Four ways of inferring the MMC

1. direct measurement of $[v]$

2. vorticity balance $$[\overline{v}] = -\frac{G+F}{f}$$

3. total energy balance $$[\overline{\omega}] = -\frac{P+Q}{\sigma}$$

4. eliminating time derivatives in governing equations

$$A(\psi) \equiv \sigma \frac{\partial^2 \psi}{\partial y^2} + f^2 \frac{\partial^2 \psi}{\partial p^2} = -f \frac{\partial}{\partial p}(G+F) + \frac{\partial}{\partial y}(P+Q)$$

Four ways of inferring the MMC

1. direct measurement of $[v]$ small residual

2. vorticity balance $[\overline{v}] = -\dfrac{G+F}{f}$ time dependence

3. total energy balance $[\overline{\omega}] = -\dfrac{P+Q}{\sigma}$ time dependence

4. eliminating time derivatives in governing equations

$$A(\psi) \equiv \sigma \frac{\partial^2 \psi}{\partial y^2} + f^2 \frac{\partial^2 \psi}{\partial p^2} = -f \frac{\partial}{\partial p}(G+F) + \frac{\partial}{\partial y}(P+Q)$$

Four ways of inferring ω in QG system

1. direct measurement of $\nabla \cdot \vec{V}$ small residual

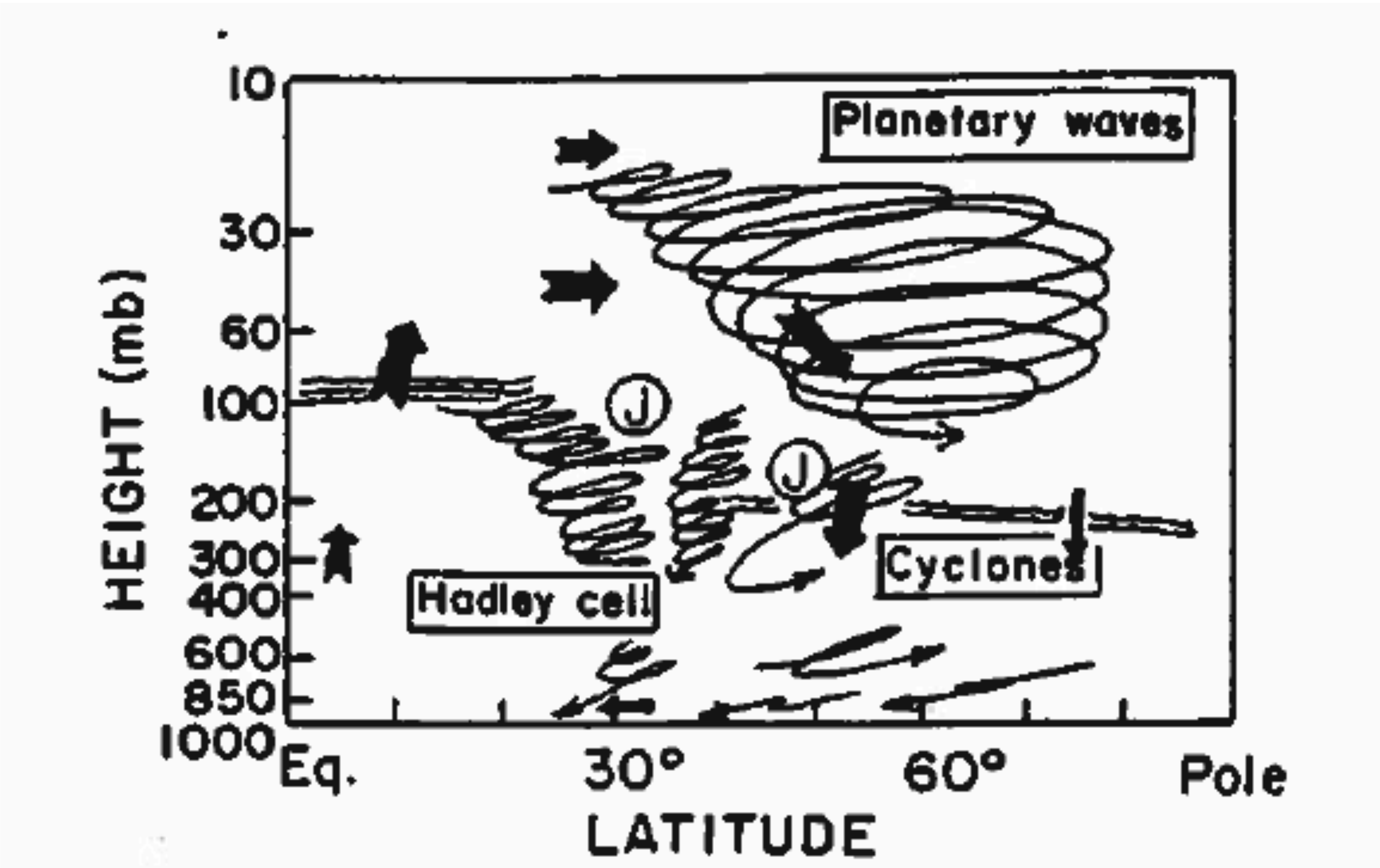
2. vorticity balance $\nabla \cdot \vec{V} = -\dfrac{\dfrac{\partial \varsigma}{\partial t} + \vec{V} \cdot \nabla \varsigma}{f + \varsigma}$ time dependence

3. total energy balance $\omega = -\dfrac{\dfrac{\partial T}{\partial t} + \vec{V} \cdot \nabla T}{\sigma}$ time dependence

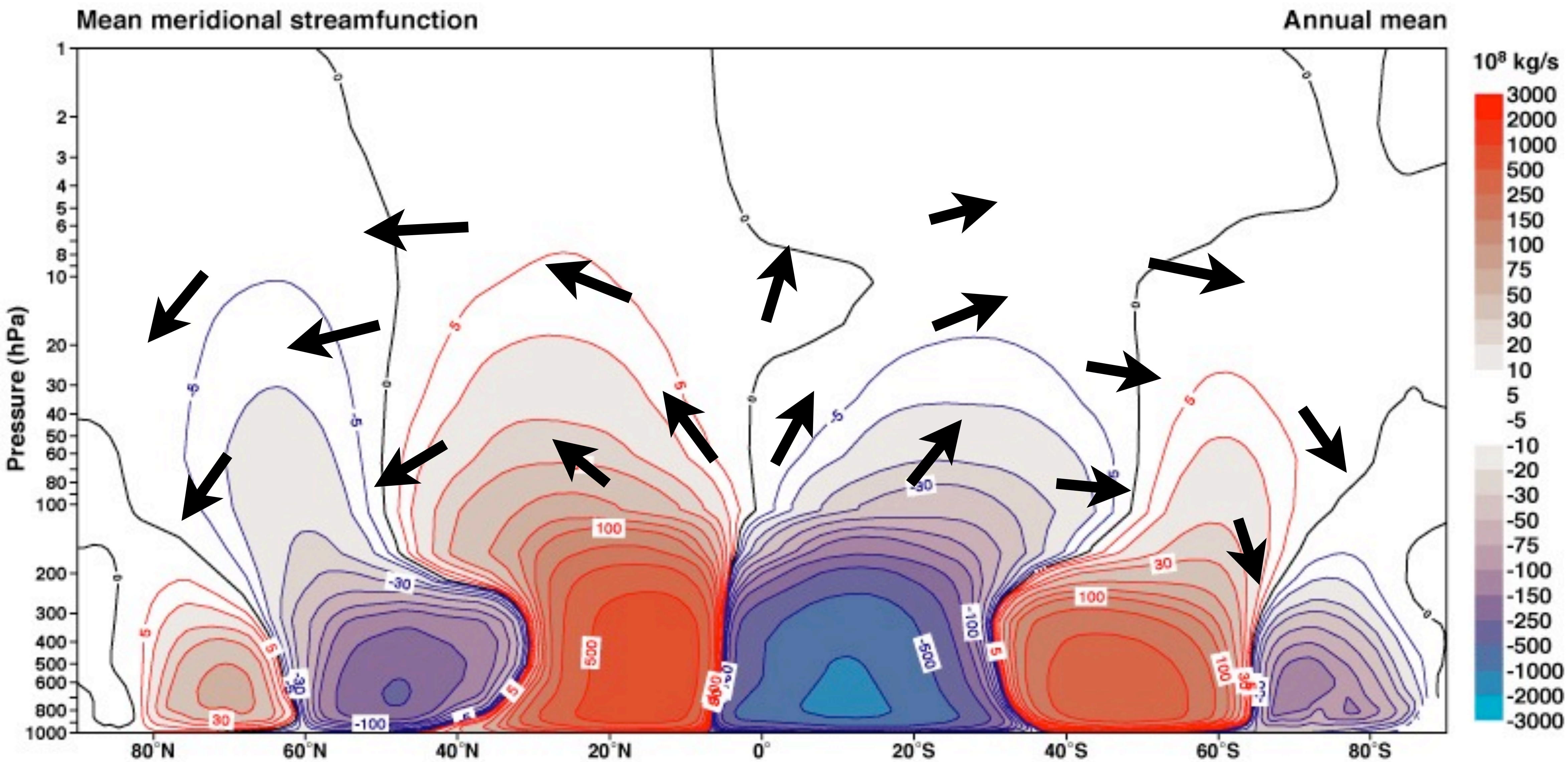
4. eliminating time derivatives in governing equations

the omega equation

Lagrangian versus Eulerian MMC



Kida JMSJ 1977

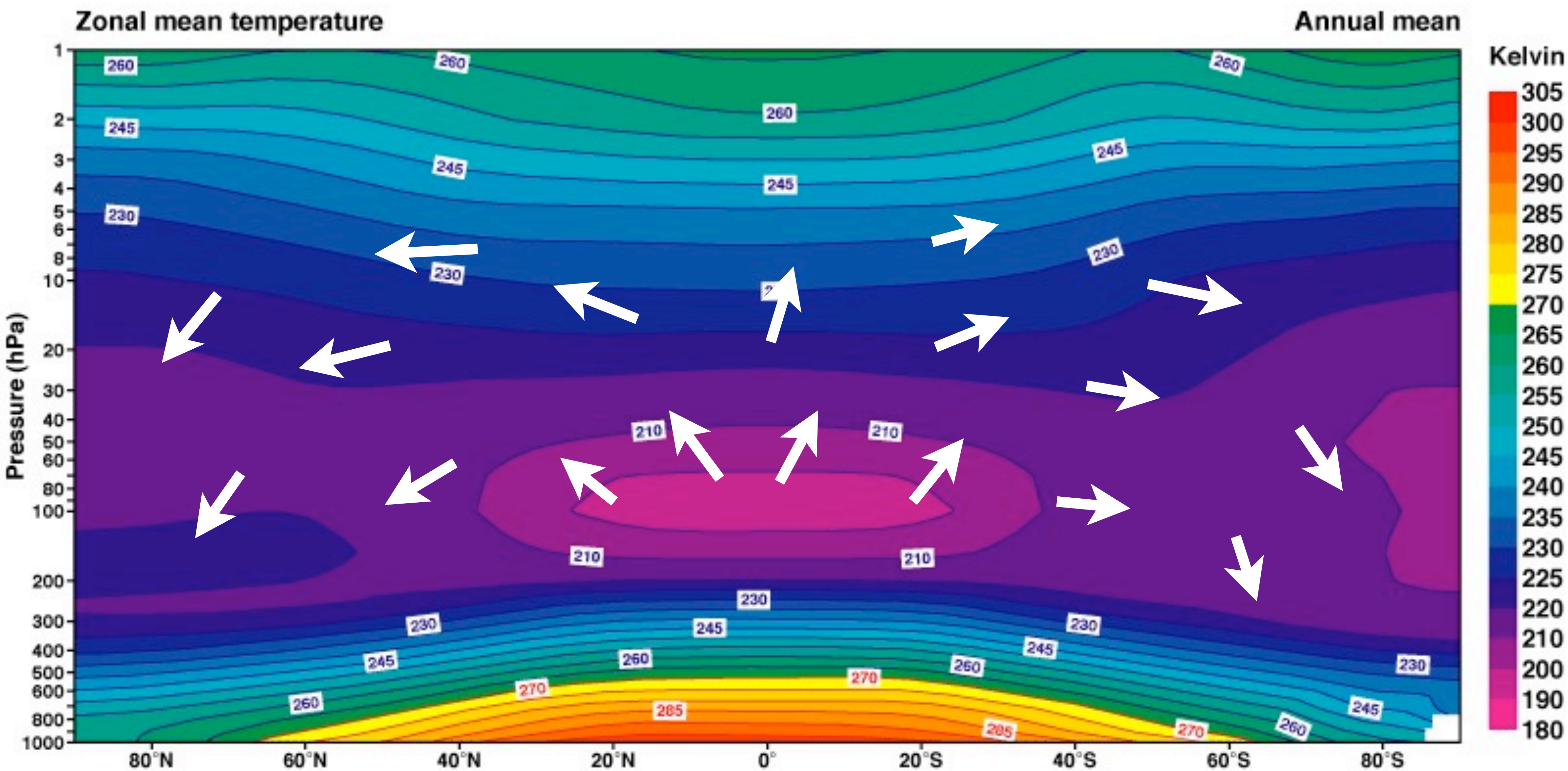


Eulerian MMC (contours) versus Lagrangian MMC (arrows)

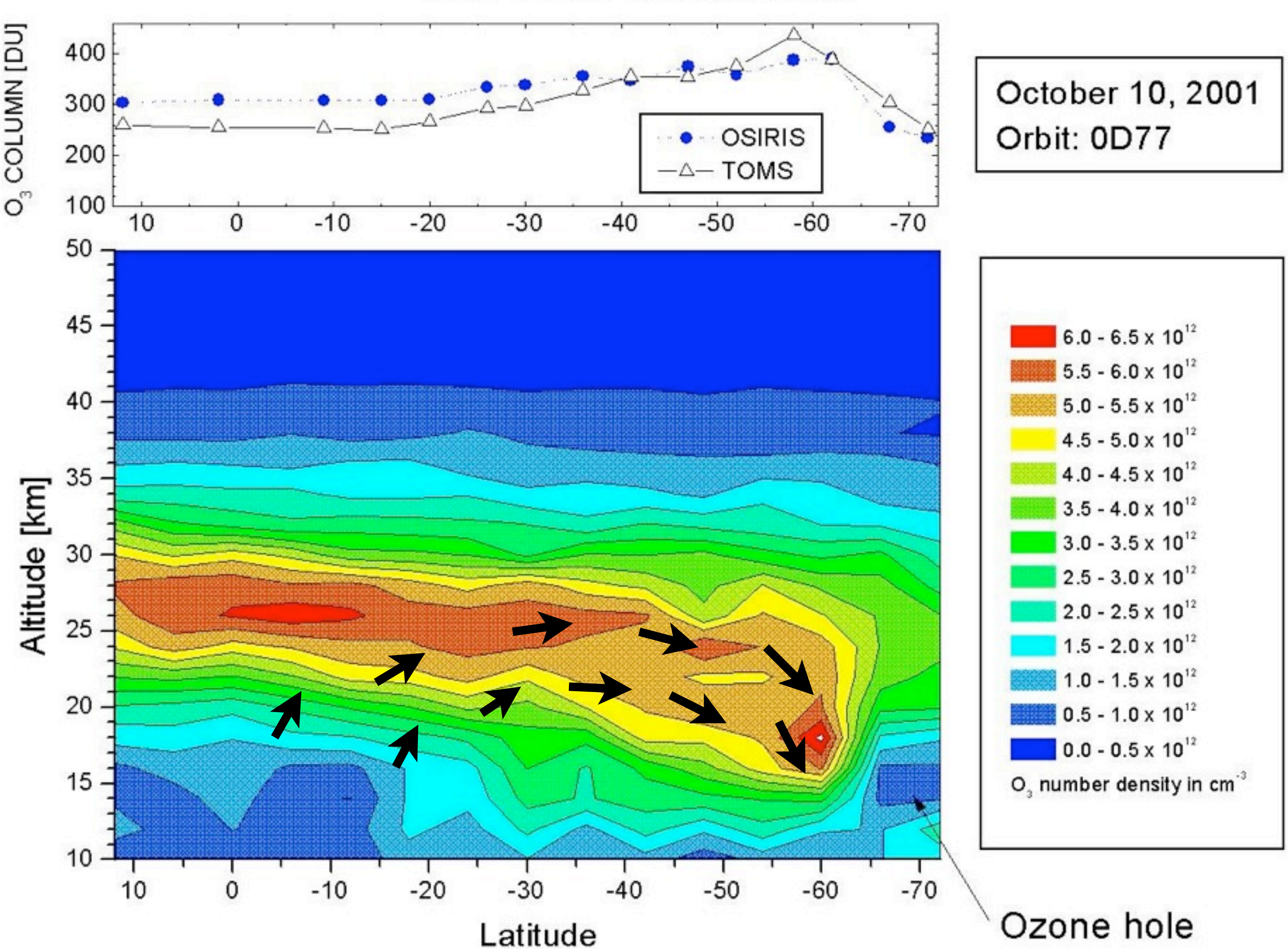
Lagrangian MMC also referred to as the Brewer-Dobson circulation

Brewer for water vapor

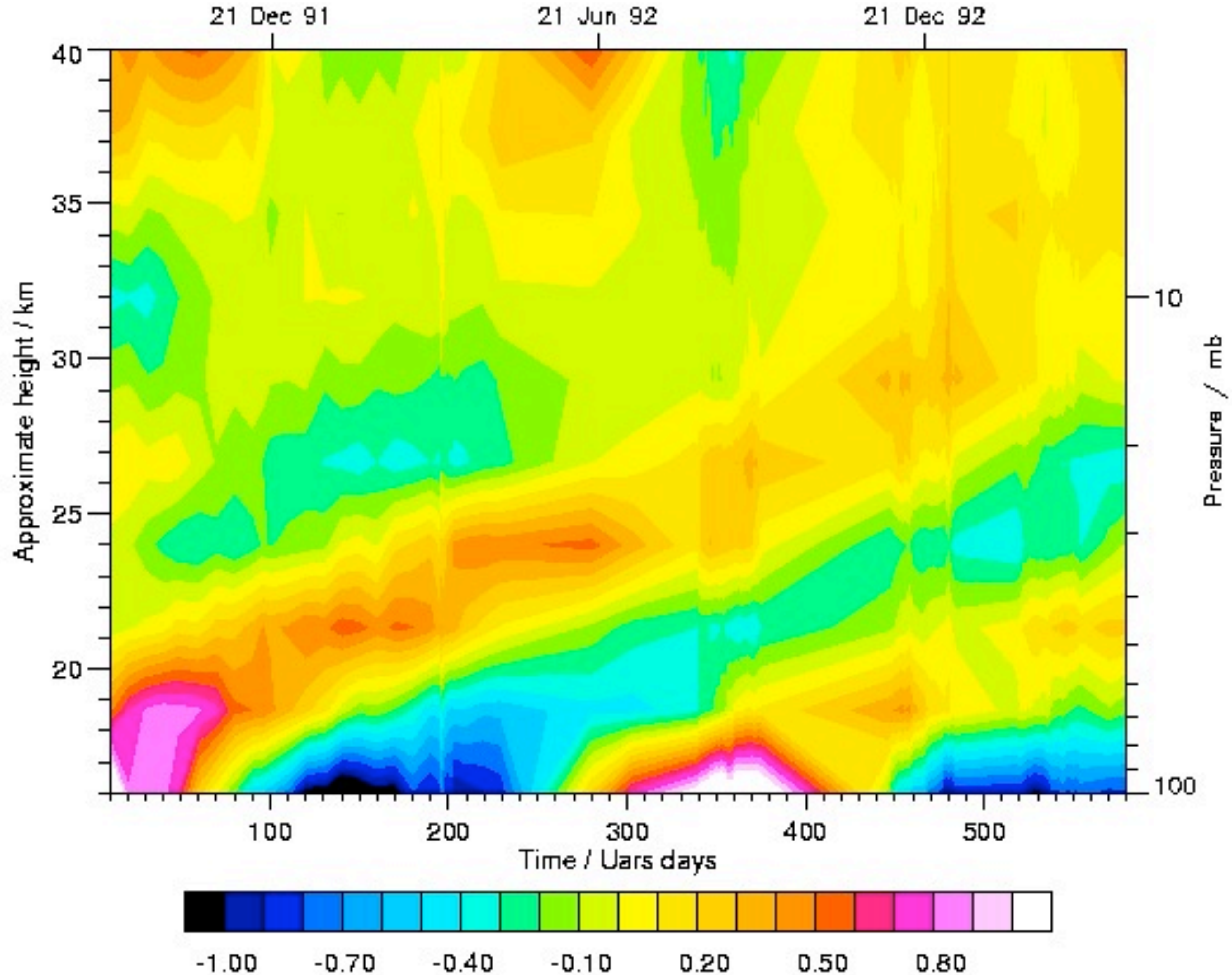
Dobson for ozone



Temperature, Lagrangian MMC (arrows)



