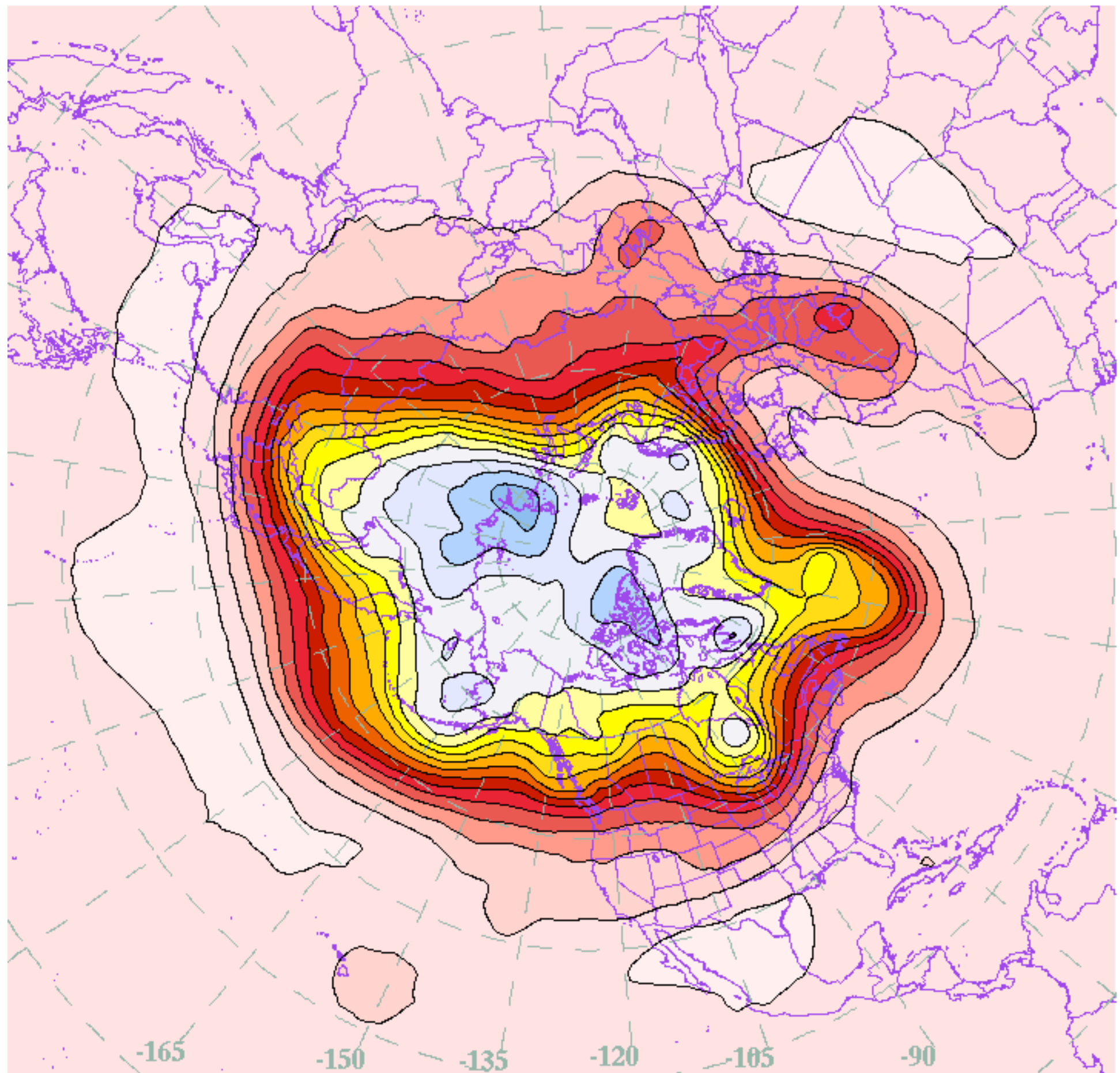


Time lapse animation  
daily fields, unfiltered  
500 hPa height field

*courtesy of David Ovens*

