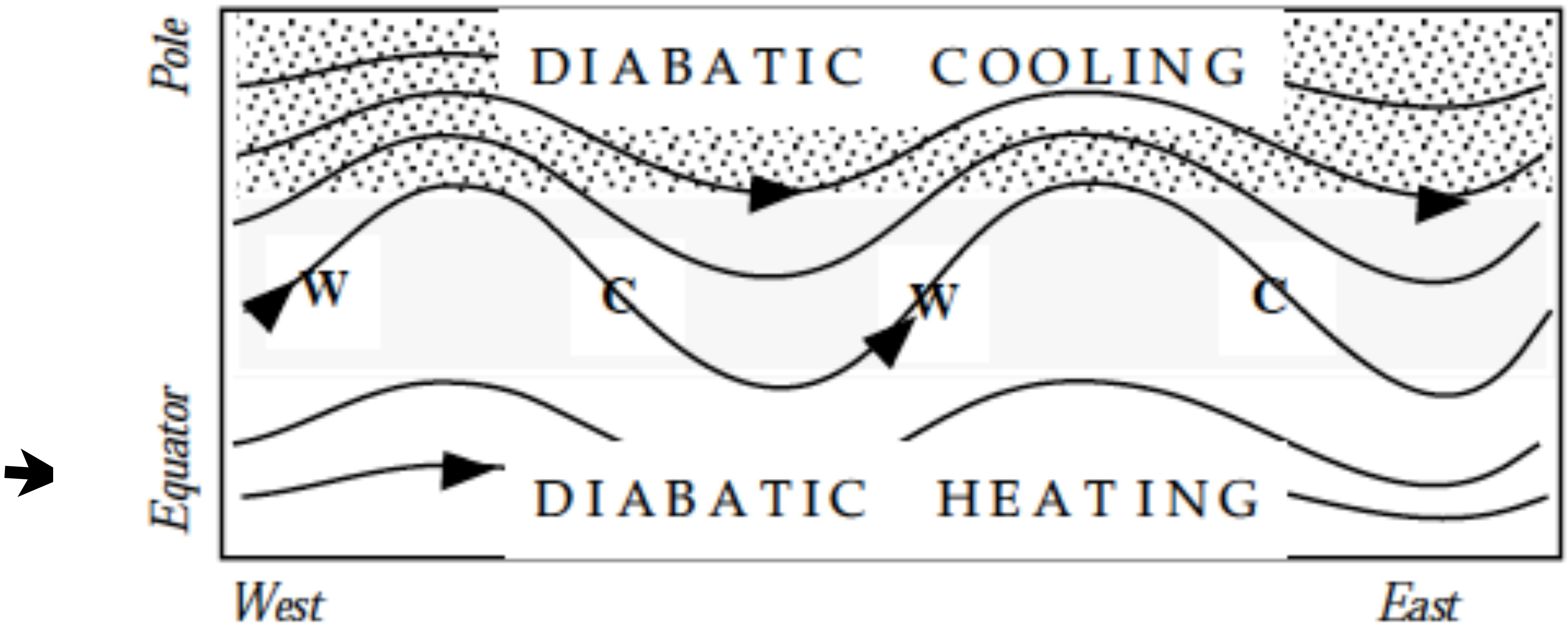
geopotential flux

Do the eddies induce diabatic heating?



$[v^*T^*]$ keeps $\partial[T]/\partial y$ from reaching thermal equilibrium and in this sense it induces $\partial[Q]/\partial y$