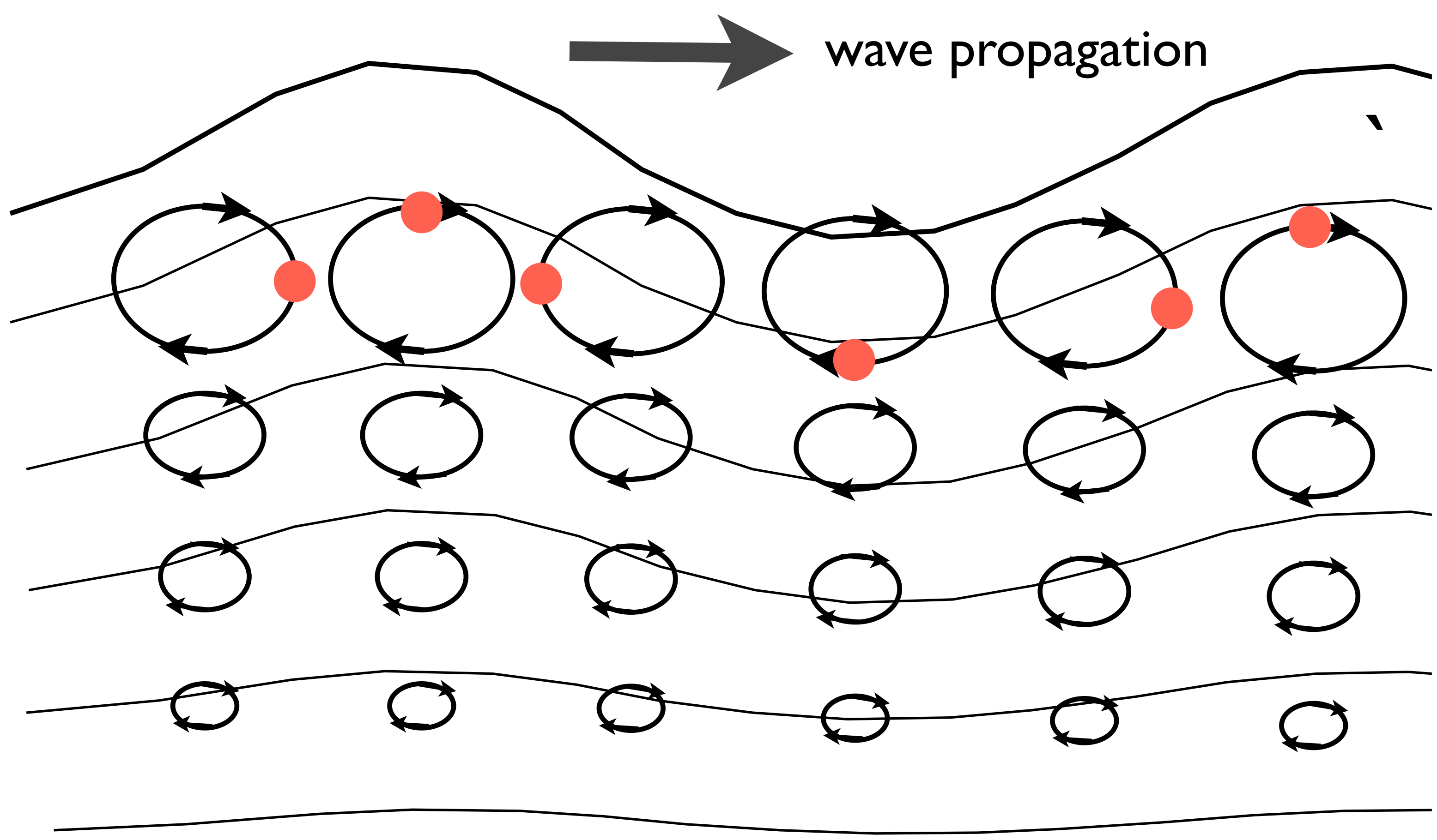
Ozone mixing ratio, Lagrangian MMC (arrows)



the “tropical tape recorder”

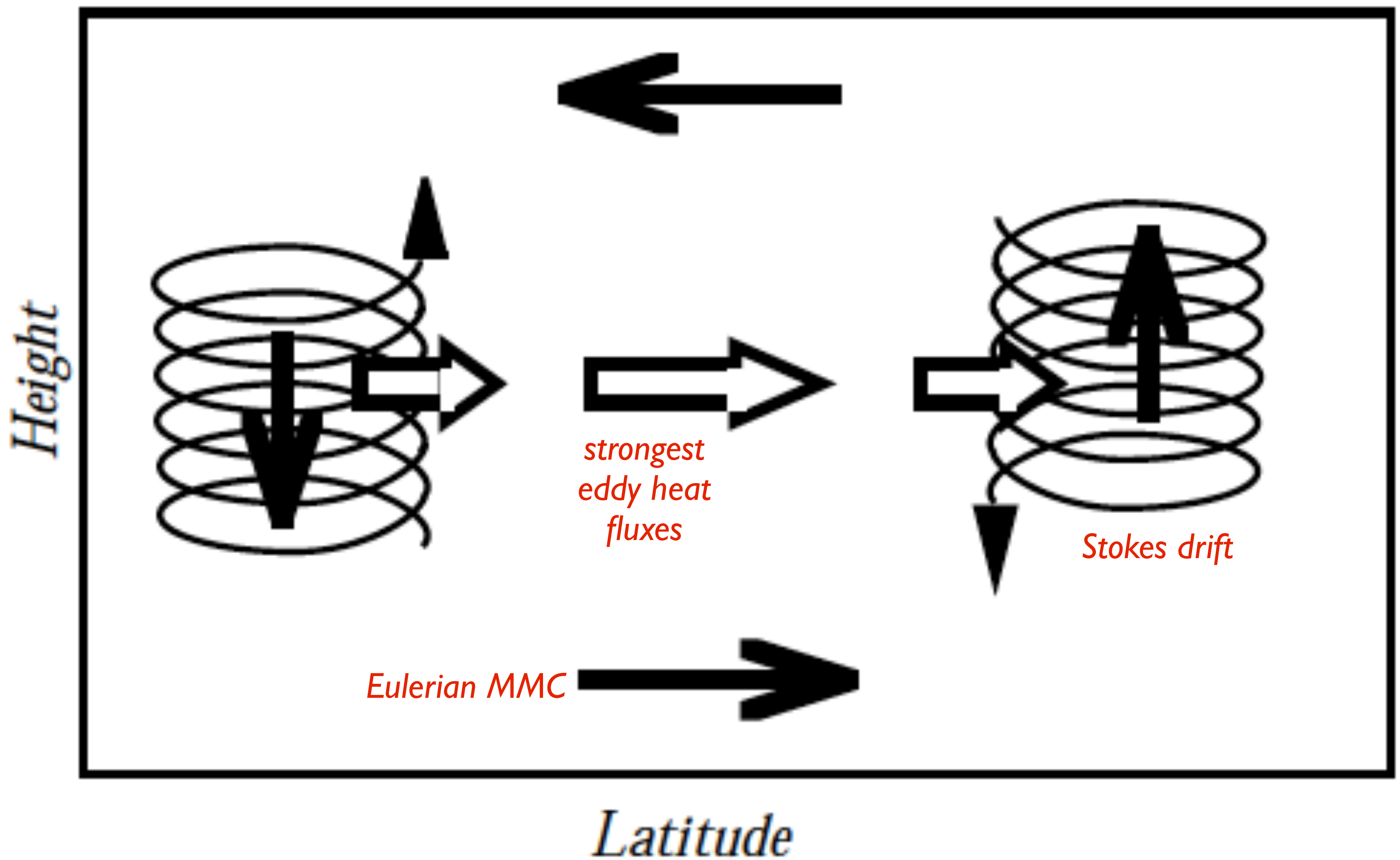
water vapor 12°S to 12°N

Stokes drift in water waves



Because of the vertical gradient of wave amplitude there is a rectified Lagrangian drift of passive tracers in the direction of the wave propagation.

Stokes drift in water waves



$$v_{Lagrangian} = v_{Eulerian} + v_{Stokes\,drift}$$

$$\frac{\partial[u]}{\partial t} = f[v]^* + \nabla \cdot \vec{E} + [F_x]$$

$$\frac{\partial[\alpha]}{\partial t} = \sigma[\omega]^* + [Q]$$

TEM equations

$$[v]^* \equiv [v] + \frac{\partial}{\partial p}\frac{[v^*\alpha^*]}{\sigma}$$

$$[\omega]^* \equiv [\omega] - \frac{\partial}{\partial y}\frac{[v^*\alpha^*]}{\sigma} = [\omega] + \frac{P}{\sigma}$$

residual MMC

$$\vec{E} \equiv \left(-[u^*v^*]\vec{j}, -f\frac{[v^*\alpha^*]}{\sigma}\vec{k}\right)$$

Eliassen-Palm flux

total eddy-forcing of [u]
incl. eddy-induced MMC

TEM (prognostic)equations

$$\frac{\partial[u]}{\partial t} = f[v]^* + \nabla\cdot\vec{E} + [F_x]$$

canceling heat flux contributions appear in $f[v]^*$ and $\nabla\cdot\vec{E}$ terms

$$\frac{\partial[\alpha]}{\partial t} = \sigma[\omega]^* + [Q]$$

heat flux contribution implicit in $\sigma[\omega]$ term

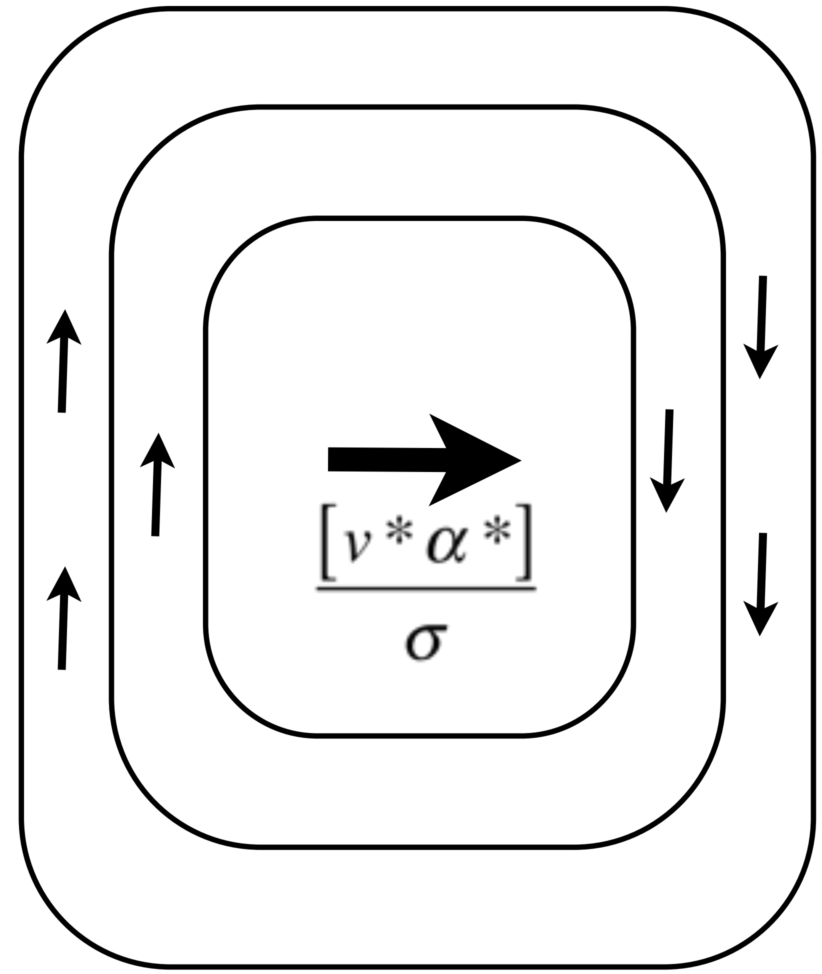
$f[v]^*$ indirect influence of diabatic heating

$\nabla\cdot\vec{E}$ total eddy forcing

residual circulation

$$[v]^* \equiv [v] + \frac{\partial}{\partial p}\frac{\left[v^*\alpha^*\right]}{\sigma}$$

$$[\omega]^* \equiv [\omega] - \frac{\partial}{\partial y}\frac{\left[v^*\alpha^*\right]}{\sigma}$$

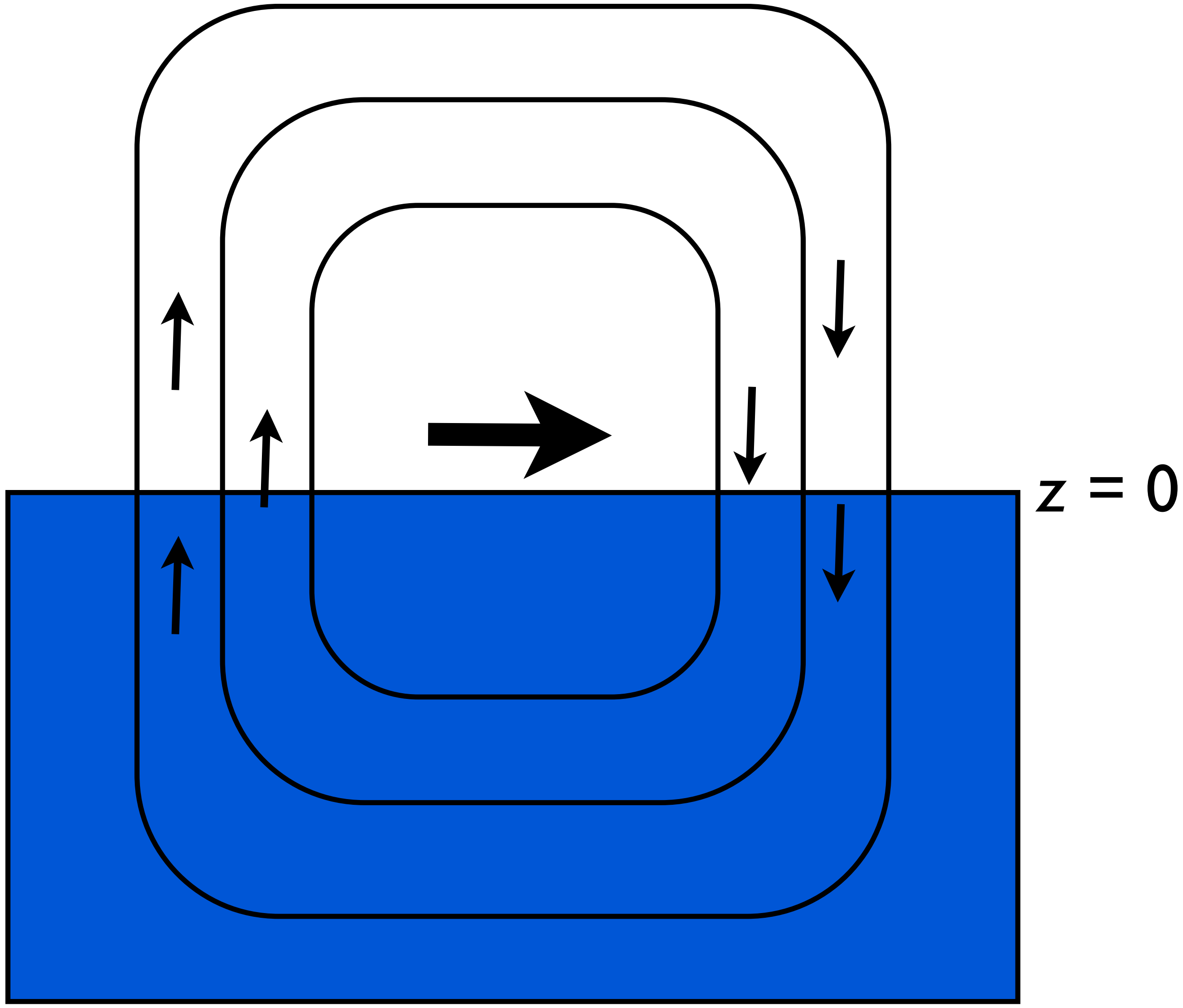


The terms involving the eddy heat fluxes are analogous to a correction for the Stokes drift.

residual circulation

$$[v]^* \equiv [v] + \frac{\partial}{\partial p}\frac{[v^*\alpha^*]}{\sigma}$$

$$[\omega]^* \equiv [\omega] - \frac{\partial}{\partial y}\frac{[v^*\alpha^*]}{\sigma}$$

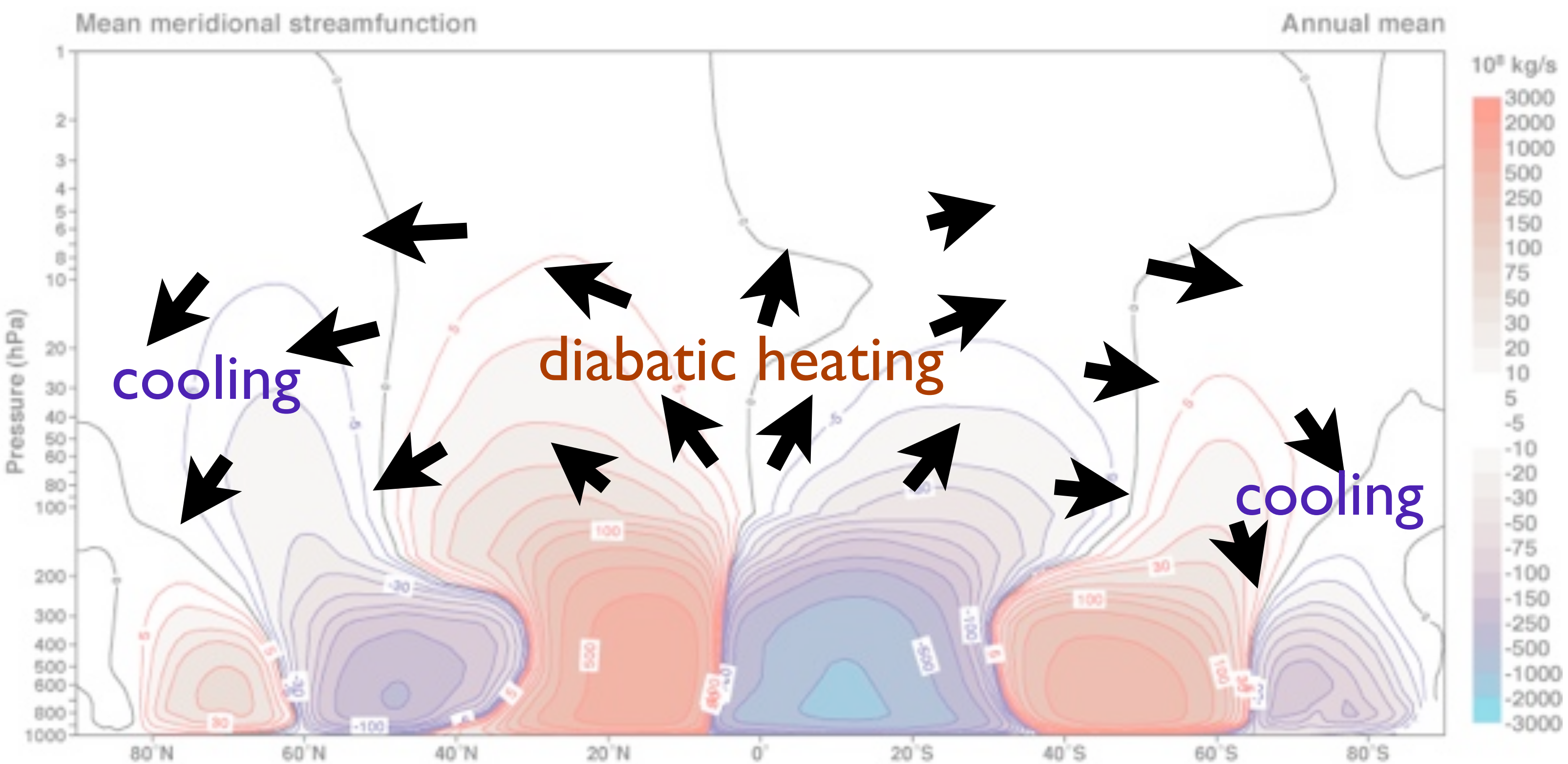


Note that $[v]^*$ doesn't vanish at the Earth's surface. Hence, the residual MMC bears little relation to the real Lagrangian MMC in the lower troposphere.