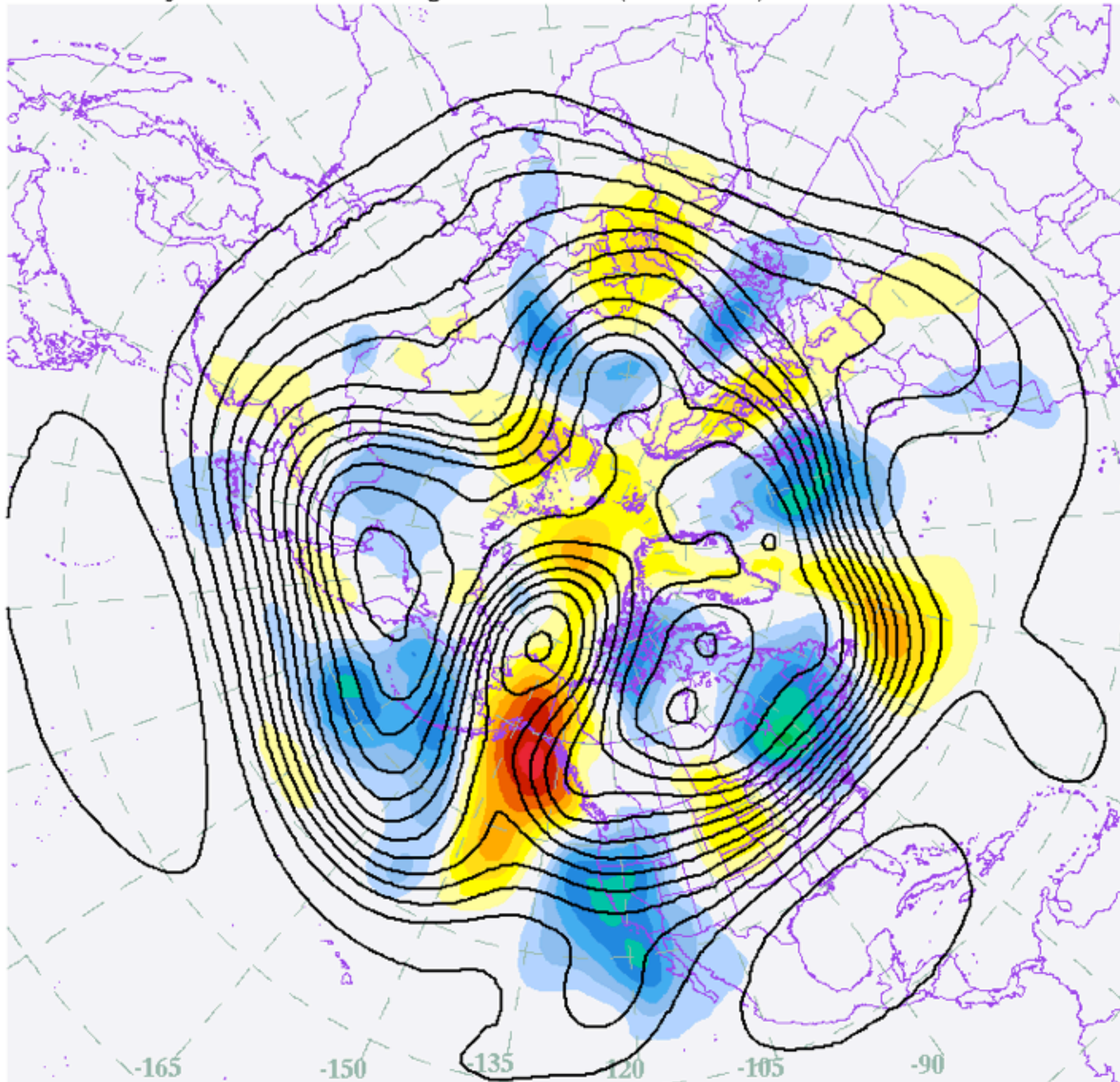


Lowpass filter: 5-day running mean

Highpass filter: departure from....