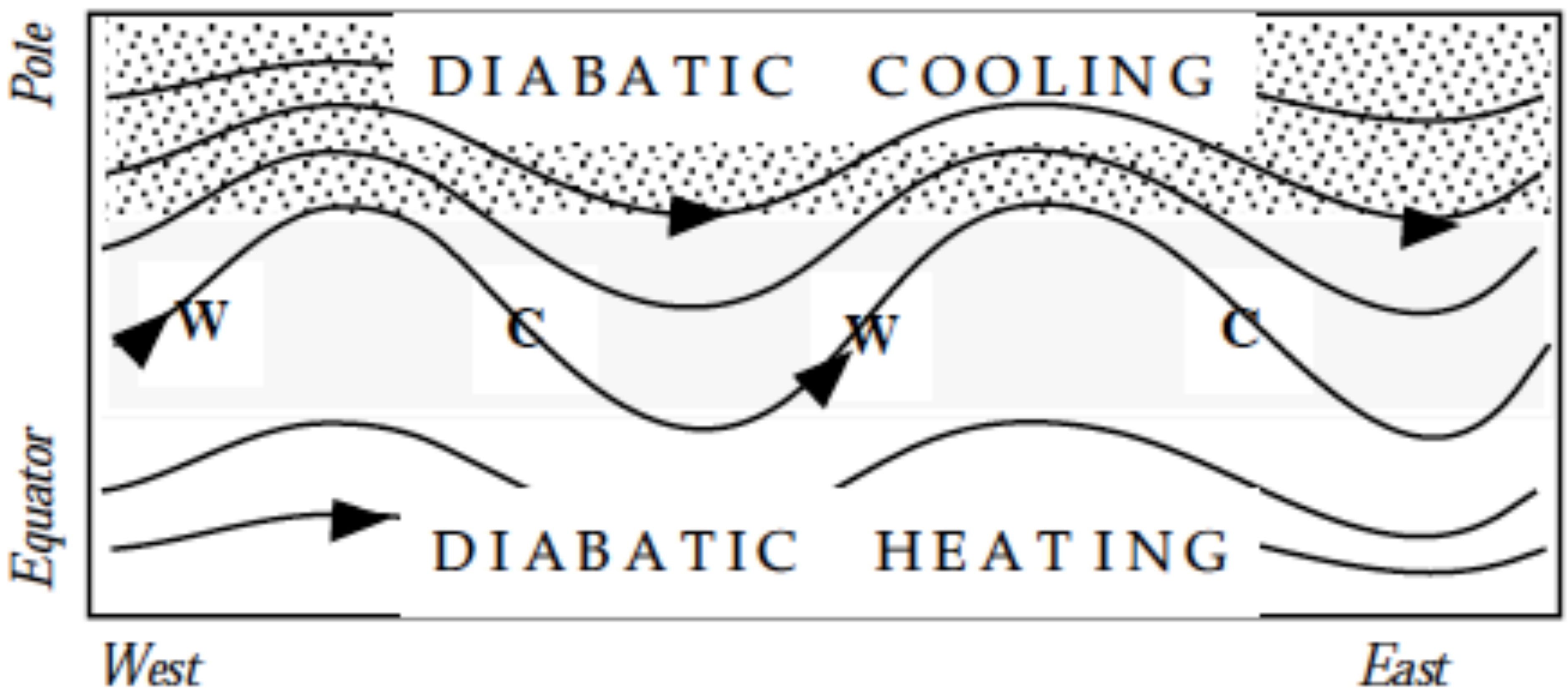
or does diabatic heating induce eddy heat transports



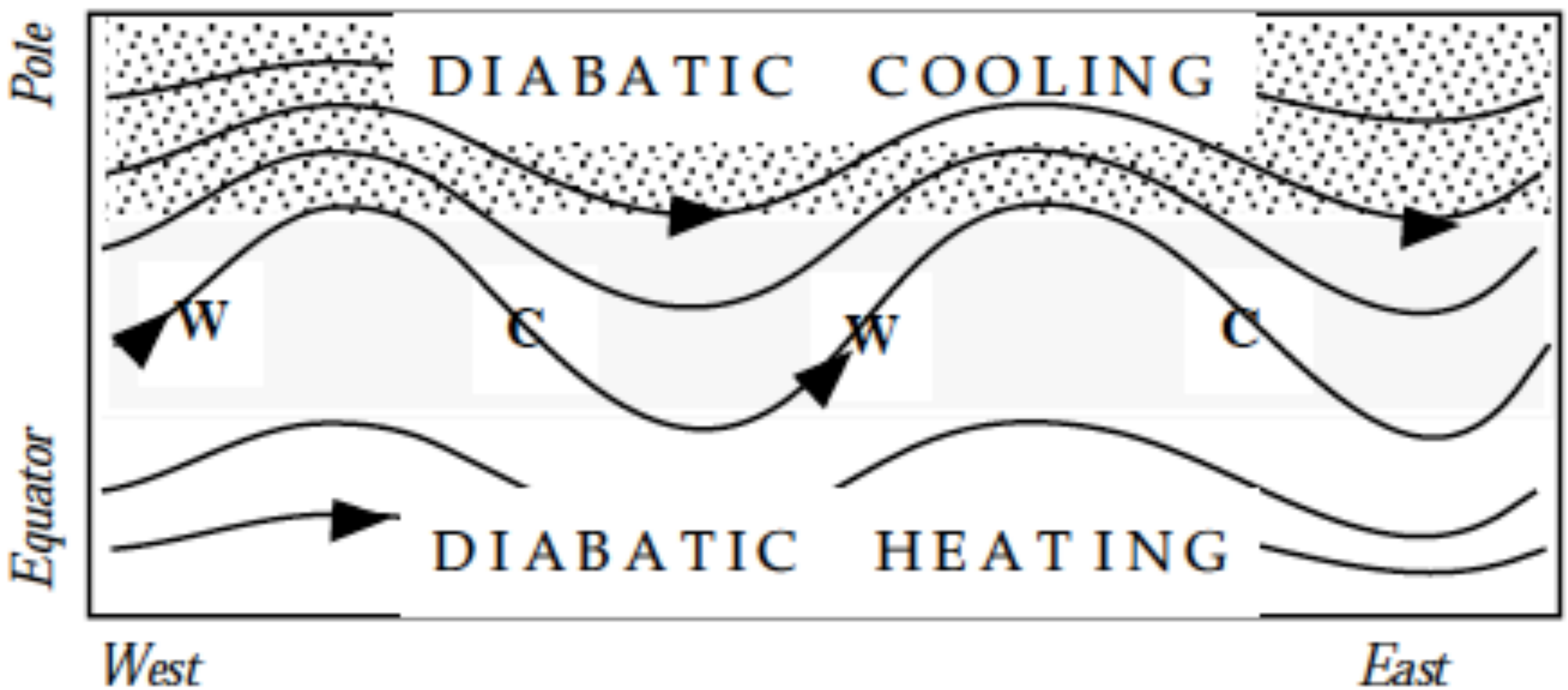
or

$\partial[Q]/\partial y$ causes poleward moving air to be warmer than equatorward moving air, thereby inducing $[v^*T^*]$

It depends on how you think about it.