But far above the bottom boundary the *residual circulation* does resemble the *Lagrangian-mean meridional circulation.*

Under steady state conditions

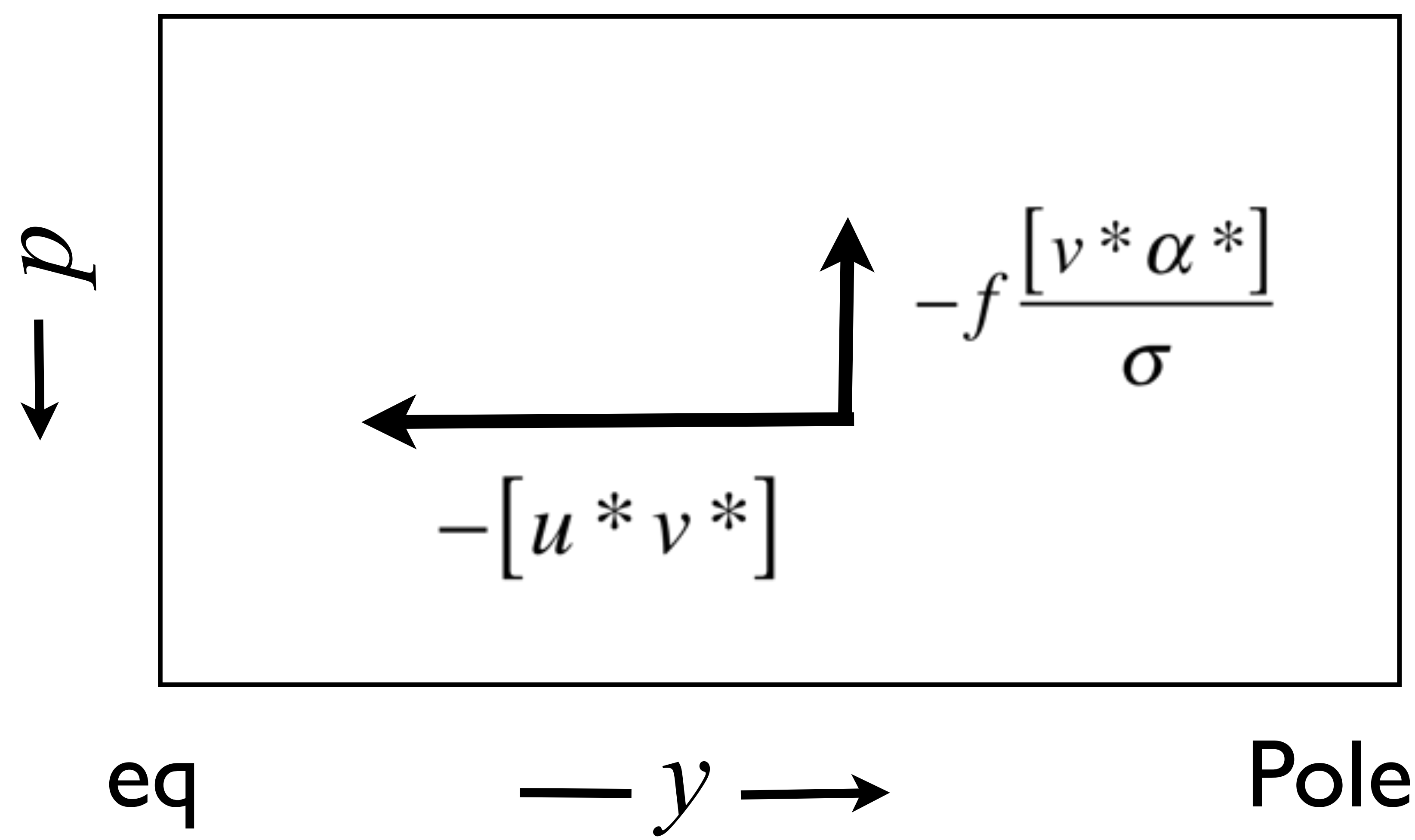
$$\sigma\overline{[\omega]}^* = [\overline{Q}]$$



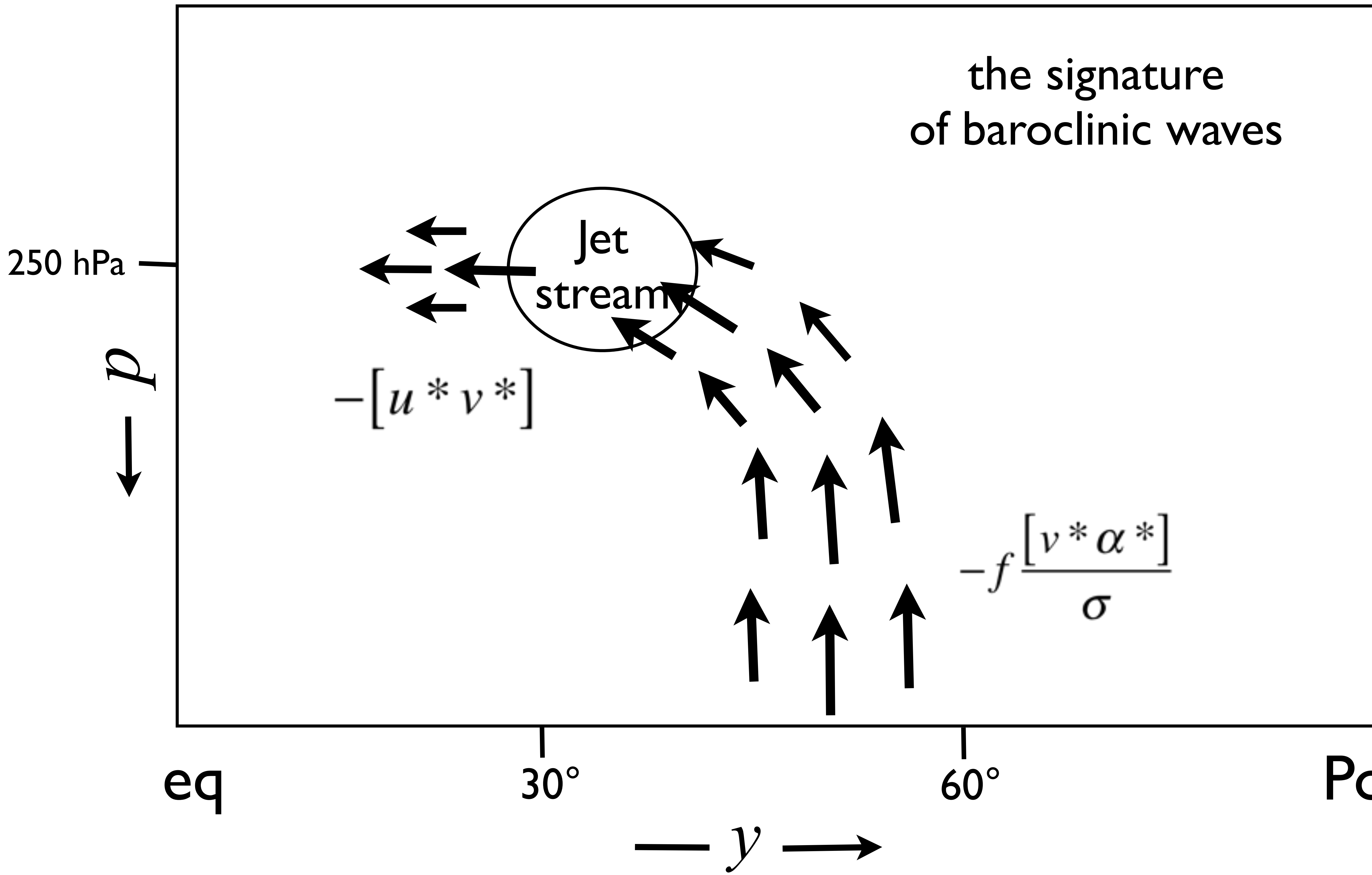
Hence, the residual circulation is sometimes referred to as the *diabatically-driven circulation*

The Eliassen-Palm flux

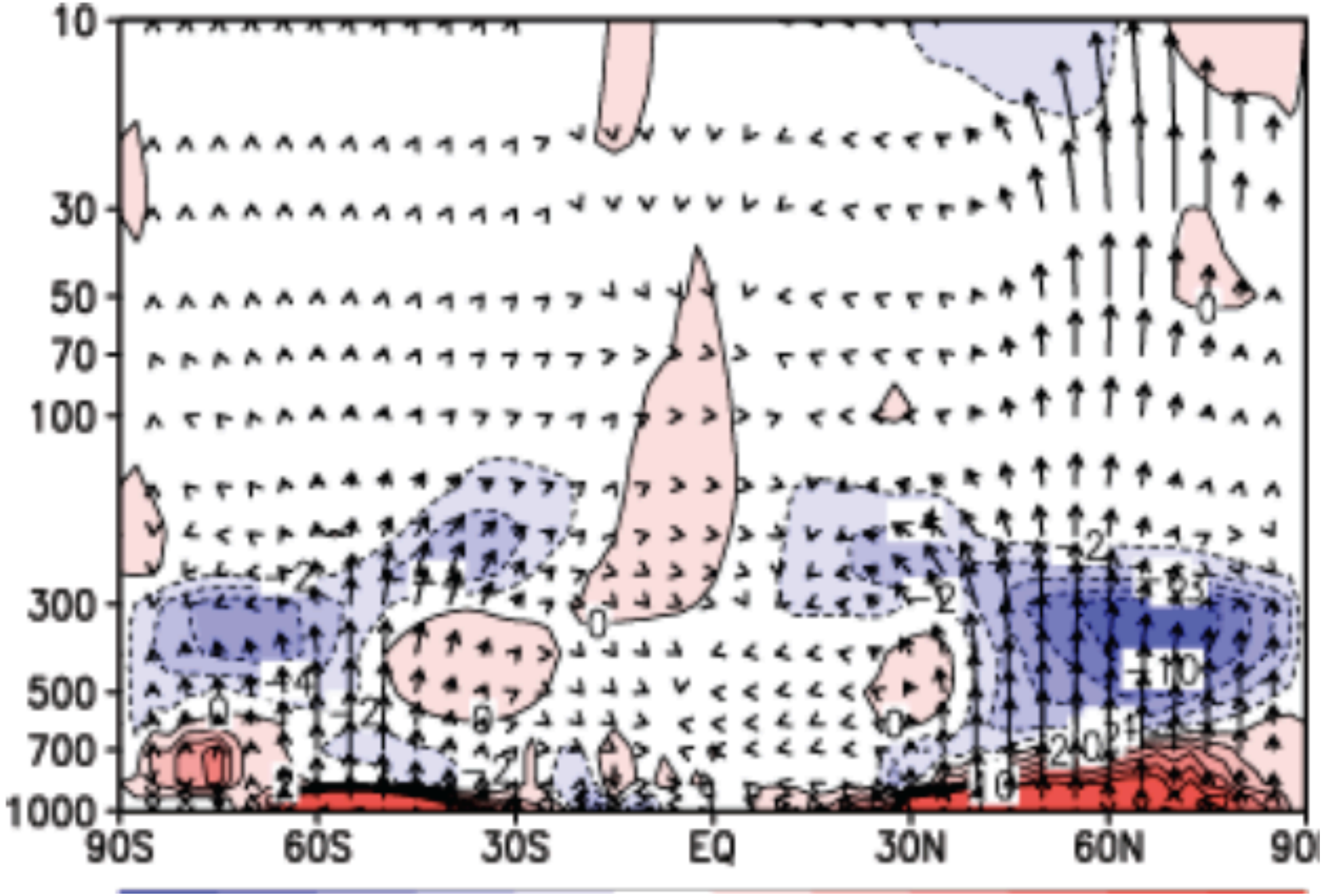
$$\vec{E} \equiv \left(-[u^* v^*]\vec{j}, -f\frac{[v^*\alpha^*]}{\sigma}\vec{k} \right)$$

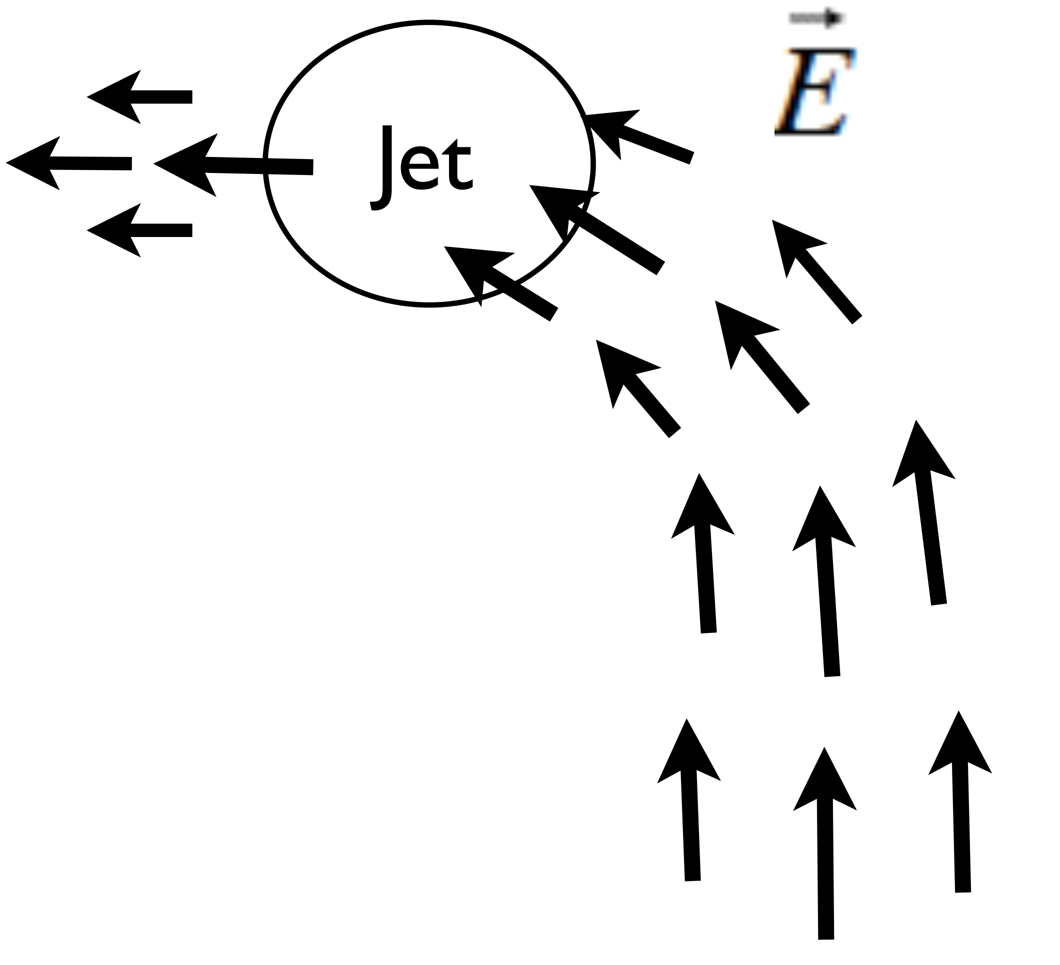


Eliassen-Palm flux



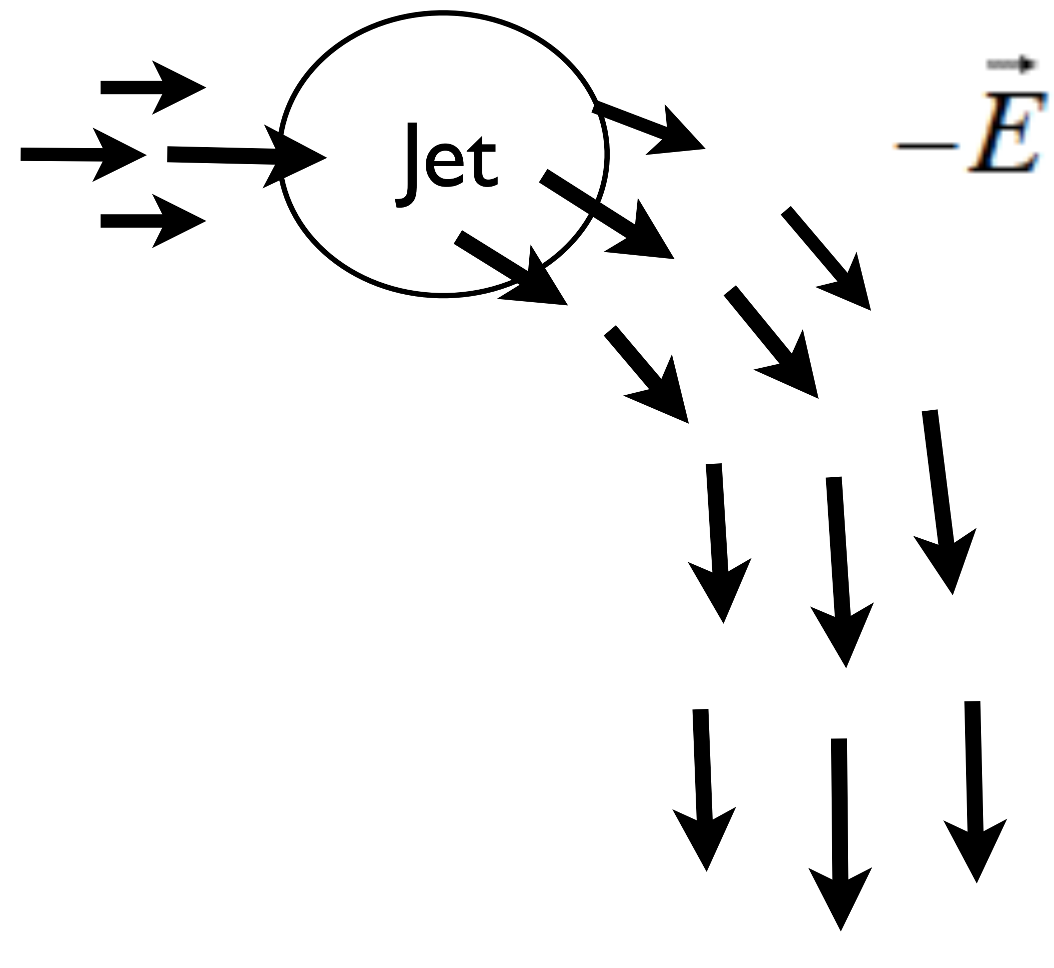
Annual mean EP fluxes JRA





EP flux

flux of wave activity



[u] flux

virtual flux of momentum

$$\vec{E} \equiv \left(-[u^* v^*]\vec{j}, -f \frac{[v^* \alpha^*]}{\sigma} \vec{k} \right)$$

$$\nabla \cdot \vec{E} = -\frac{\partial}{\partial y}[u^* v^*] - f \frac{\partial}{\partial p} \frac{[v^* \alpha^*]}{\sigma}$$