5-day centered 500 MB Heights/Anomalies (dekameters) valid 00Z 01 Dec 2007

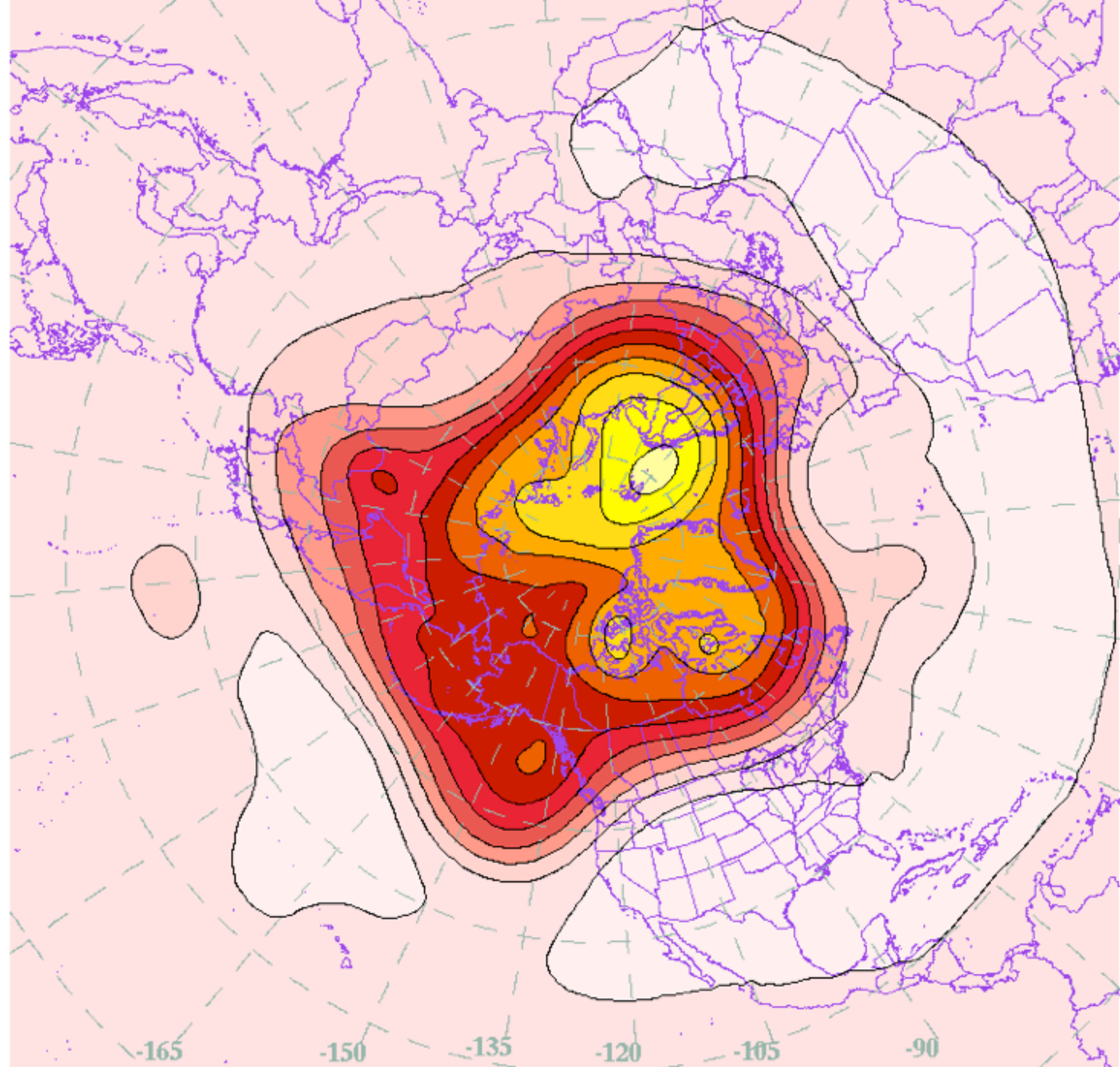


Univ. of Washington Dept. of Atm. Sci.