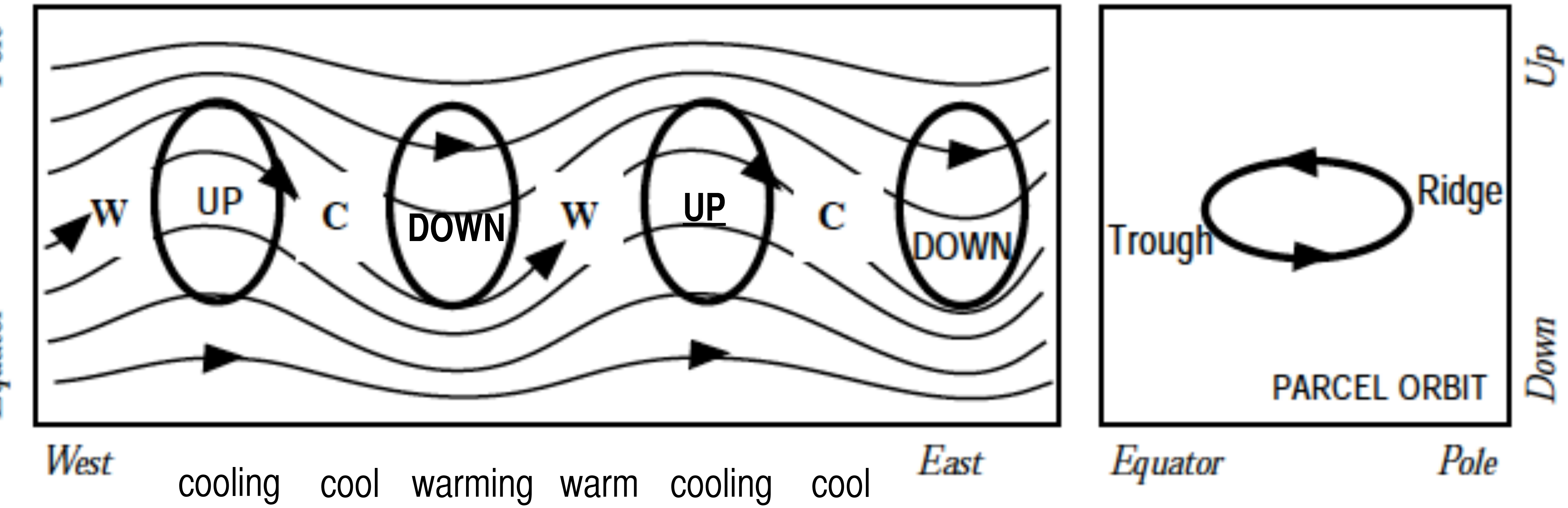
It depends on how you think about it.



but in any case, simple stirring does not produce poleward heat transports in the absence of diabatic heating.

Is it possible to have poleward eddy heat transports in adiabatic flow?

Here's how it can happen

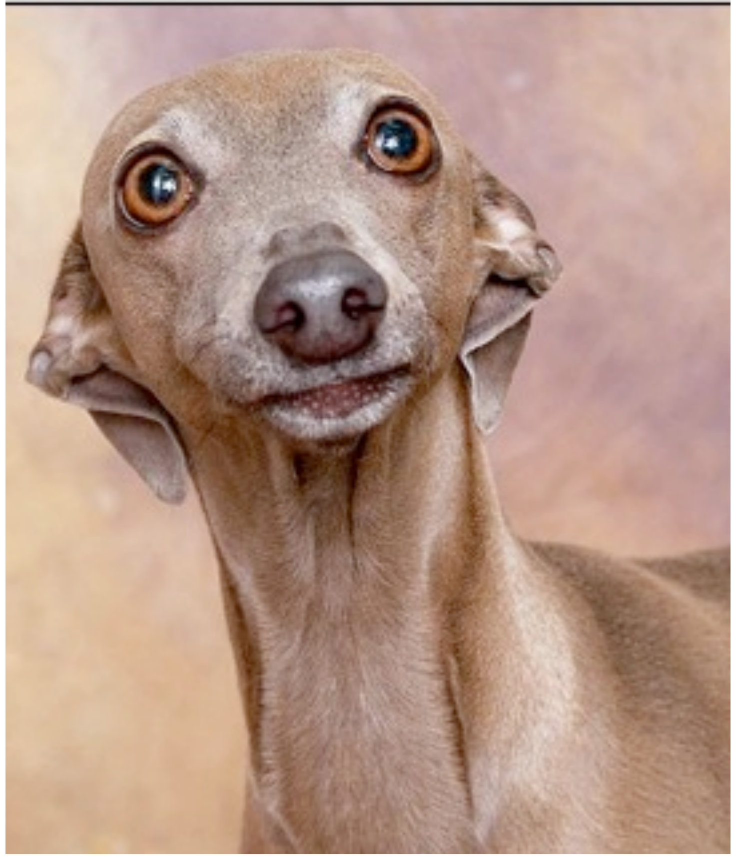


$$\frac{DT}{Dt} = \sigma\omega$$

As air parcels move through their elliptical orbits in the meridional plane (right) they conserve potential temperature

The motion is adiabatic and air parcels have cyclic orbits so the waves have no effect upon the slopes of the isentropes in the meridional plane.