$$\nabla \cdot \vec{E} = [q^* v^*]$$

$$\nabla \cdot \vec{E} = G + G*$$

$$\nabla \cdot \vec{E} = \frac{d[u]}{dt}$$

eddy forcing

Eliassen-Palm flux: another interpretation

$\vec{E}$ a measure of the flux of *wave activity* in the meridional plane

related to the group velocity $\vec{E} = \vec{c}_g A$

$$\frac{\partial A}{\partial t} + \nabla \cdot \vec{E} = D$$

$A = {}^1/_2[q]_y\left[\eta^{*2}\right]$ is a measure of *wave activity*

η is the meridional displacement of potential vorticity contours

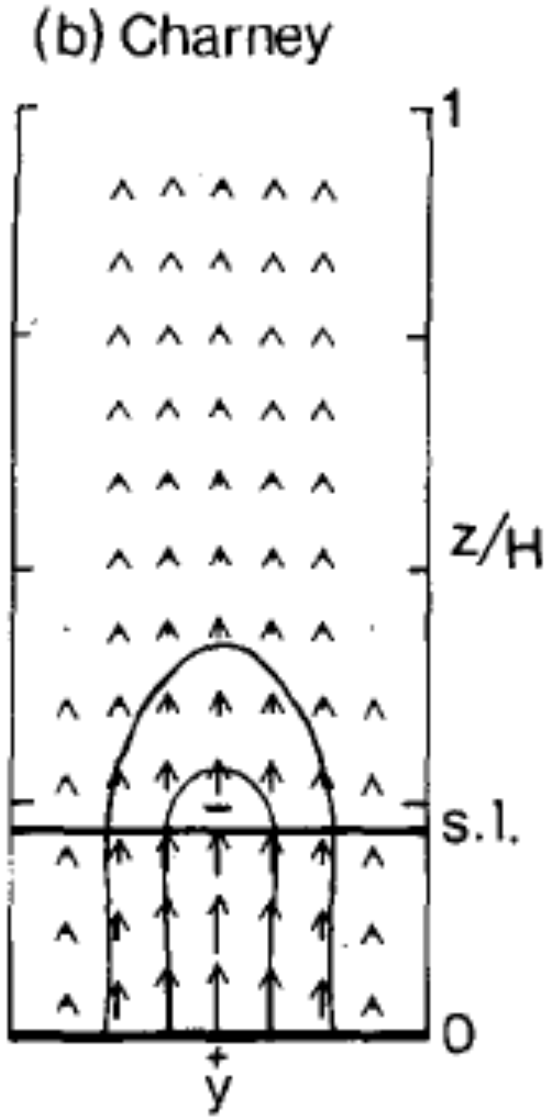
D is the generation or dissipation of wave activity

$$\frac{\partial A}{\partial t} + \nabla \cdot \vec{E} = D$$

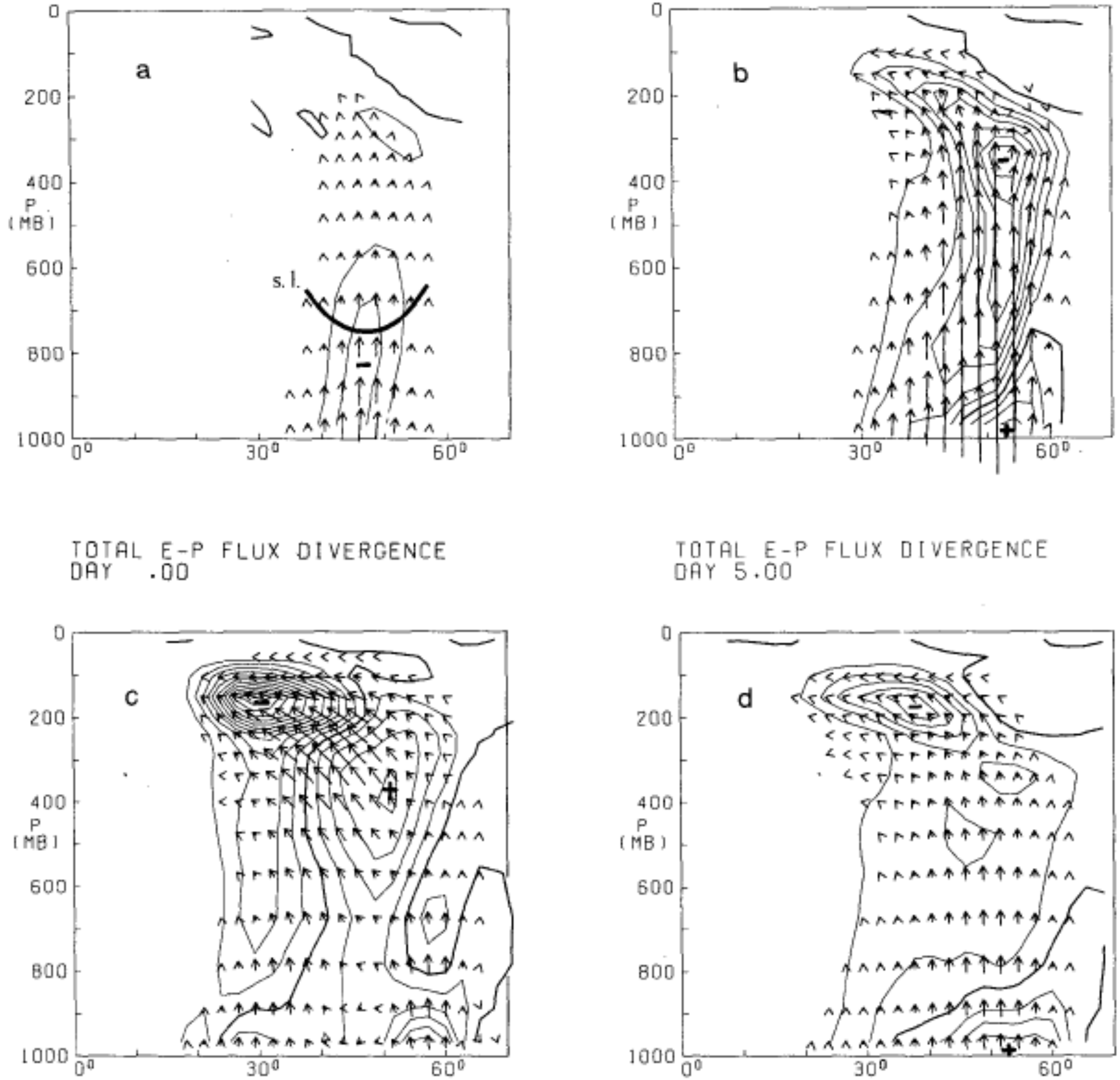
$D = 0$ $\vec{E}$ traces the flow of wave activity from where the eddies have been to where they are going

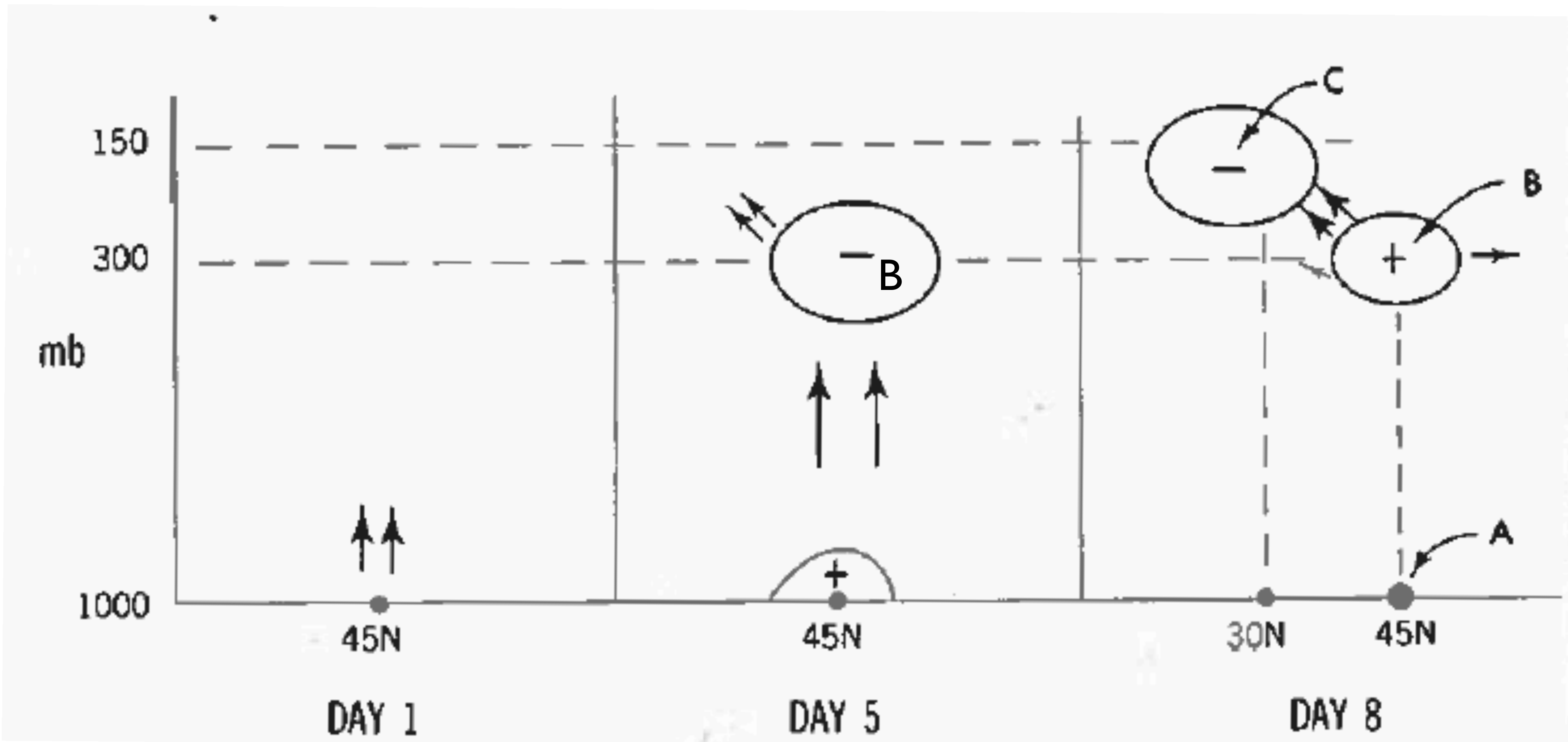
$\frac{\partial A}{\partial t} = 0$ $\vec{E}$ traces the flow of wave activity from the region of generation to the region of dissipation

Edmon, Hoskins and McIntyre, JAS 1980



baroclinic wave life cycle





linear growth stage

transition to nonlinear

nonlinear barotropic

$$\frac{\partial A}{\partial t} + \nabla \cdot \vec{E} = D$$

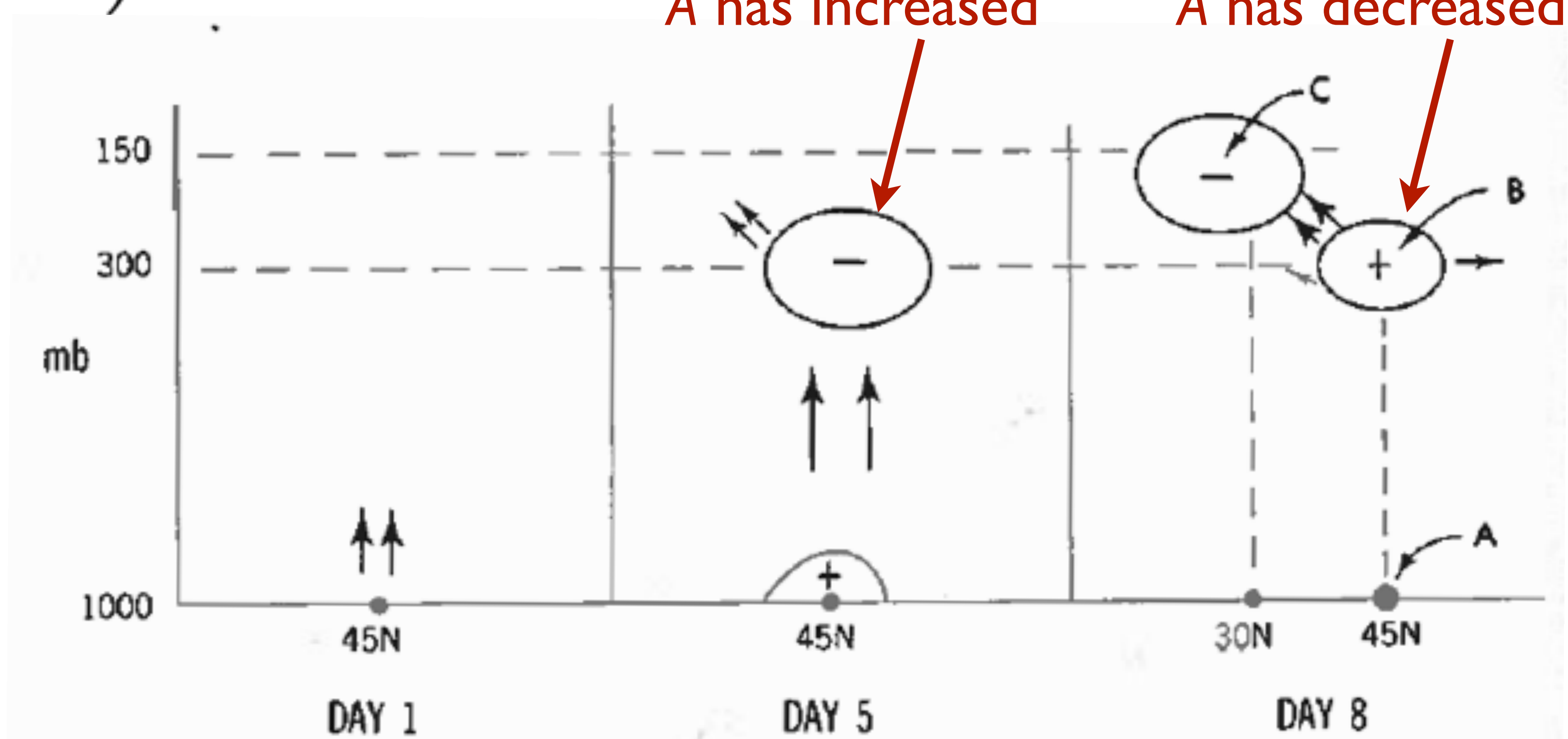
$$\frac{\partial [u]}{\partial t} = \nabla \cdot \vec{E}$$

Hence,

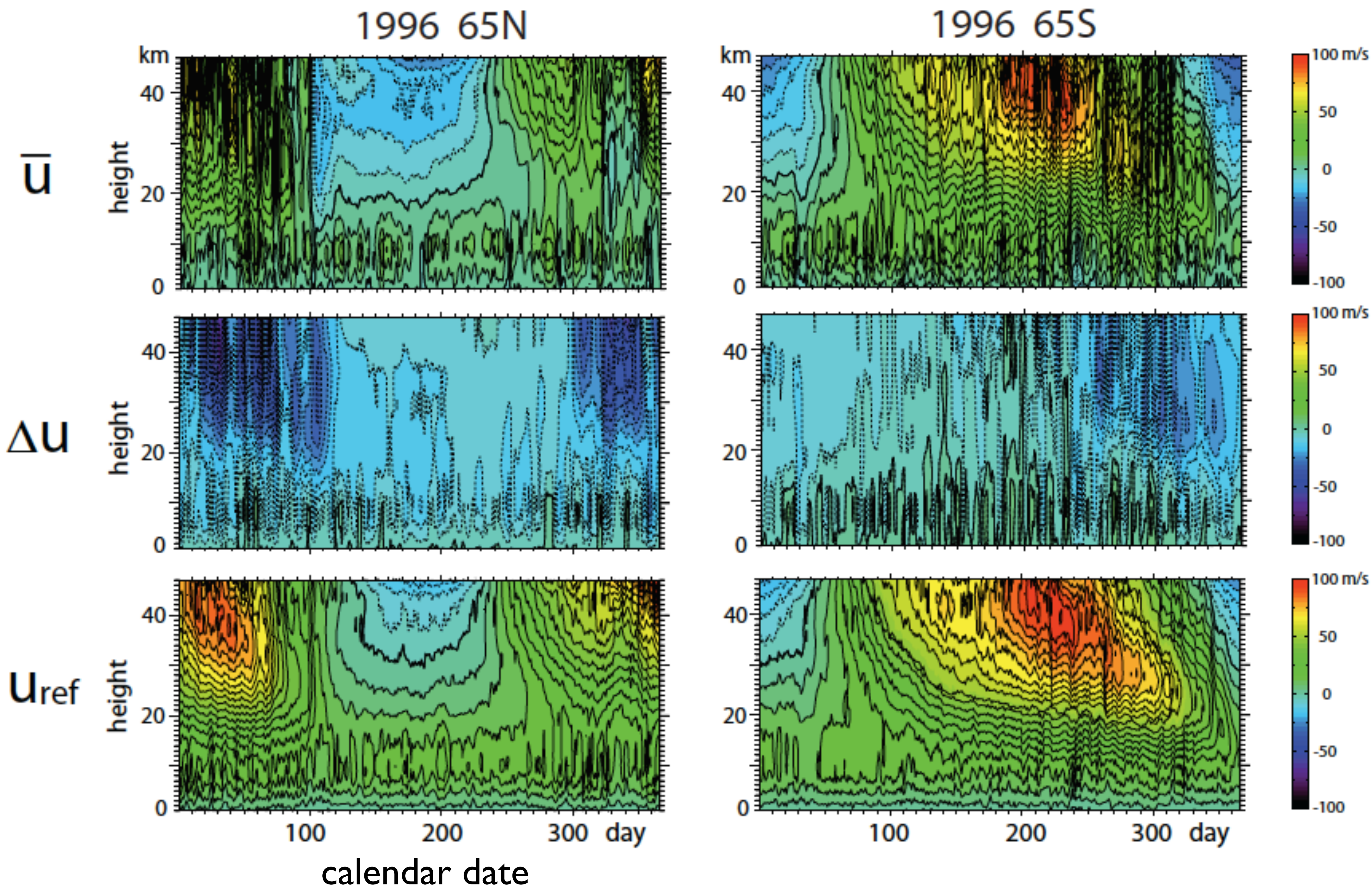
$$\frac{\partial}{\partial t}([u] + A) = D$$

[u] has decreased
A has increased

[u] has increased
A has decreased



Nakamura and Solomon, Fig. 8



u_{ref} analogous to $[u] + A$

$$\frac{\partial A}{\partial t} + \nabla \cdot \vec{E} = D$$

If $\frac{\partial A}{\partial t} = 0$ and $D = 0$

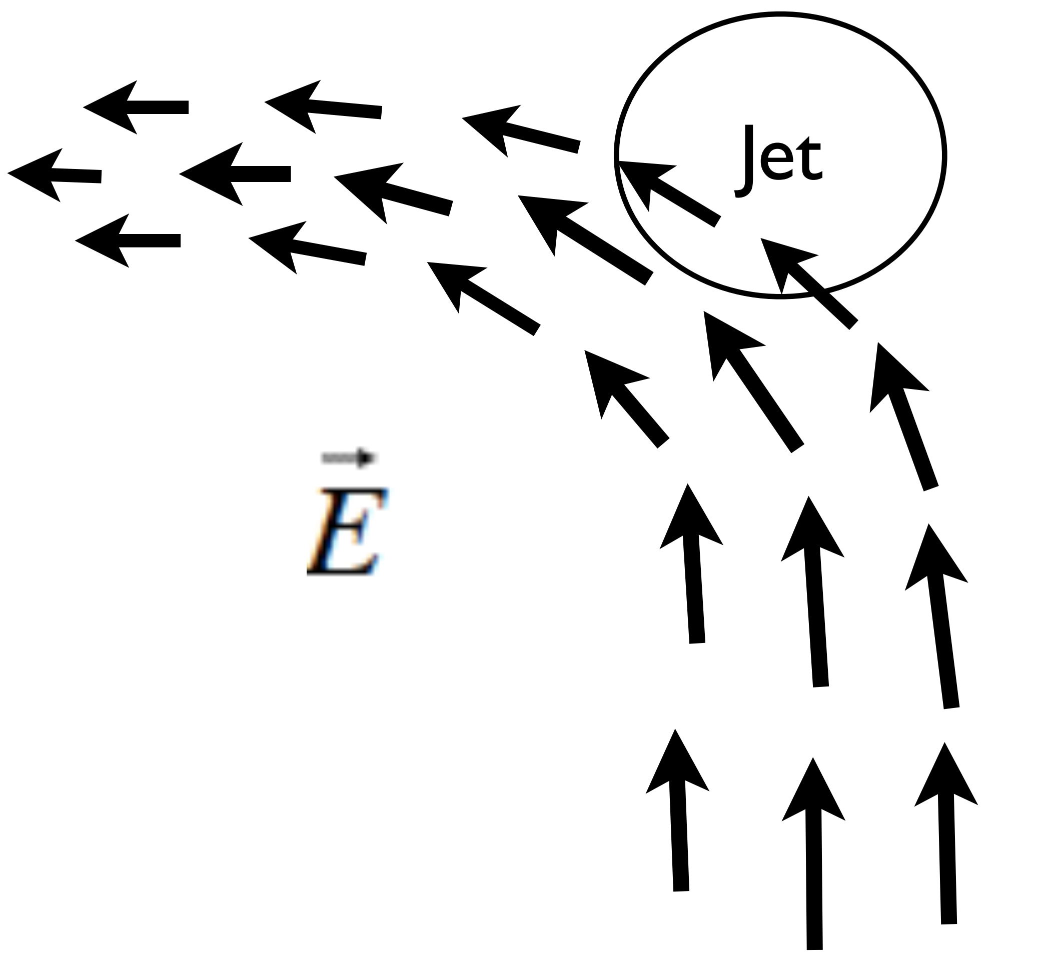
it follows that $\nabla \cdot \vec{E} = 0$

So what?