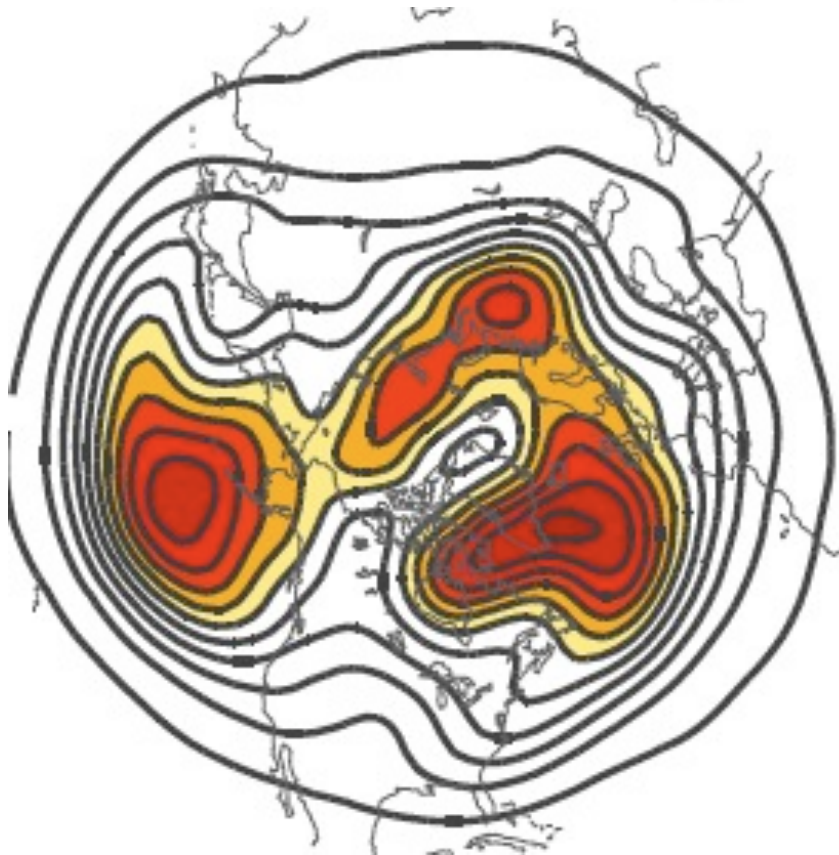
Time lapse animation  
lowpass filtered  
500 hPa height field

*courtesy of David Ovens*

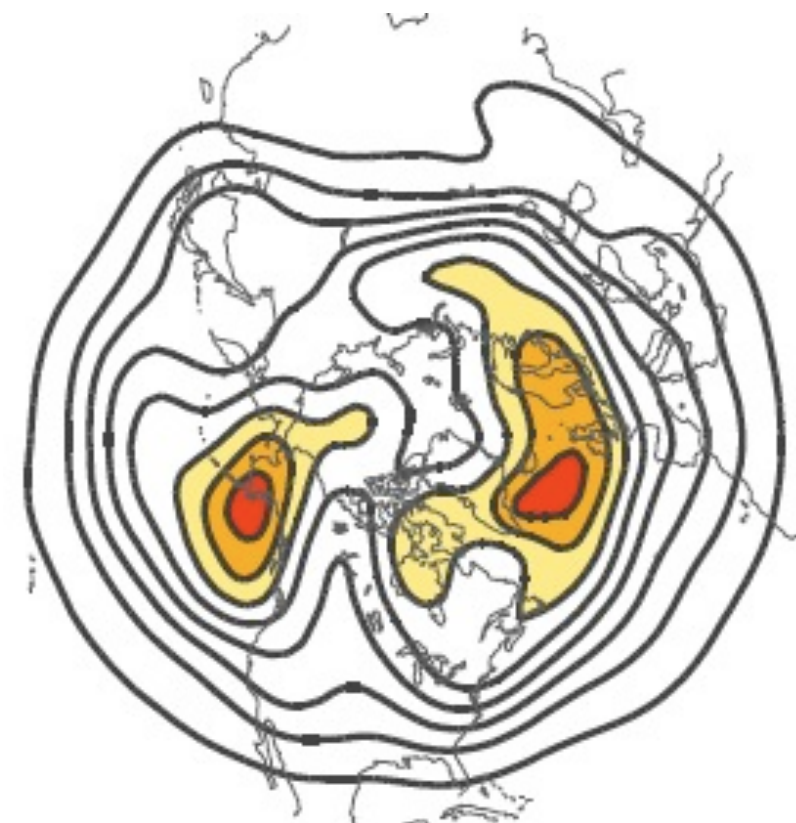


# 500 hPa height variance DJF

30 day low pass ( $Z_{30}$ )