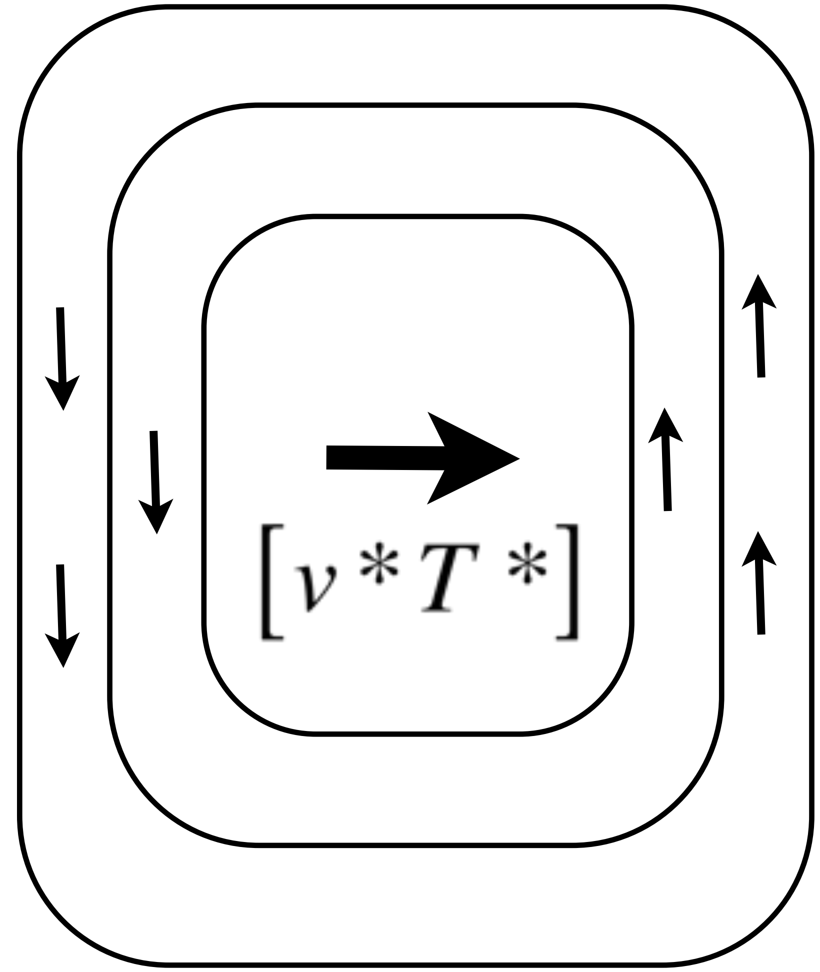
Eddy heat fluxes that have no effect on the temperature field! How can that be?



Eddy heat fluxes that have no effect on the temperature field! How can that be?

It's simple: The eddies induce a mean meridional circulation in which the adiabatic cooling in the ascending branch cancels the warming due to the convergence of the eddy heat flux poleward of the storm track.........

Here's how it works.