In the absence of transience and dissipation, the eddies do not interact with the zonal flow.

They call that the *non-interaction theorem*.
It takes 4 pages of equations to prove it.
But like I said, so what?

Consider, for example, the polar night jet.....

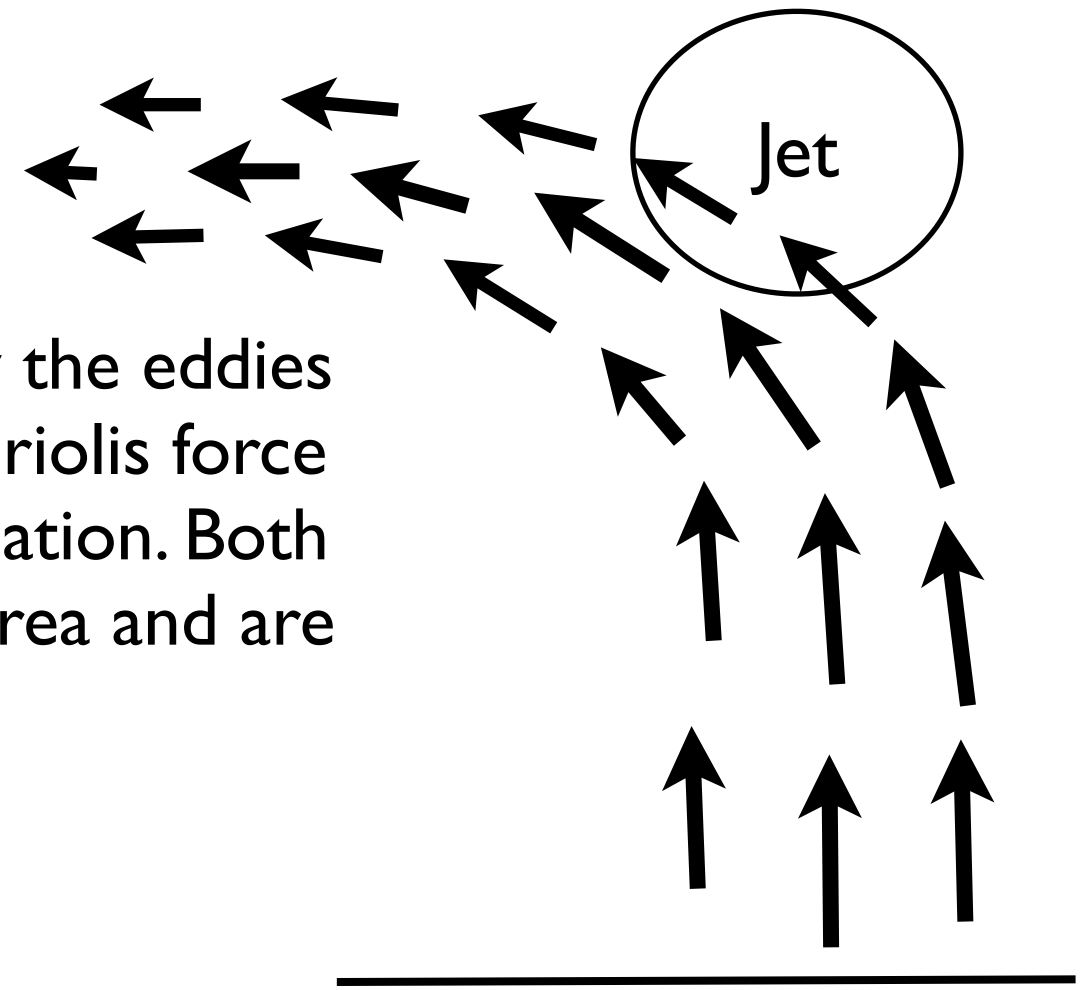


Most of the time the EP fluxes turn equatorward in or just below it and

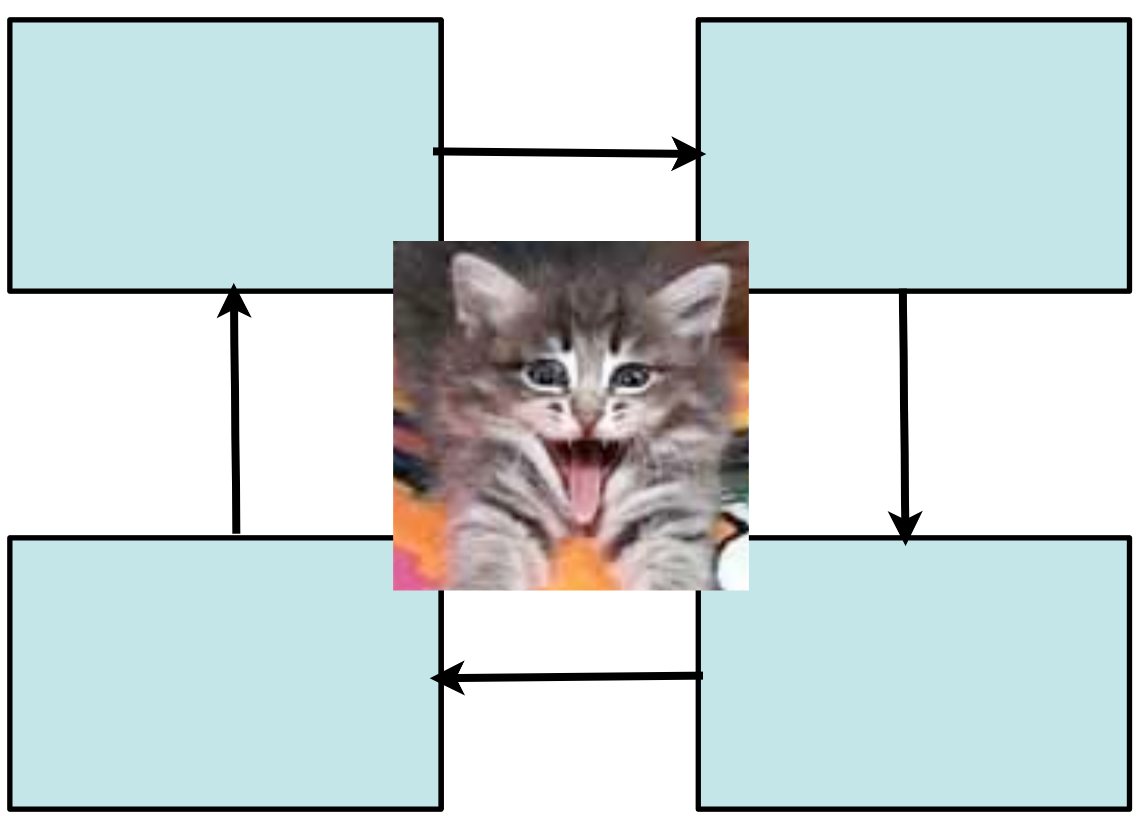
$$\nabla \cdot \vec{E} = 0$$

There is no interaction between the waves and the mean flow.

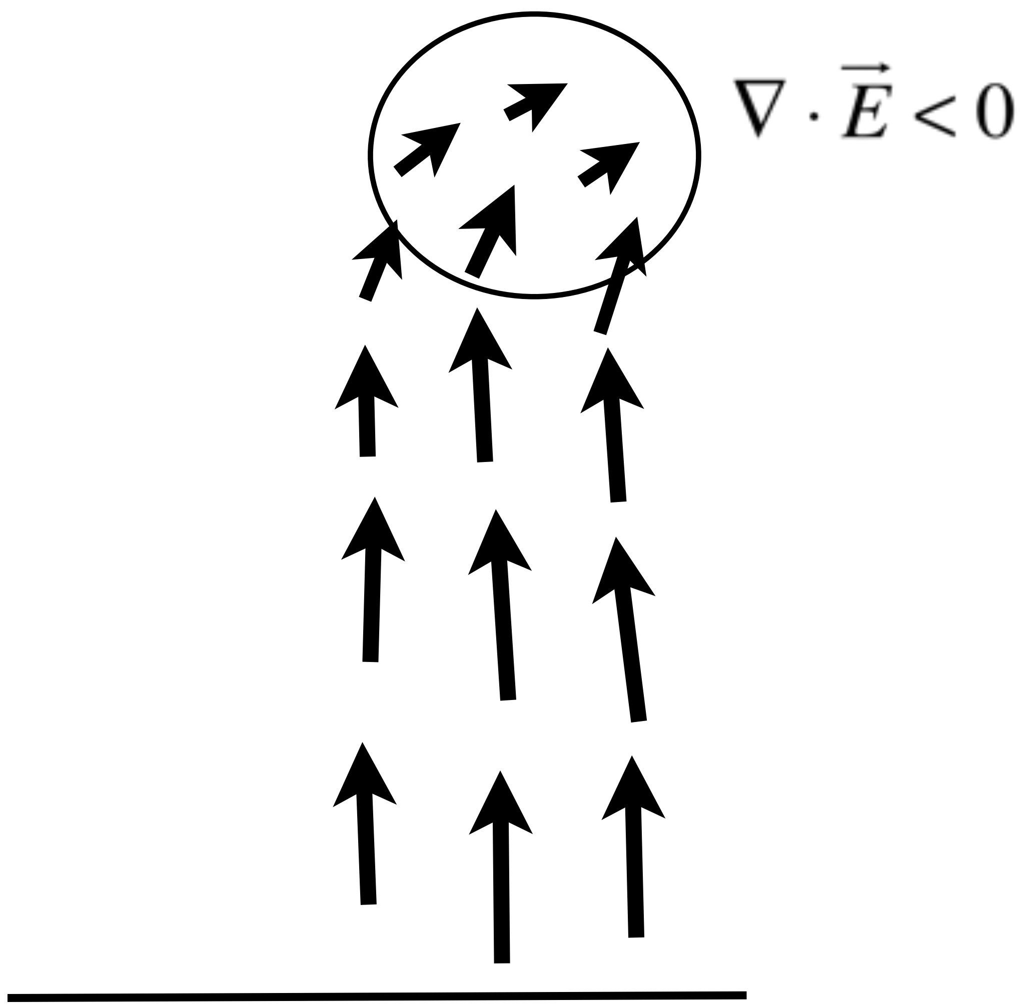
$\nabla \cdot \vec{E} < 0$



Easterly tendency induced by the eddies balanced by the eastward Coriolis force induced by the diabatic circulation. Both are distributed over a wide area and are therefore weak.

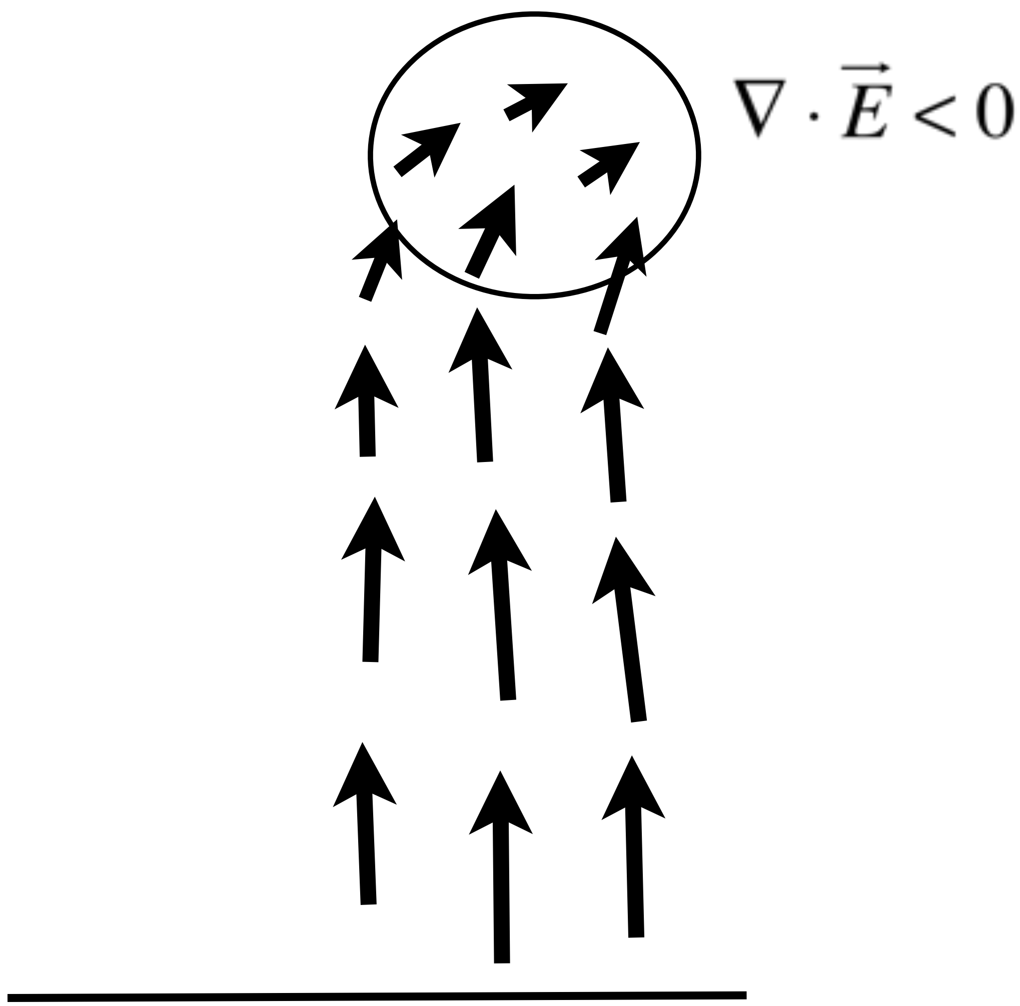


That's when we get awesome conversions, but nothing happens because $\nabla \cdot \vec{E} = 0$ near the jet.



The convergence is concentrated within a small area and is therefore much stronger than the diabatically-induced westerly acceleration.

But sometimes the fluxes converge into the jet, producing a sudden warming.



The jet gets zapped!!

What's so special about $\nabla \cdot \vec{E} < 0$?

Haven't you ever heard of *wave breaking* ?

and critical levels?

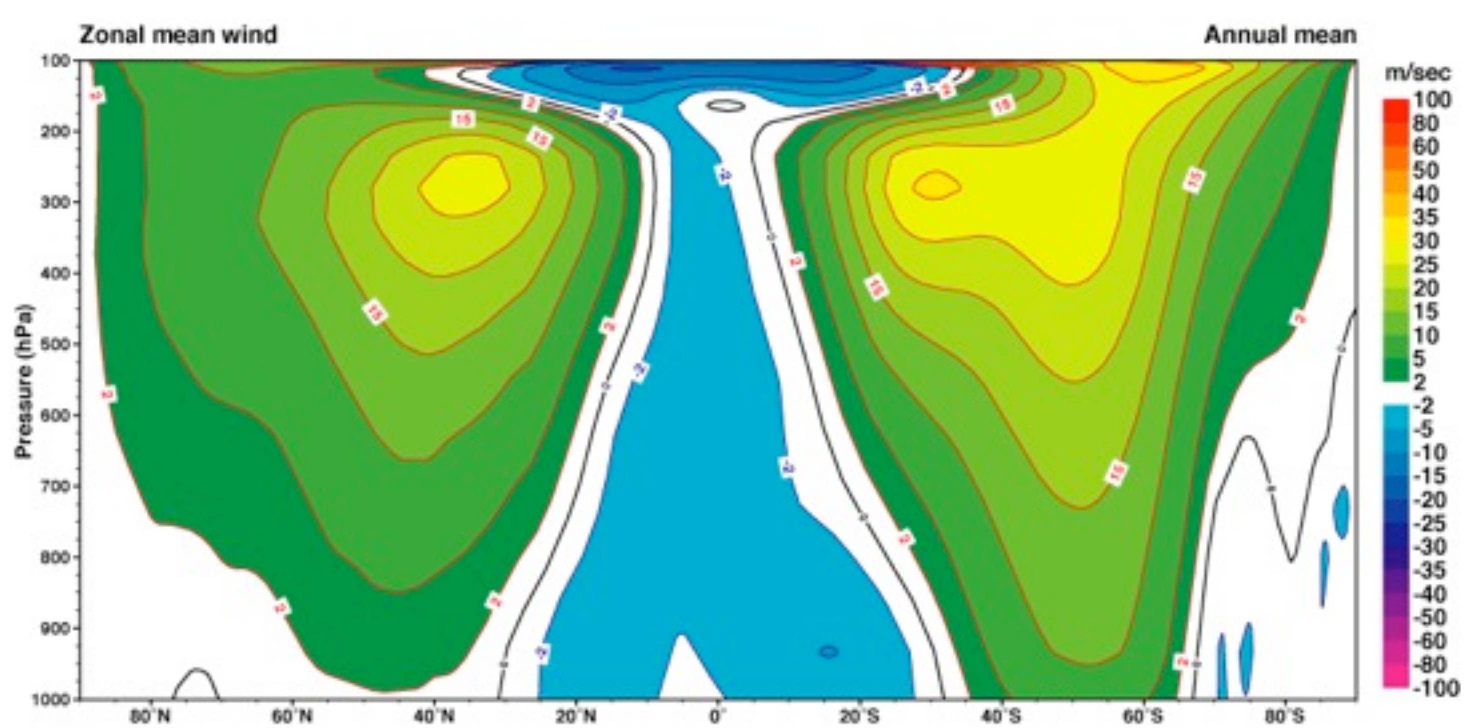
A critical level is where the doppler-shifted phase speed of a wave is zero; i.e., where $u = c$.

We can speak of a critical level, a critical latitude, or a critical line in the meridional plane along which $u = c$.

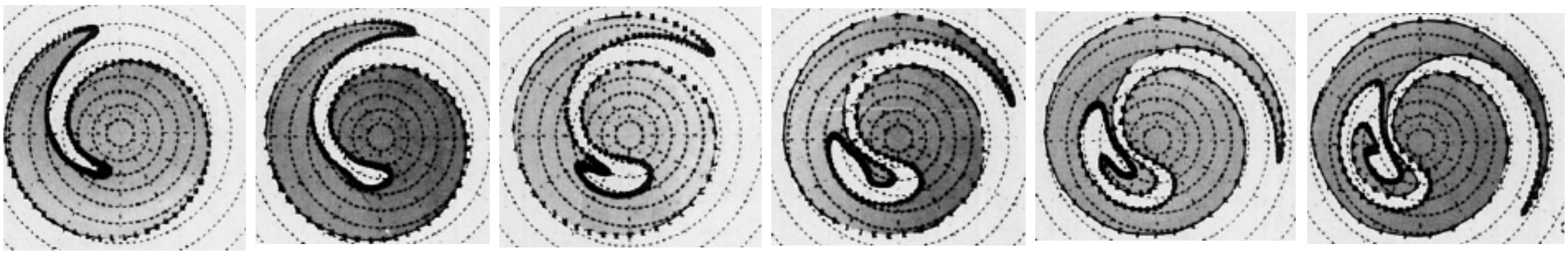
The steering level is a critical level.