6–30 day band pass ( $z_{\text{int}}$ )

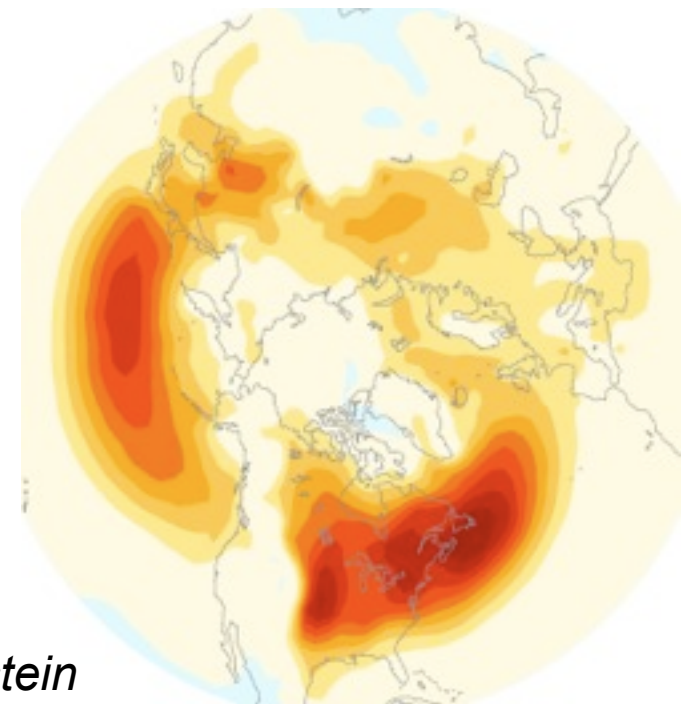


6 day high pass ( $z_{\text{HP}}$ )