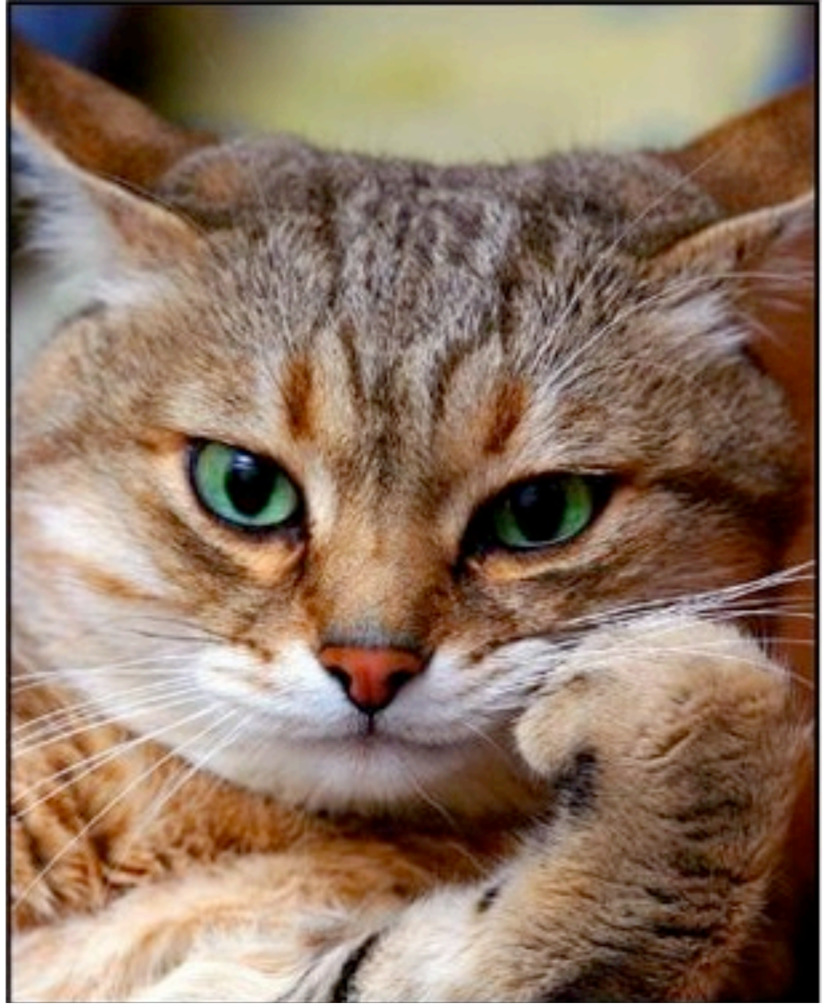


For perfect cancellation $\sigma[\omega] = -\frac{\partial}{\partial y}[v^*T^*]$

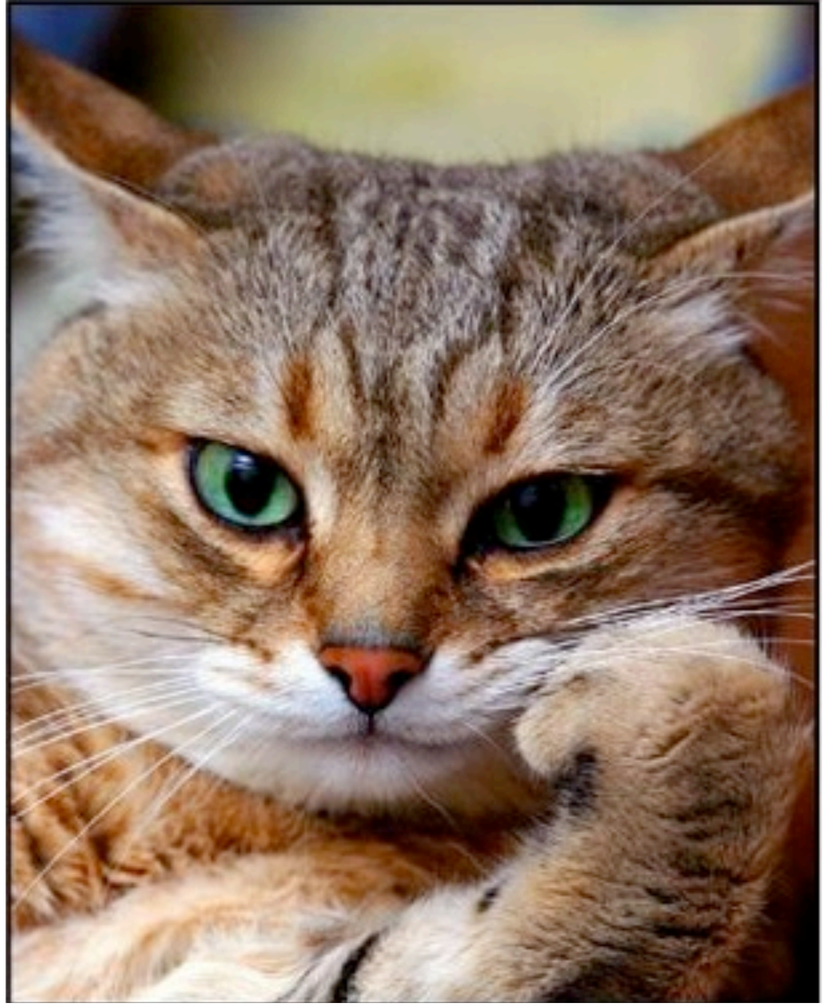
If the MMC are represented as the gradient of a streamfunction ψ with $[\omega] = -\frac{\partial \psi}{\partial y}$ and $[v] = \frac{\partial \psi}{\partial p}$

then the eddy heat flux is the streamfunction for the MMC

i.e., $\psi = [v^*T^*]$



Next thing you know he'll be telling us that the Ferrell cell is eddy-induced!