For stationary waves the critical line corresponds to the zero isotach.

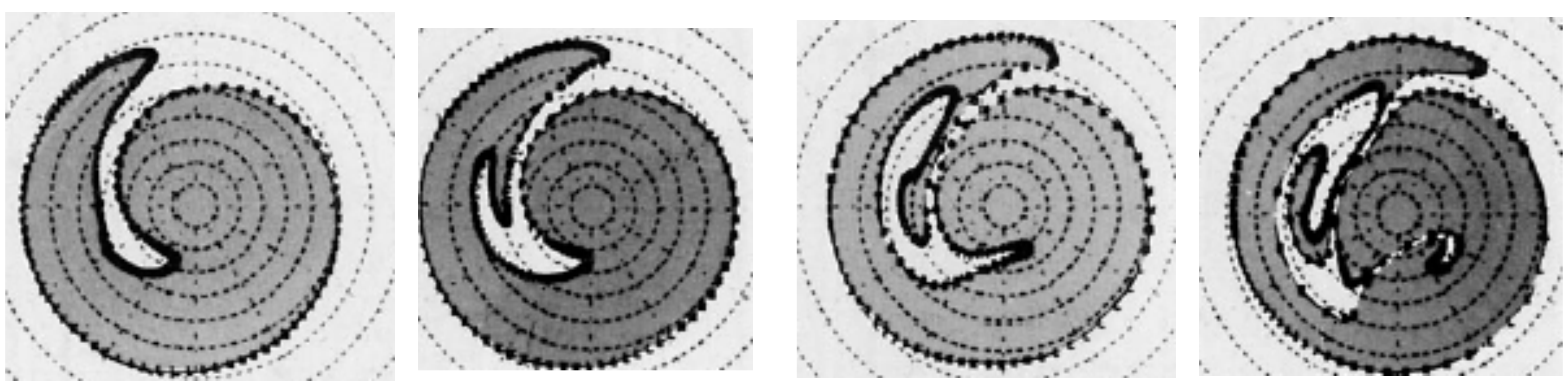
For baroclinic waves it corresponds roughly to the 10 m/s isotach.



As waves approach a critical line in the meridional plane, the period of the air trajectories in the waves approaches infinity and the motion ceases to be wavelike.

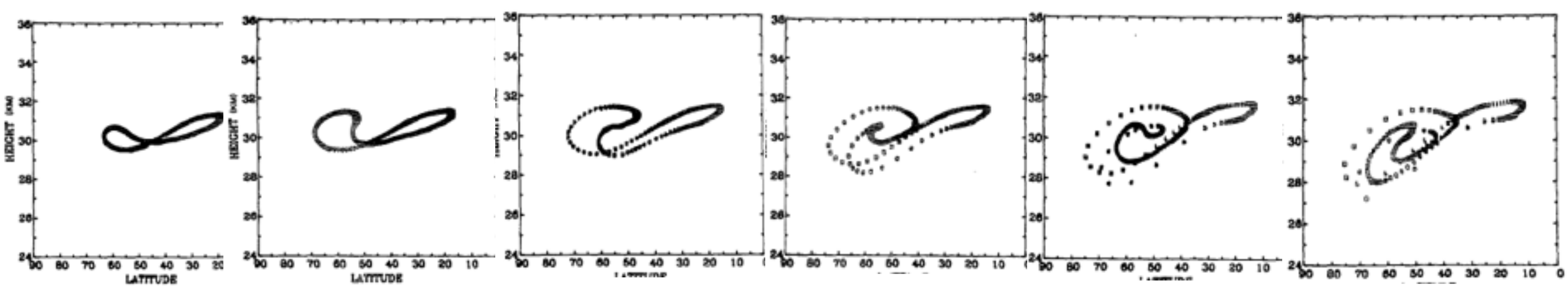


linear



After C. P. F. Hsu, JAS, 1981

nonlinear



Waves approach a critical lines in the meridional plane exponentially and as they do so they become more subject to dissipation (Newtonian cooling, eddy diffusion).

Recall that

$$[v^*\Phi^*] = -(u-c)[u^*v^*]$$

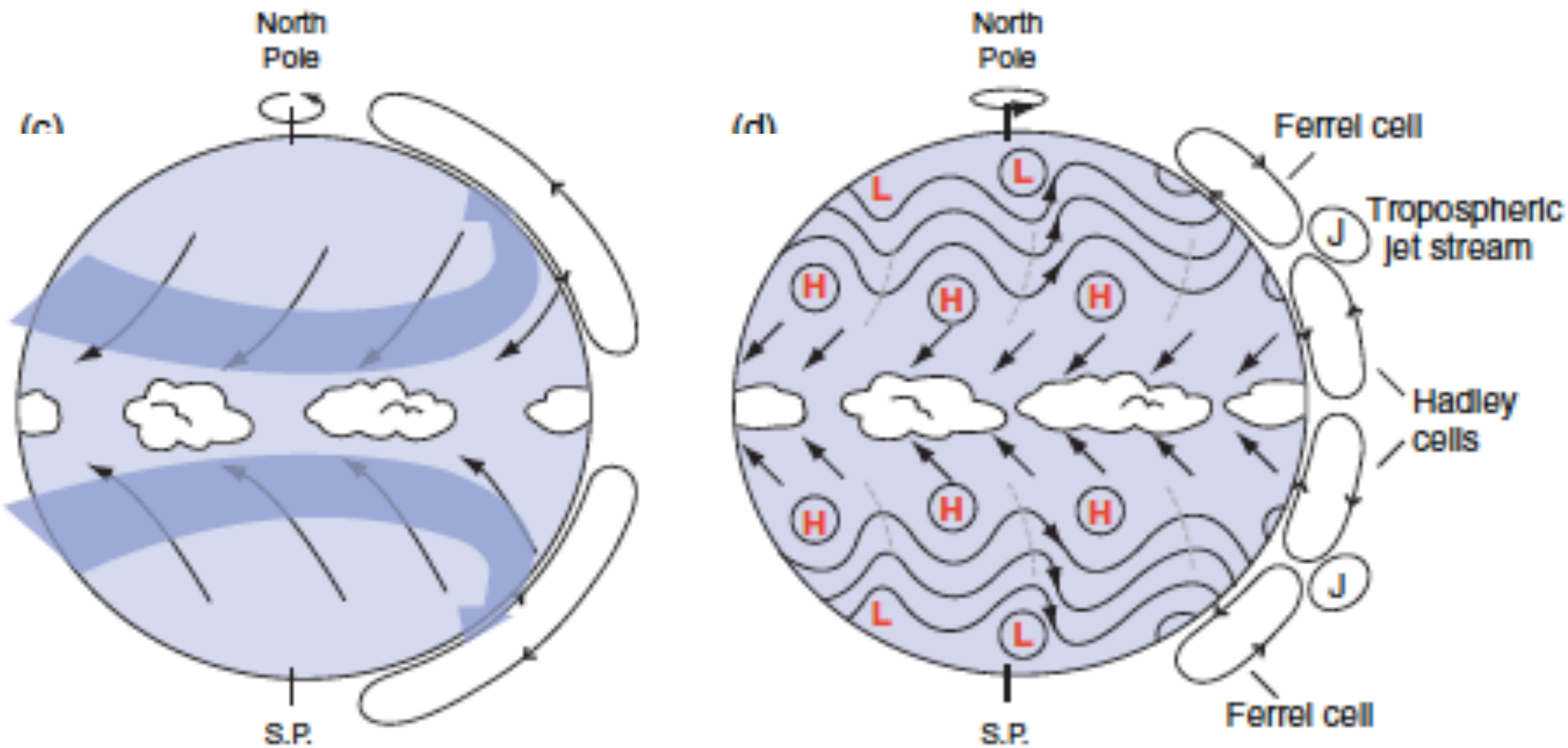
$$[\omega^*\Phi^*] = (u-c)[v^*T^*]$$

Hence, the work term approaches zero as the waves approach the critical line

Waves cannot disperse across a critical line

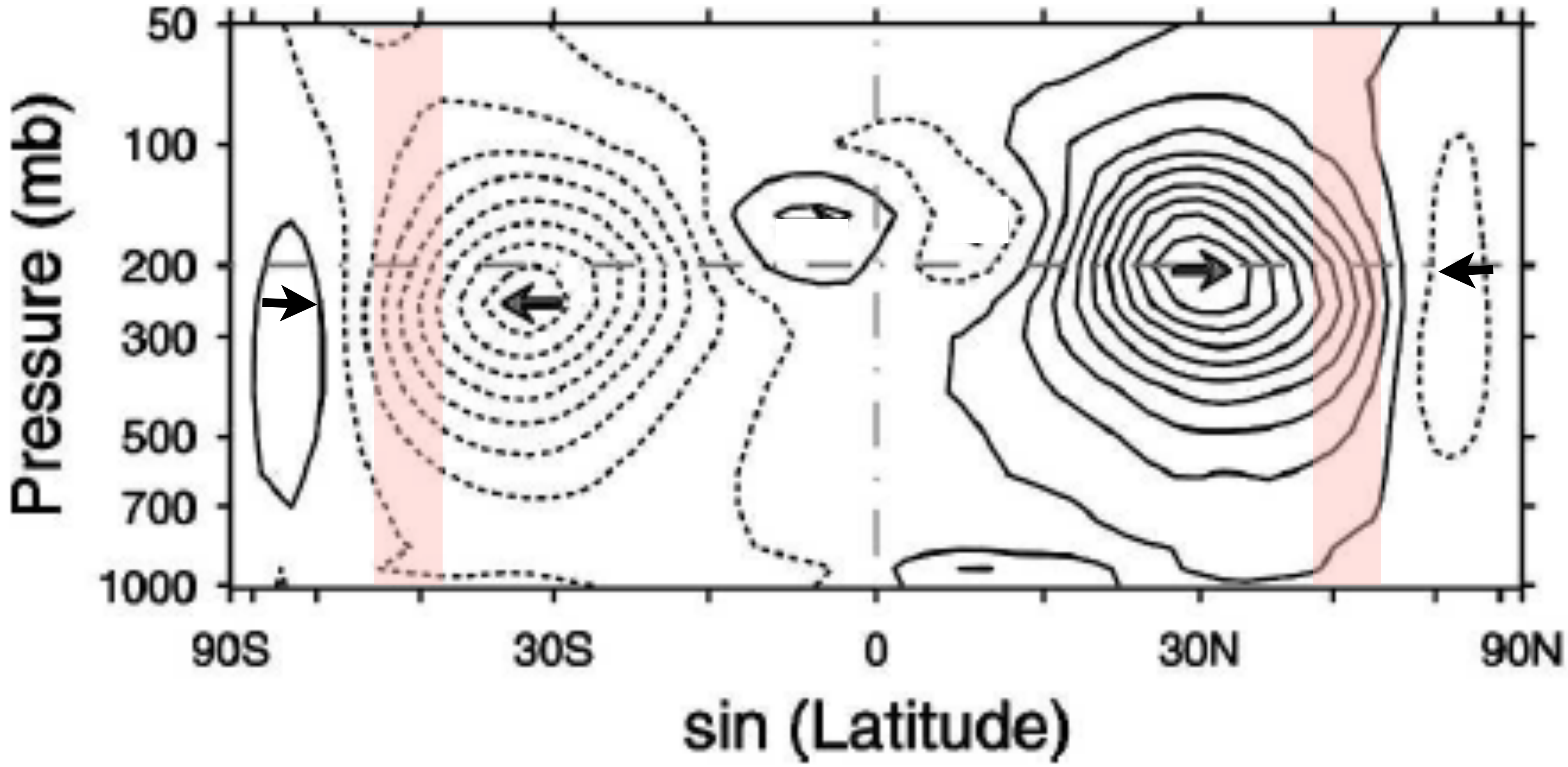
Under certain conditions waves may be reflected from a critical line

Now we are in a better position to explain why there are surface westerlies in the extratropics and easterly "trade winds" in the tropics



Atmosphere with flow driven by meridional gradient of diabatic heating (left). The meridional temperature gradient and vertical wind shear strengthen until baroclinic instability develops in extratropical latitudes. Wave activity disperses equatorward out of the storm track and zonal momentum is transported poleward (right).

Climatological-(annual) mean eddy flux of westerly momentum



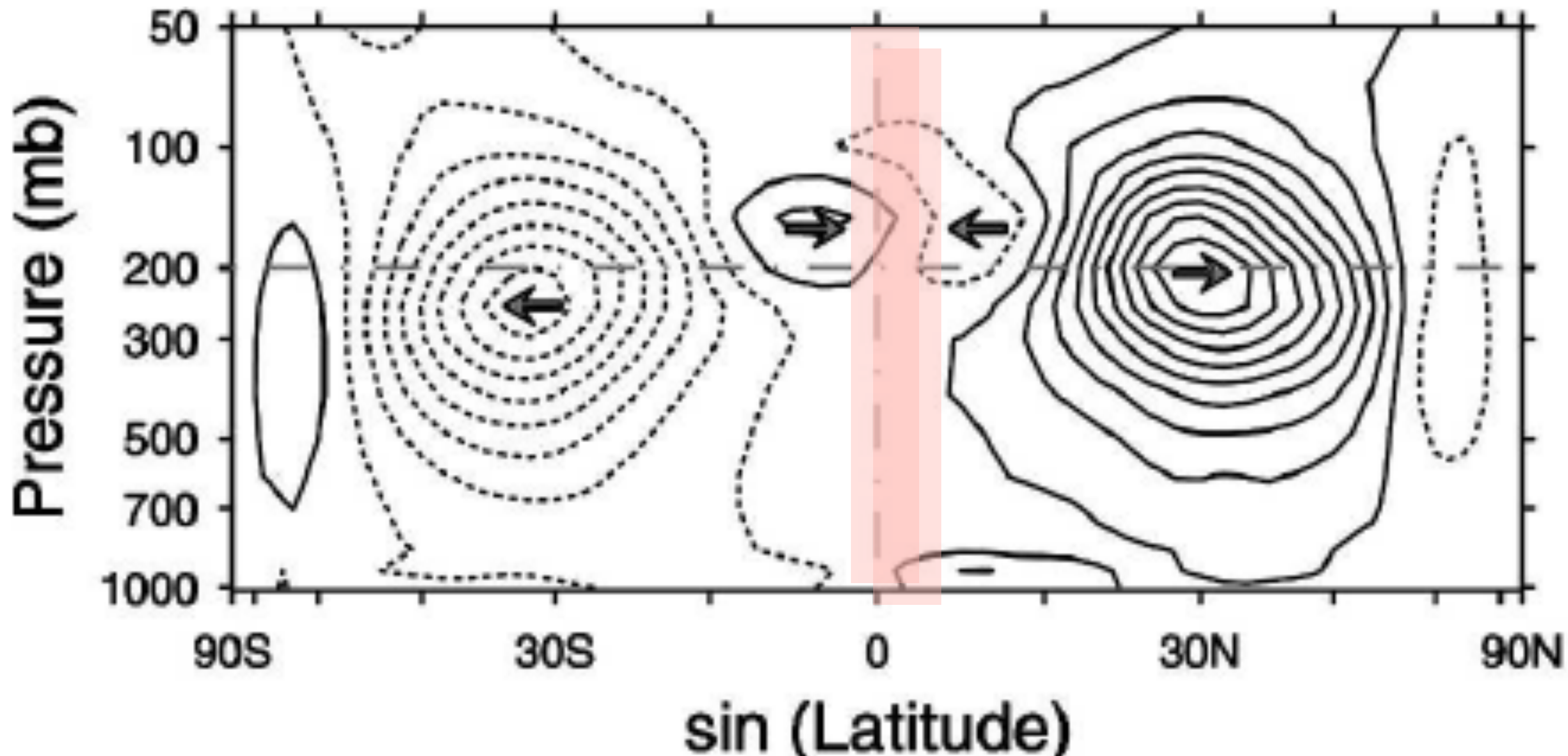
NCEP reanalysis data

Dima, Kraucunas and Wallace JAS 2005

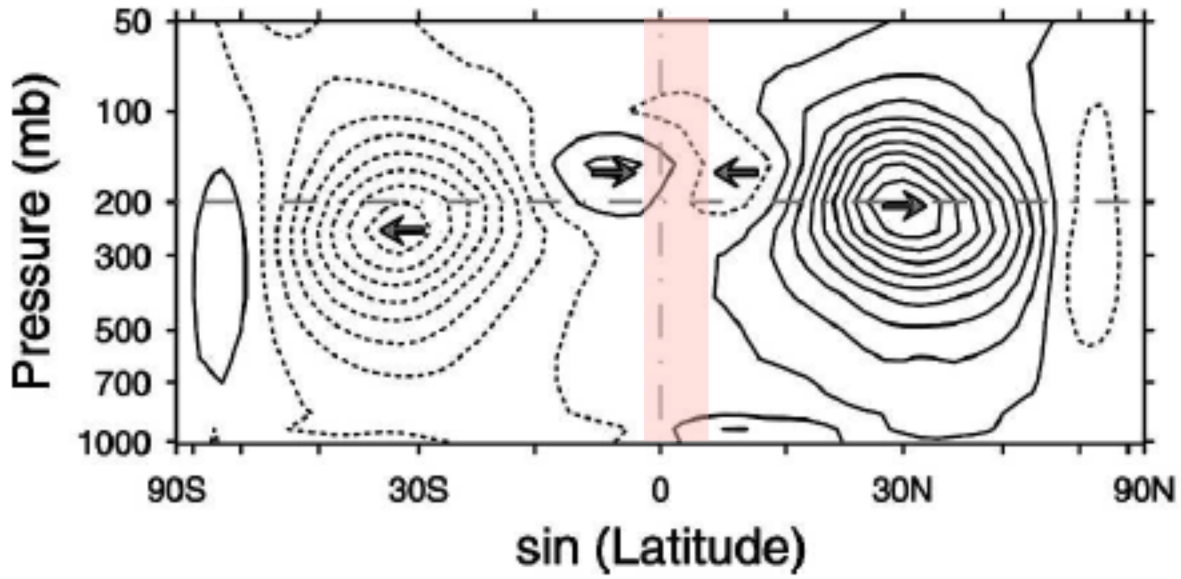
Shaded bands indicate extratropical storm tracks.

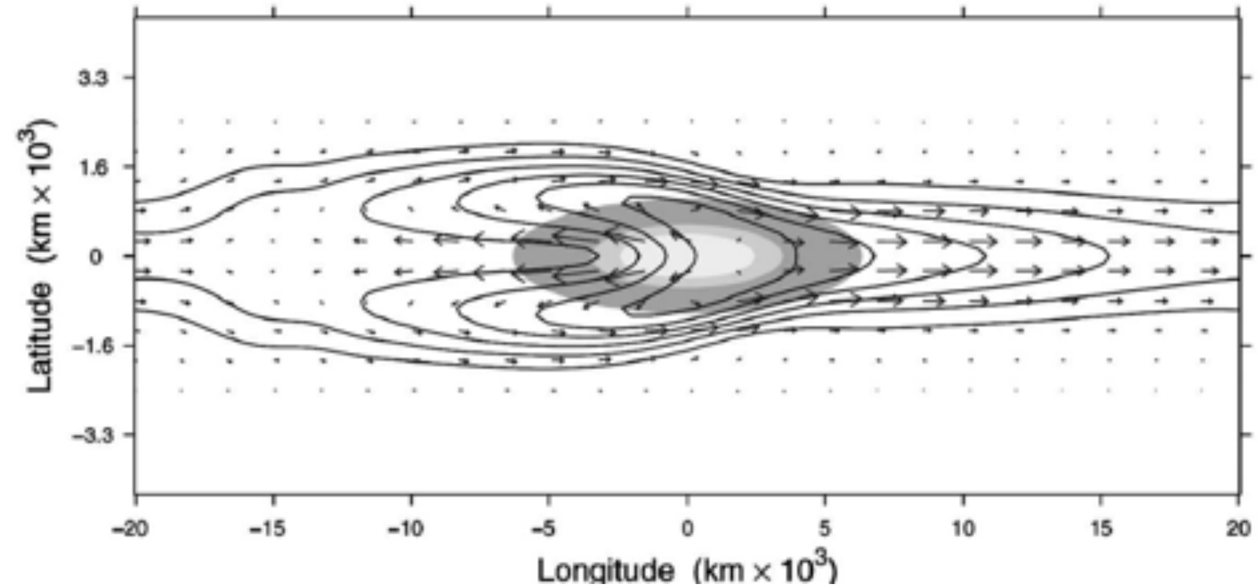
Note flux of momentum out of the tropics and into the extratropical storm tracks.