*after Rennert and Wallace JAS, 2009*

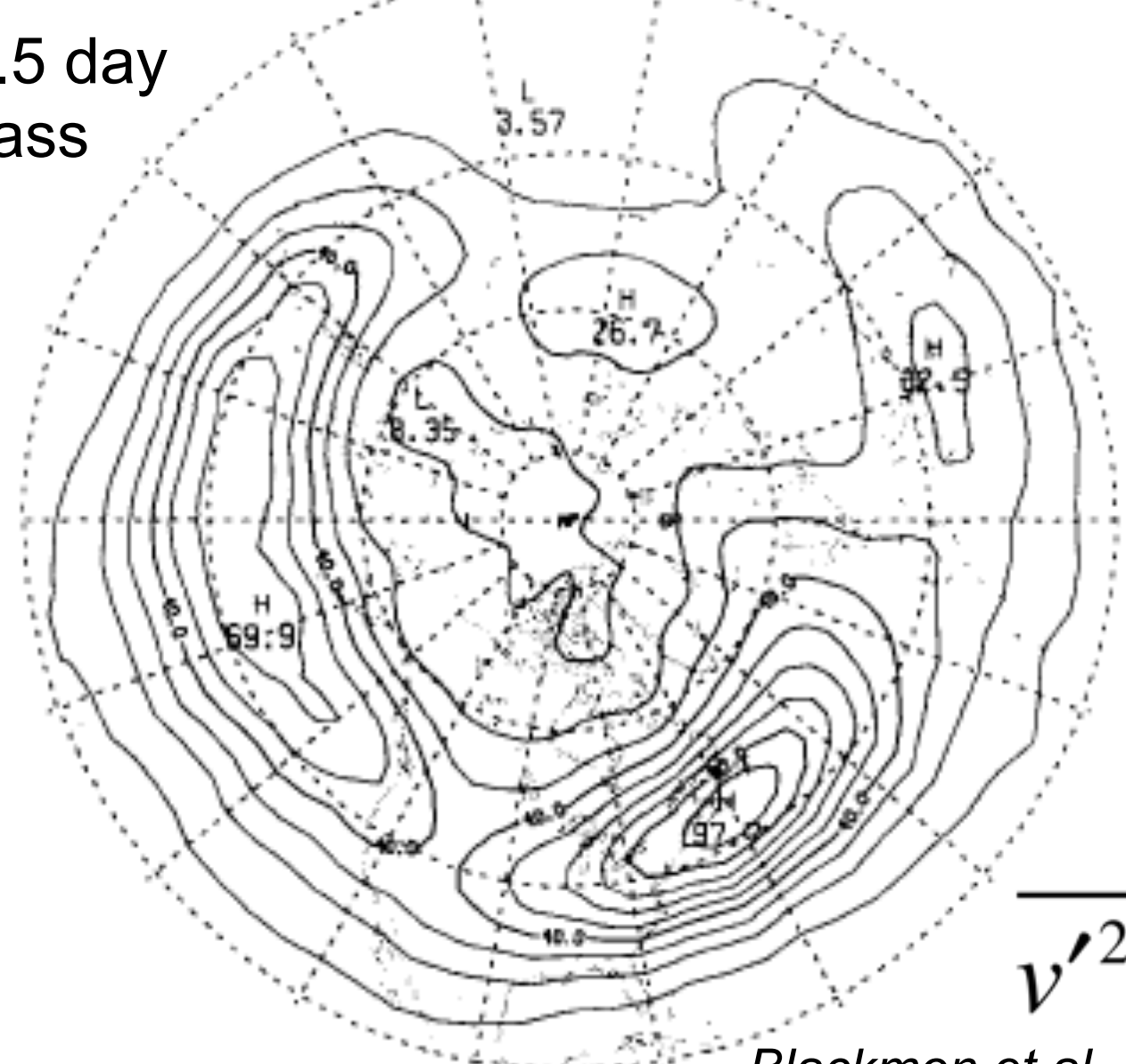