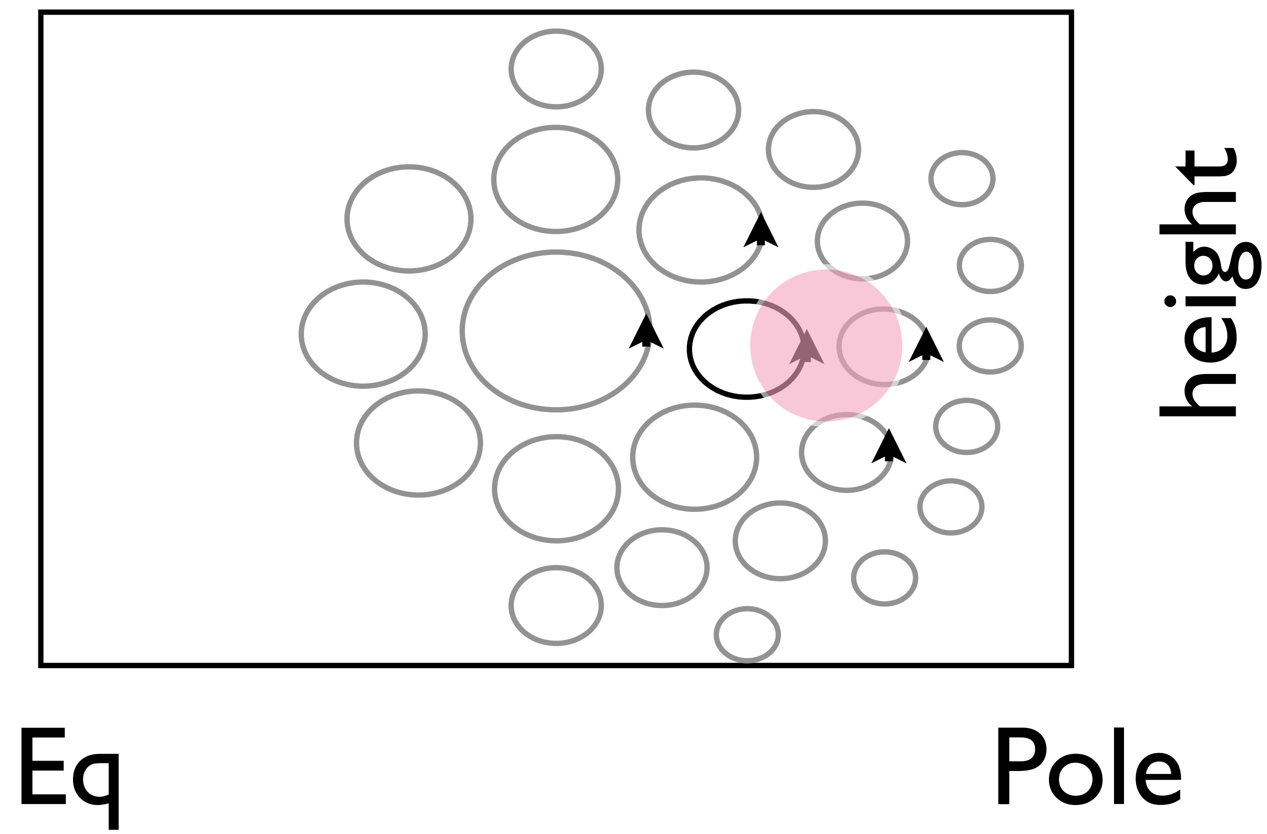
So how are these eddy heat fluxes supposed to induce a MMC?

It’s simple: you just need to consider the orbit of air parcels in the waves and how it varies with latitude and height