What about those wiggles in the tropics?

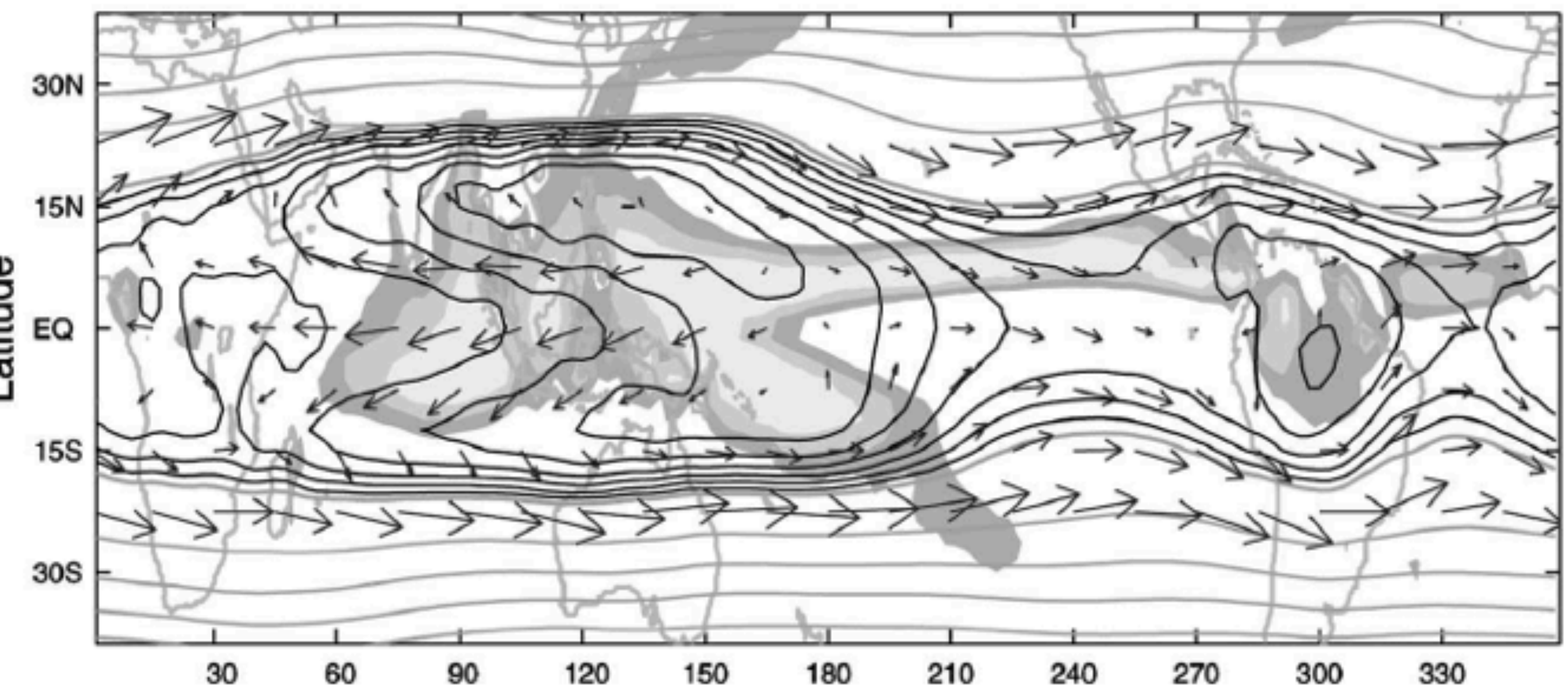


I'm glad you asked. They are the signature of the equatorial planetary waves.



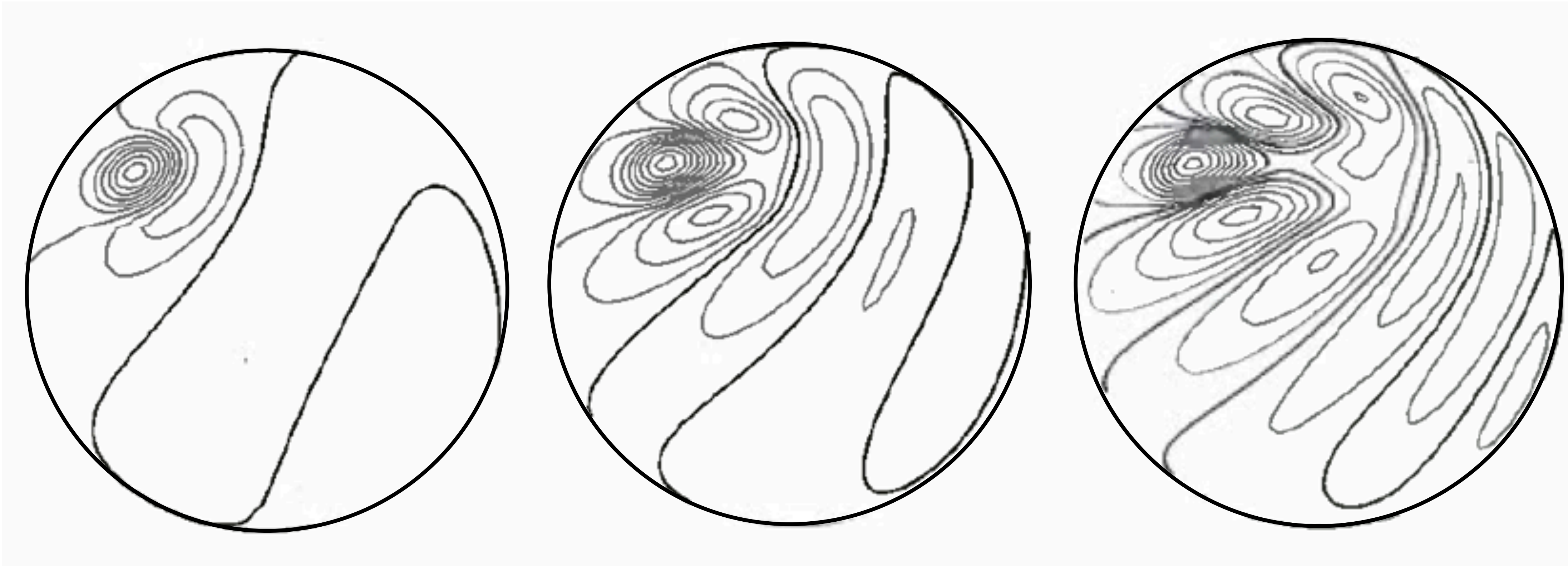


Response to idealized equatorial heat source



annual-mean 150 hPa wind and geopotential height
shading denotes precipitation

Note the flux of momentum toward the equator.



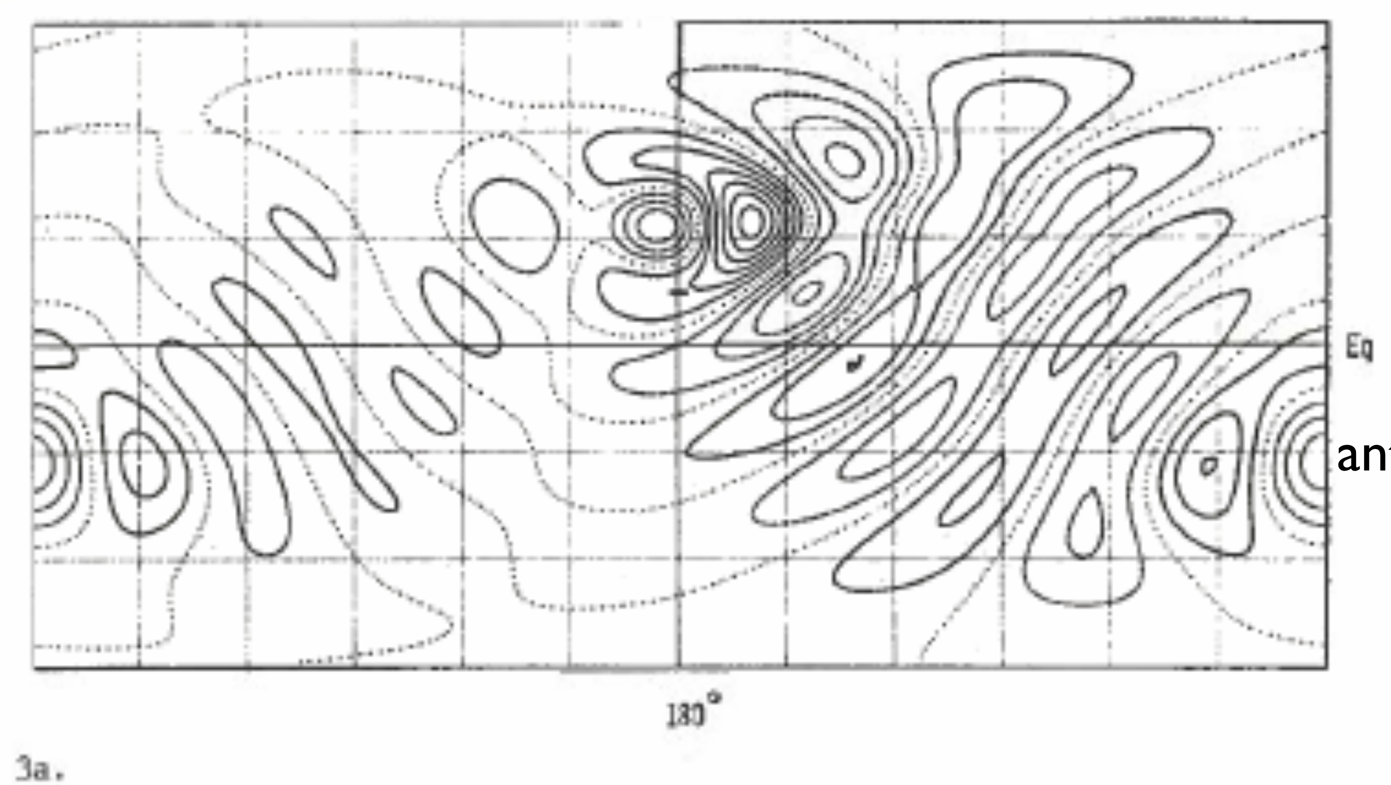
vorticity field

after B. J. Hoskins, QJRMS 1983

Wavetrain generated in a barotropic model by flow over a small circular mountain at 30°N 2, 4, and 6 days after the mountain is "turned on". Note and the apparent meridional group velocity, the tilt of the wave axes, and the implied flux of westerly momentum

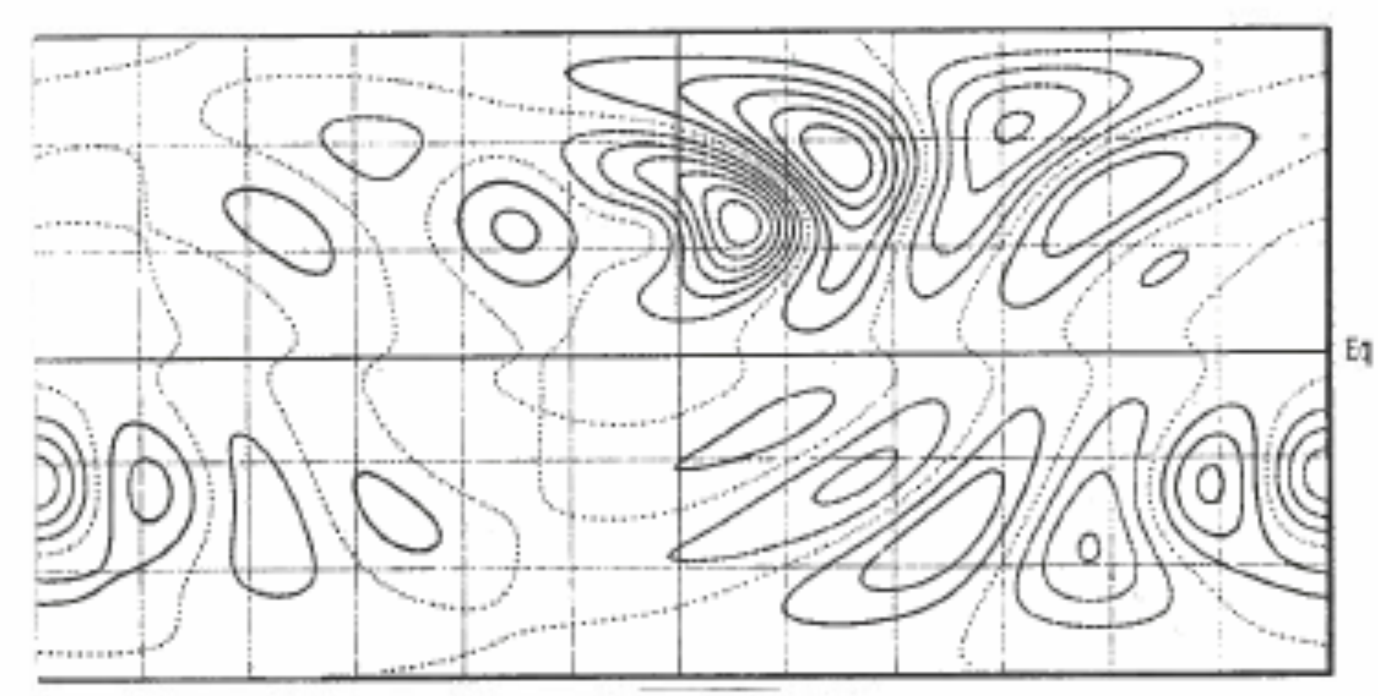
Group velocity follows great circle routes around the Earth, through the antipodal point. Note how the steady state solution depends on the amount of dissipation.

Cylindrical projection



vorticity

antipodal point

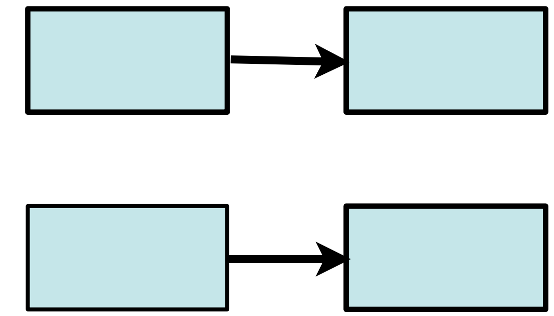


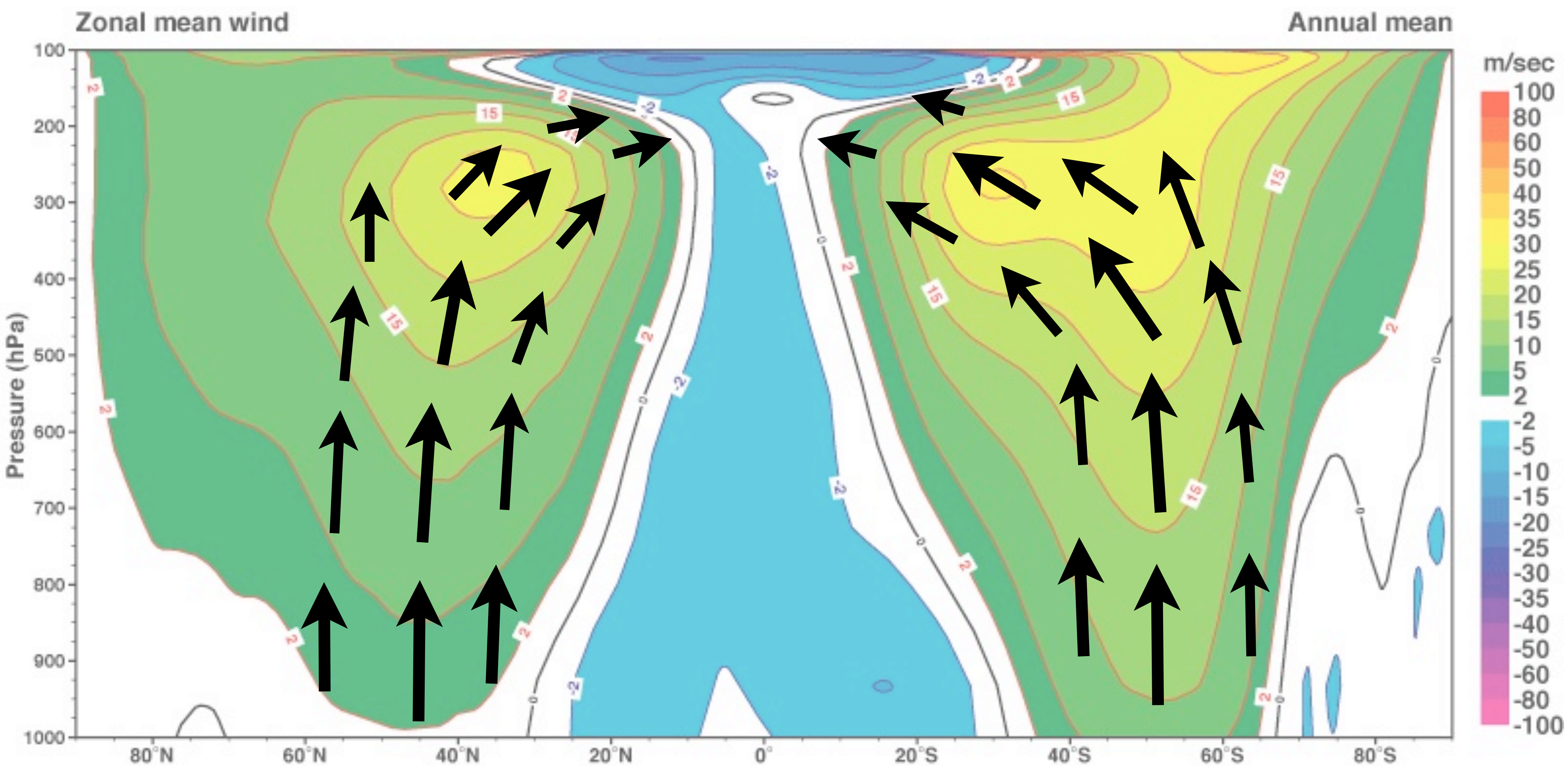
geopotential height

$$C_A + C_K = -f\frac{[v^*\alpha^*]}{\sigma}\frac{\partial[\alpha]}{\partial y} - [u^*v^*]\frac{\partial[u]}{\partial y}$$

$$= -\frac{[v^*\alpha^*]}{\sigma}\frac{\partial[u]}{\partial p} - [u^*v^*]\frac{\partial[u]}{\partial y}$$