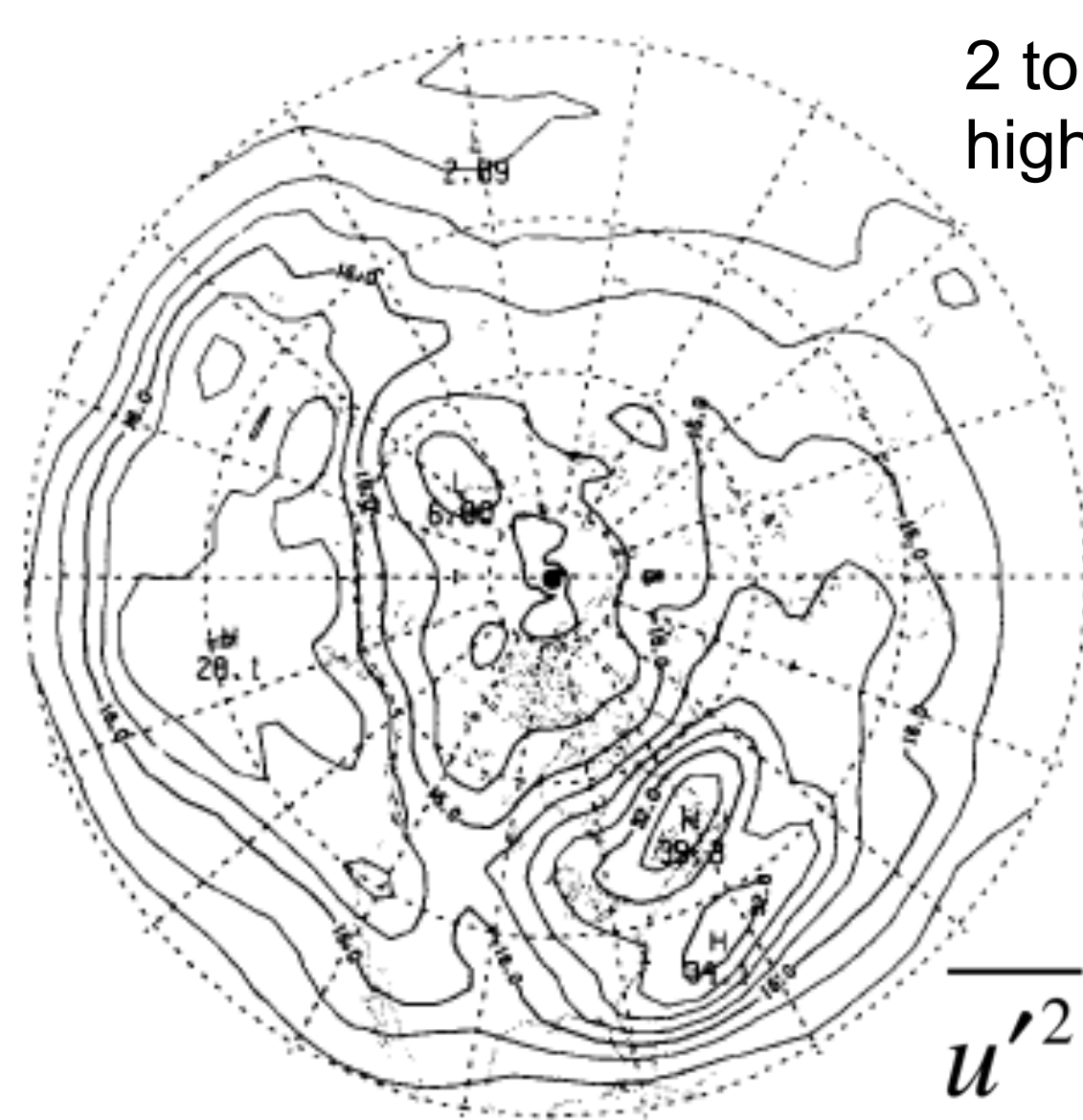
eddy heat flux  
850 hPa