Eddy orbits

looking in the upstream direction
in westerly background flow

$$u > c$$

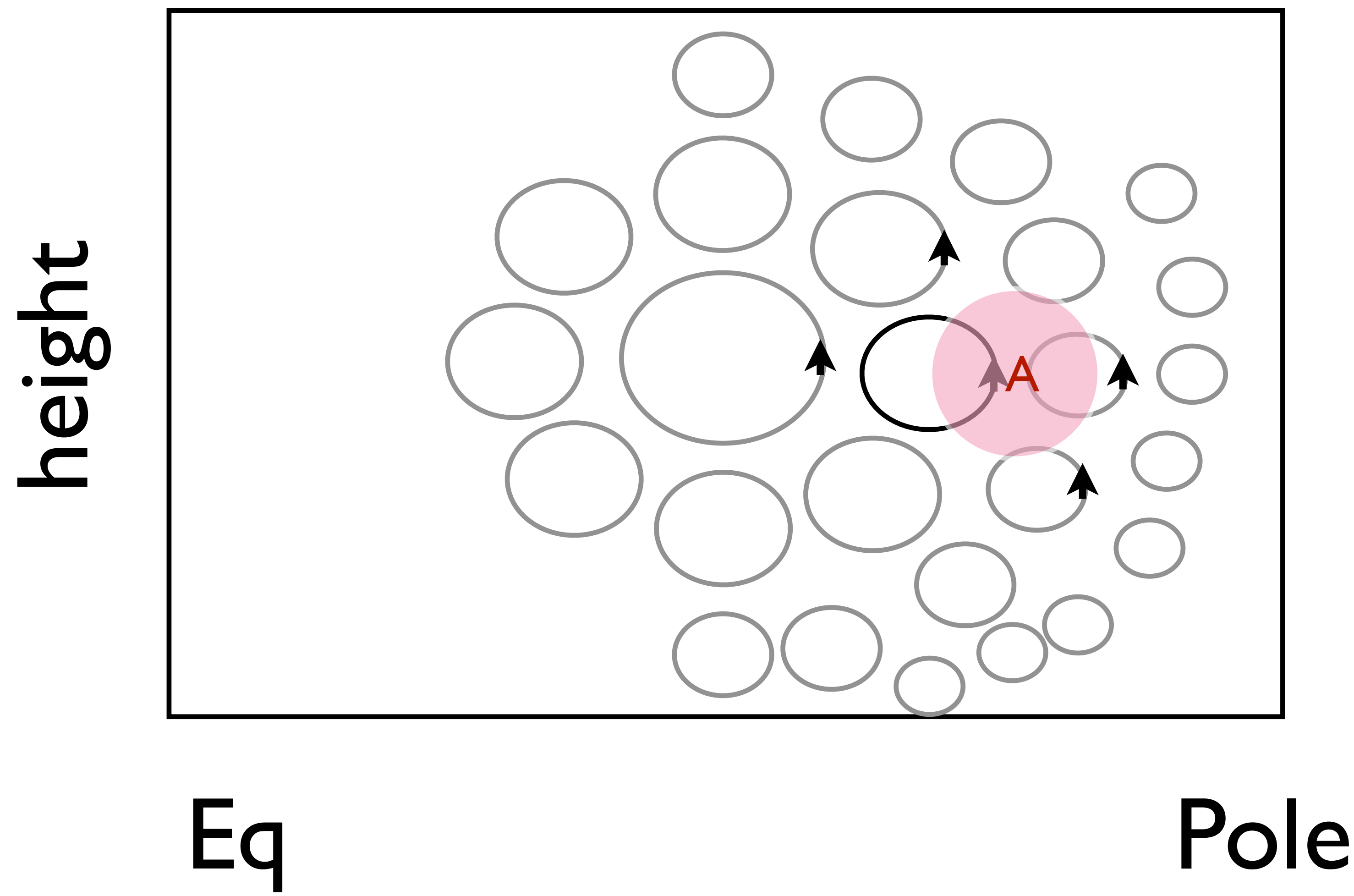


$[v^* T^*]$ assumed to be strongest in the middle of the domain

For fixed u and c the width of the orbit is proportional to $\sigma(v)$

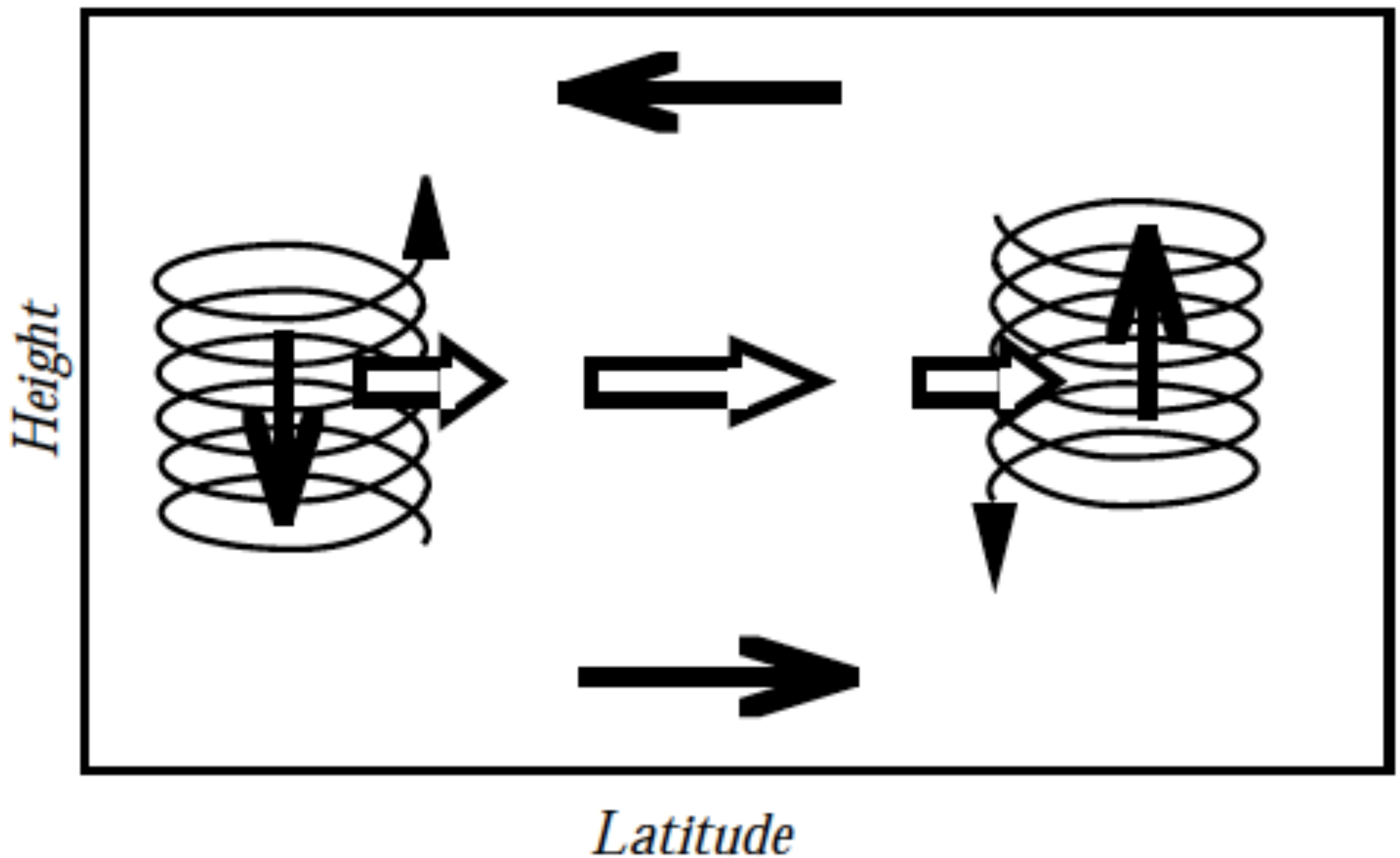
and the depth is proportional to $\sigma(T)$