$$= \vec{E}\cdot\nabla[u]$$

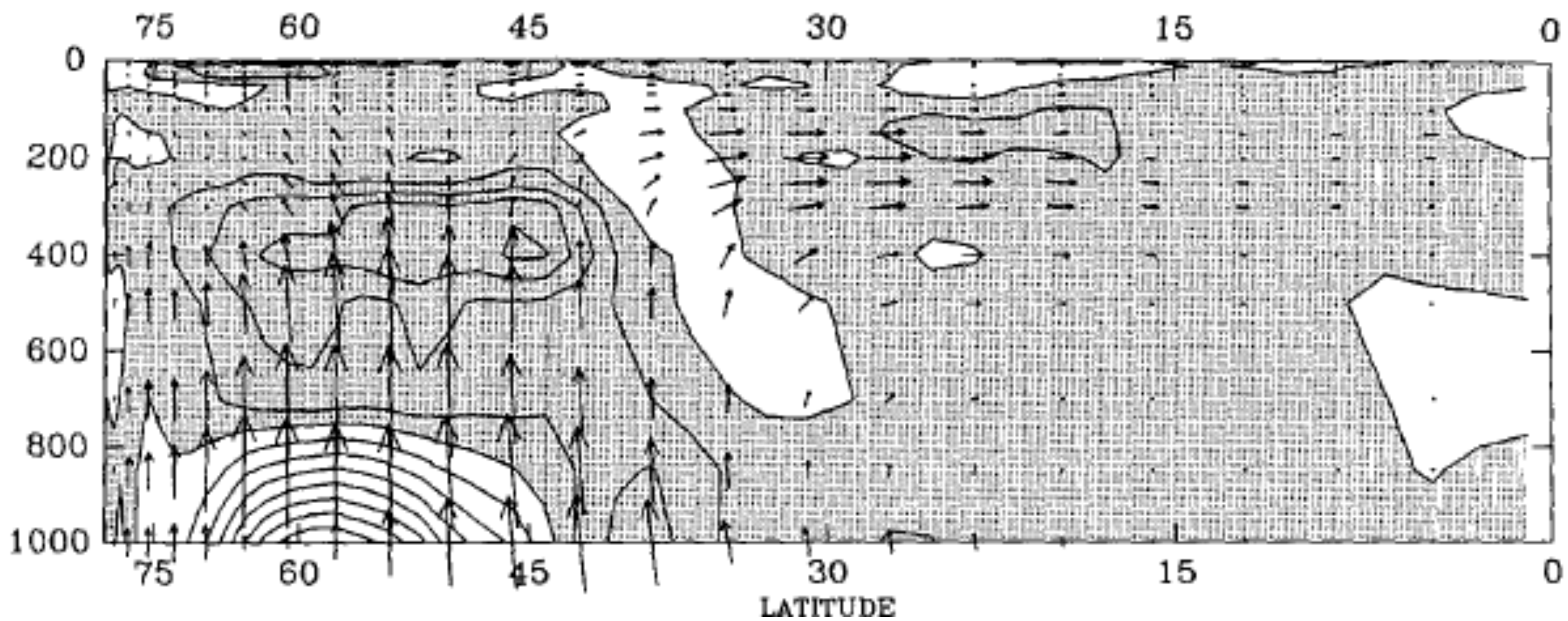




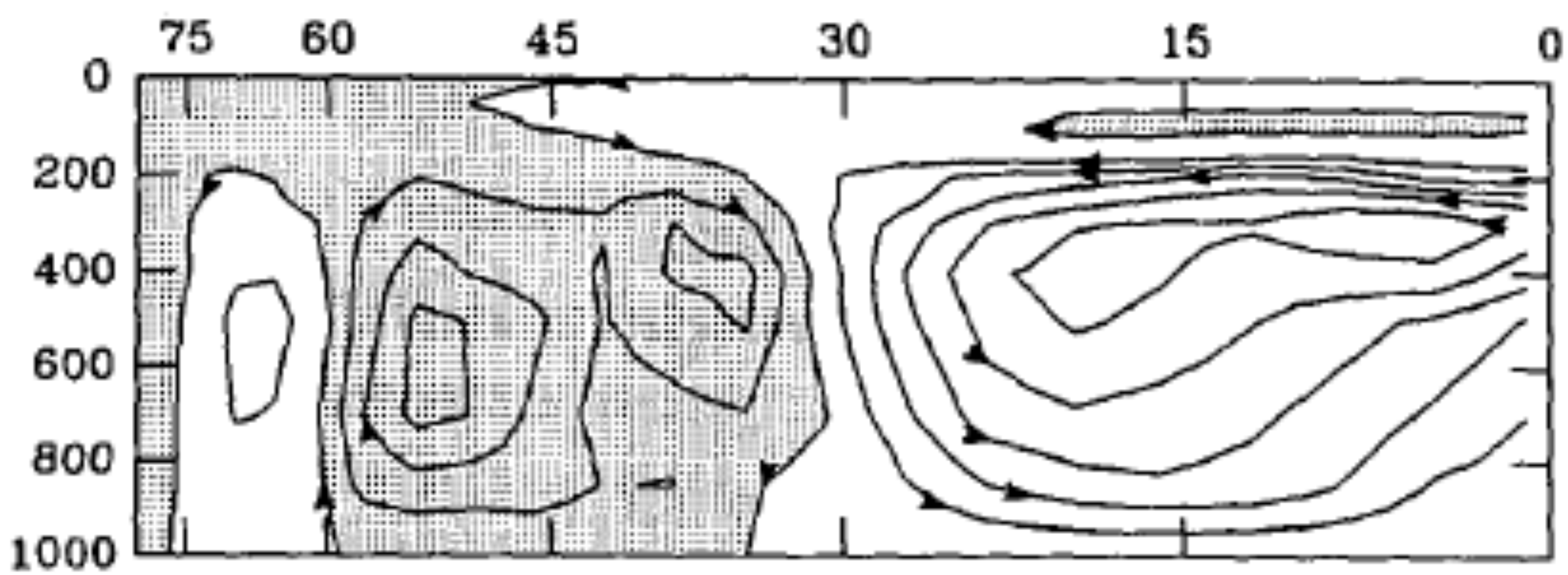
$\vec{E}$ is mainly countergradient, so eddies gain energy from zonal flow.

Role of the heat fluxes on the lower boundary

Pfeffer, JAS, 1992

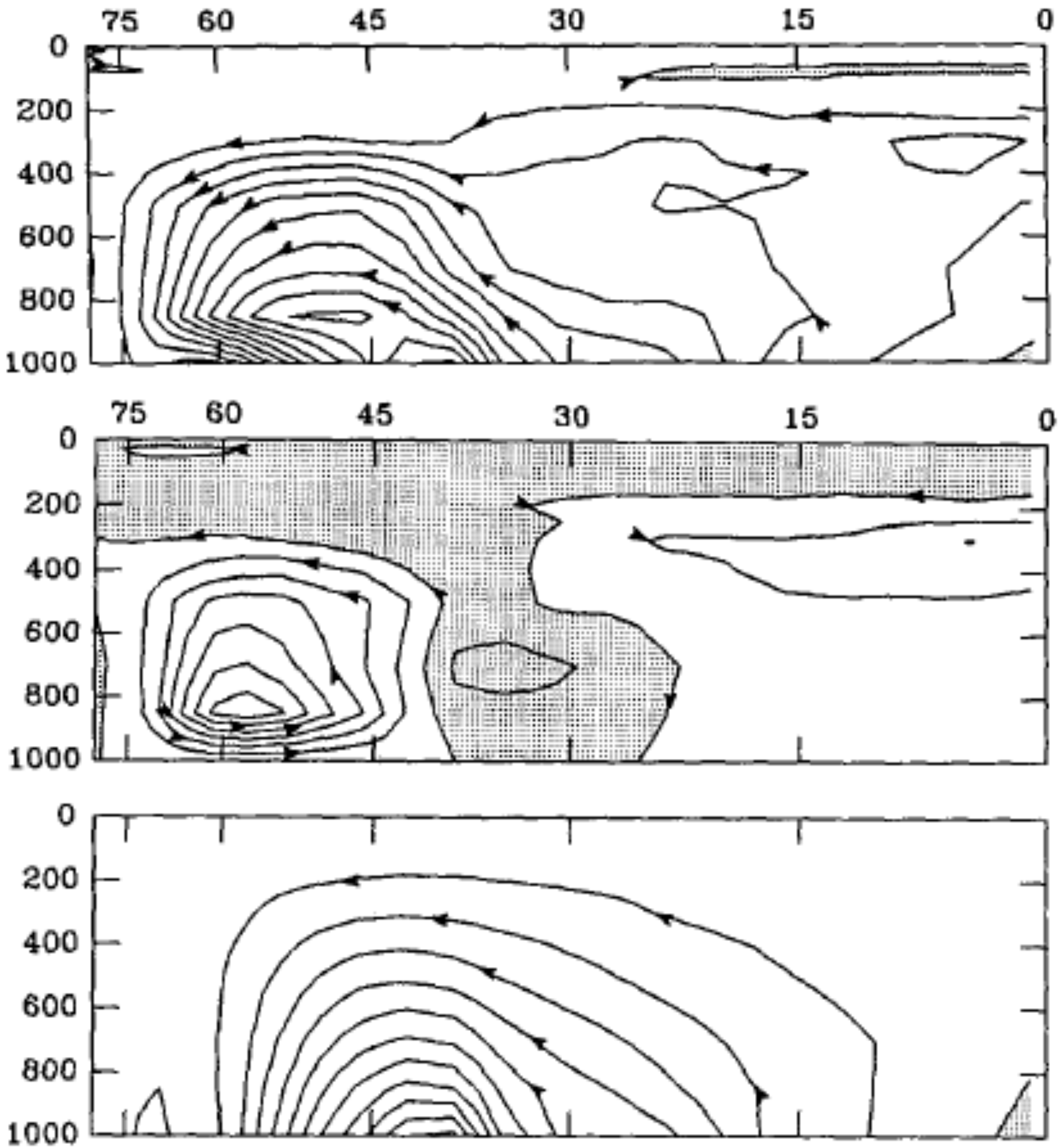


$\vec{E}, \nabla \cdot \vec{E}$



Eulerian MMC

Case study January 9-14, 1979

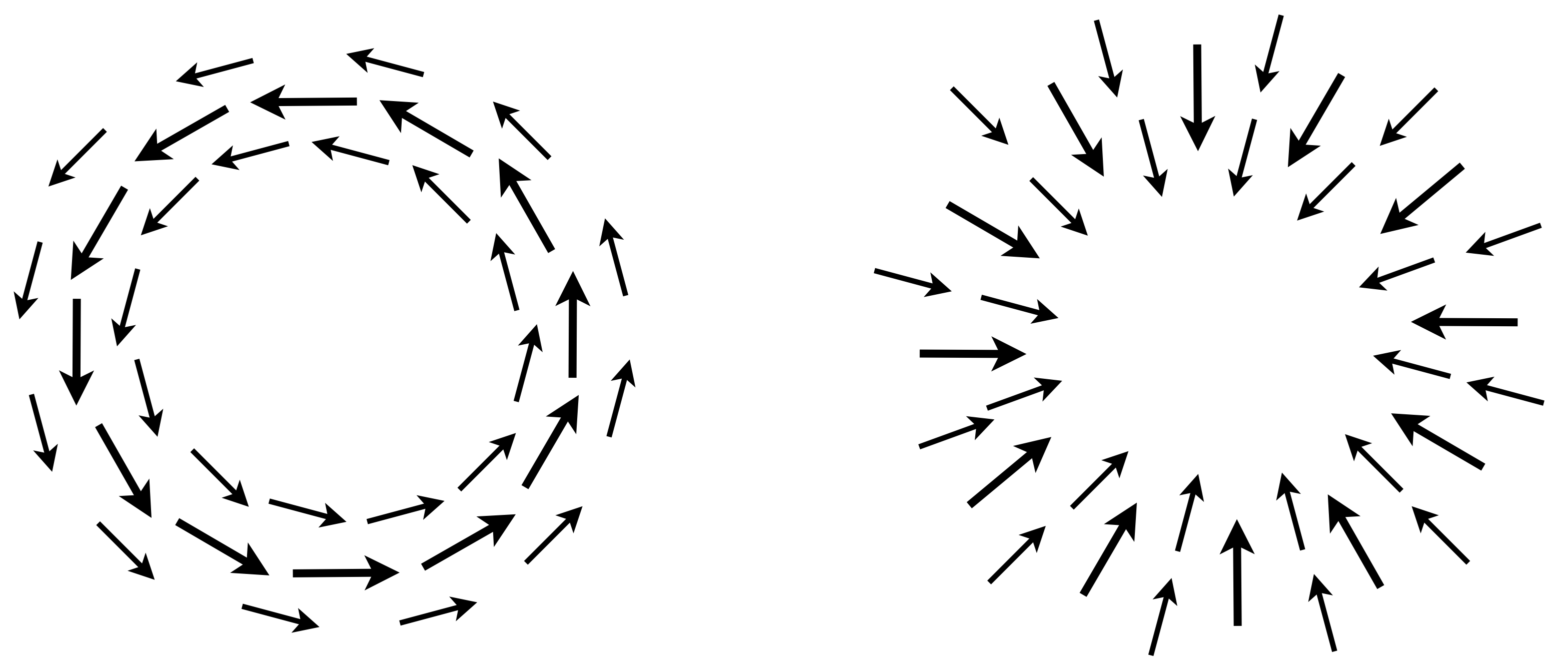


Residual MMC

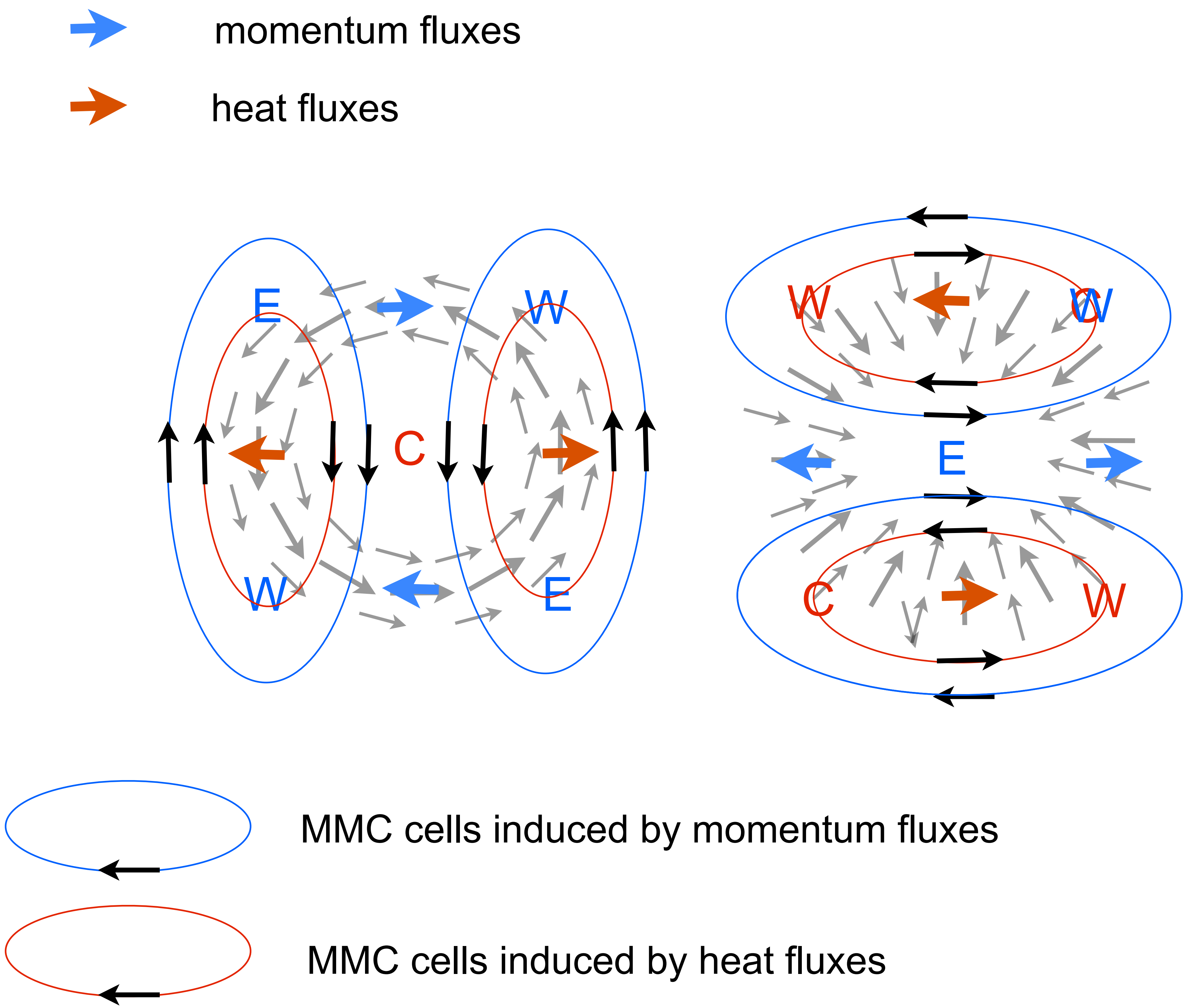
internally forced component

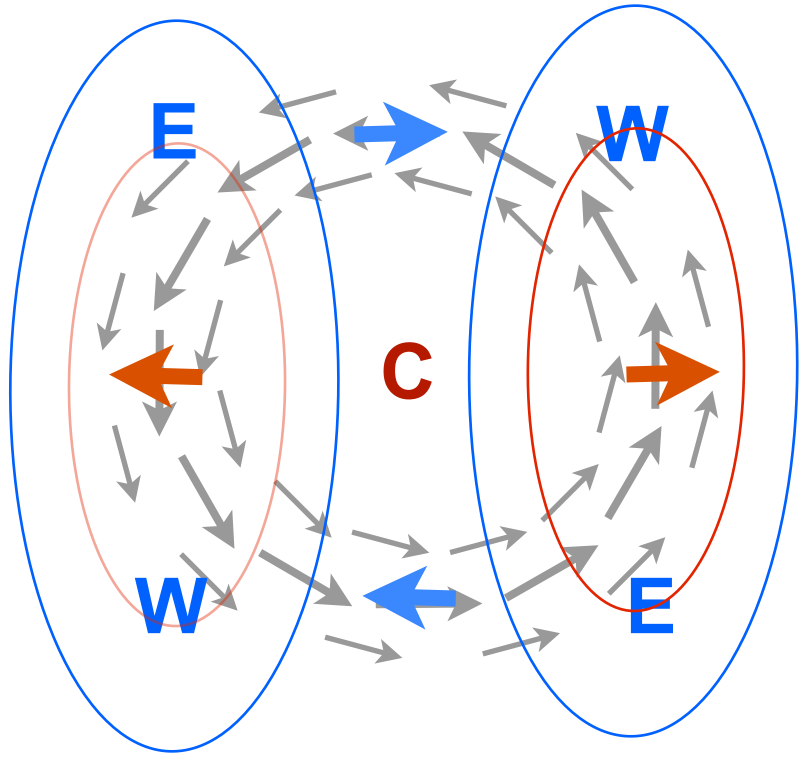
boundary forced component

Review Problem

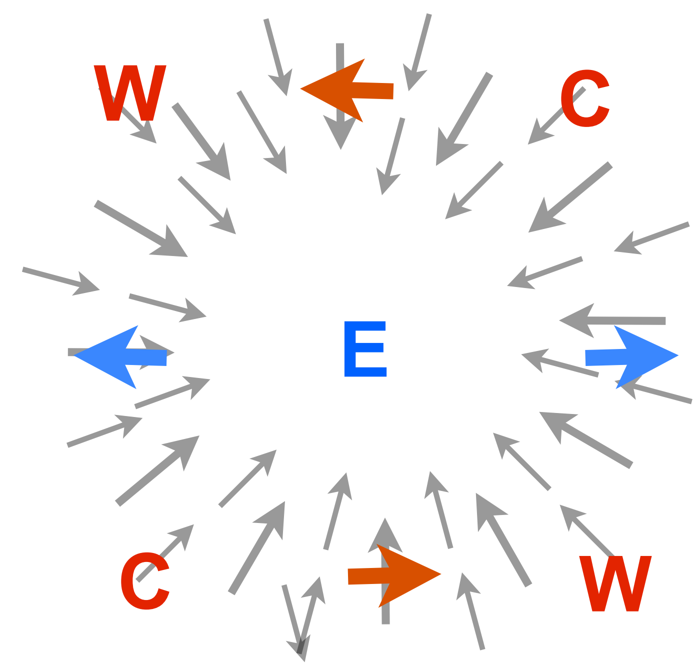


Hypothetical Eliassen-Palm flux distributions in the meridional plane. Diagnose the induced MMC and the resulting changes in zonal wind and temperature.


momentum fluxes
heat fluxes
E
W
C
W
E
W
C
W
E
C
W
MMC cells induced by momentum fluxes
MMC cells induced by heat fluxes



Eddy-induced tendencies force large departures from geostrophic balance, resulting in strong MMC but little net change in zonal wind or temperature.



Eddy-induced tendencies force geostrophic balanced, tendencies, resulting in a strong easterly acceleration where the fluxes converge and weak MMC.