and the area proportional to $[v^* T^*]$

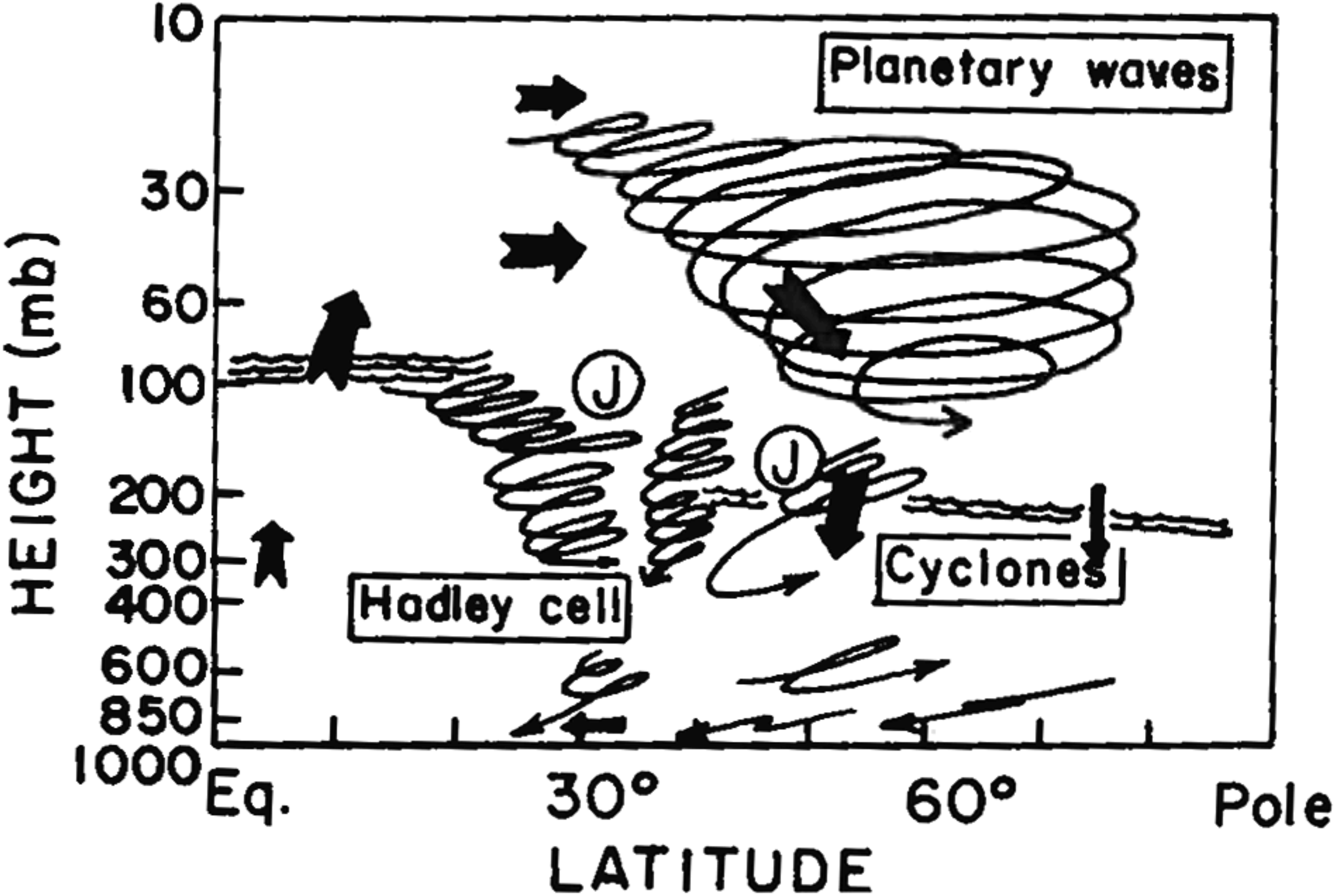


The Eulerian perspective

At A, ascent in ridges is stronger than descent in troughs because eddies on equatorward side of the pink box are stronger. Hence, there is ascent in Eulerian mean.



The Lagrangian perspective: Stokes drift



Planetary waves
J
J
Hadley cell
Cyclones
HEIGHT (mb)
10
30
60
100
200
300
400
600
850
1000
Eq.
30°
60°
Pole
LATITUDE

Eulerian versus Lagrangian mean meridional circulations

$$\psi_E + \psi_S = \psi_L$$

For eddy-induced MMC

In the absence of transience and dissipation,

$$\psi_L = 0$$