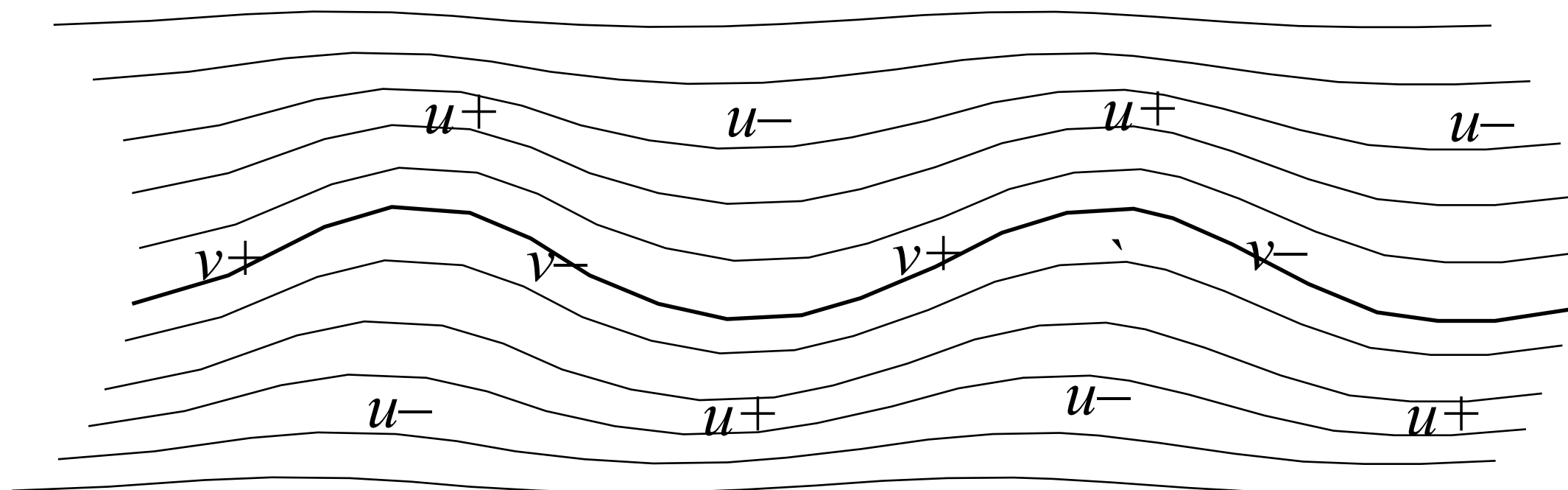
*courtesy of Justin Wettstein*



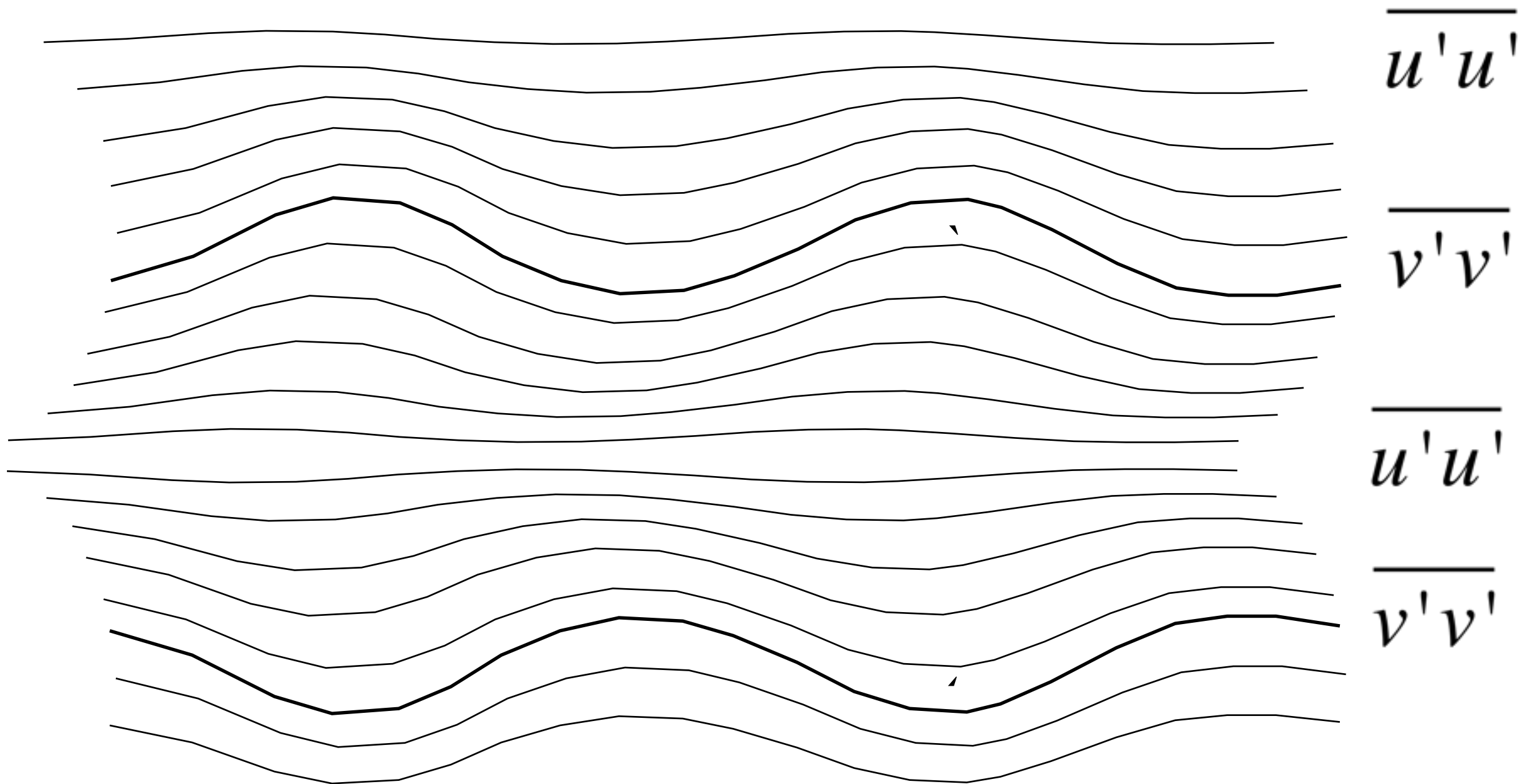
2 to 6.5 day  
highpass

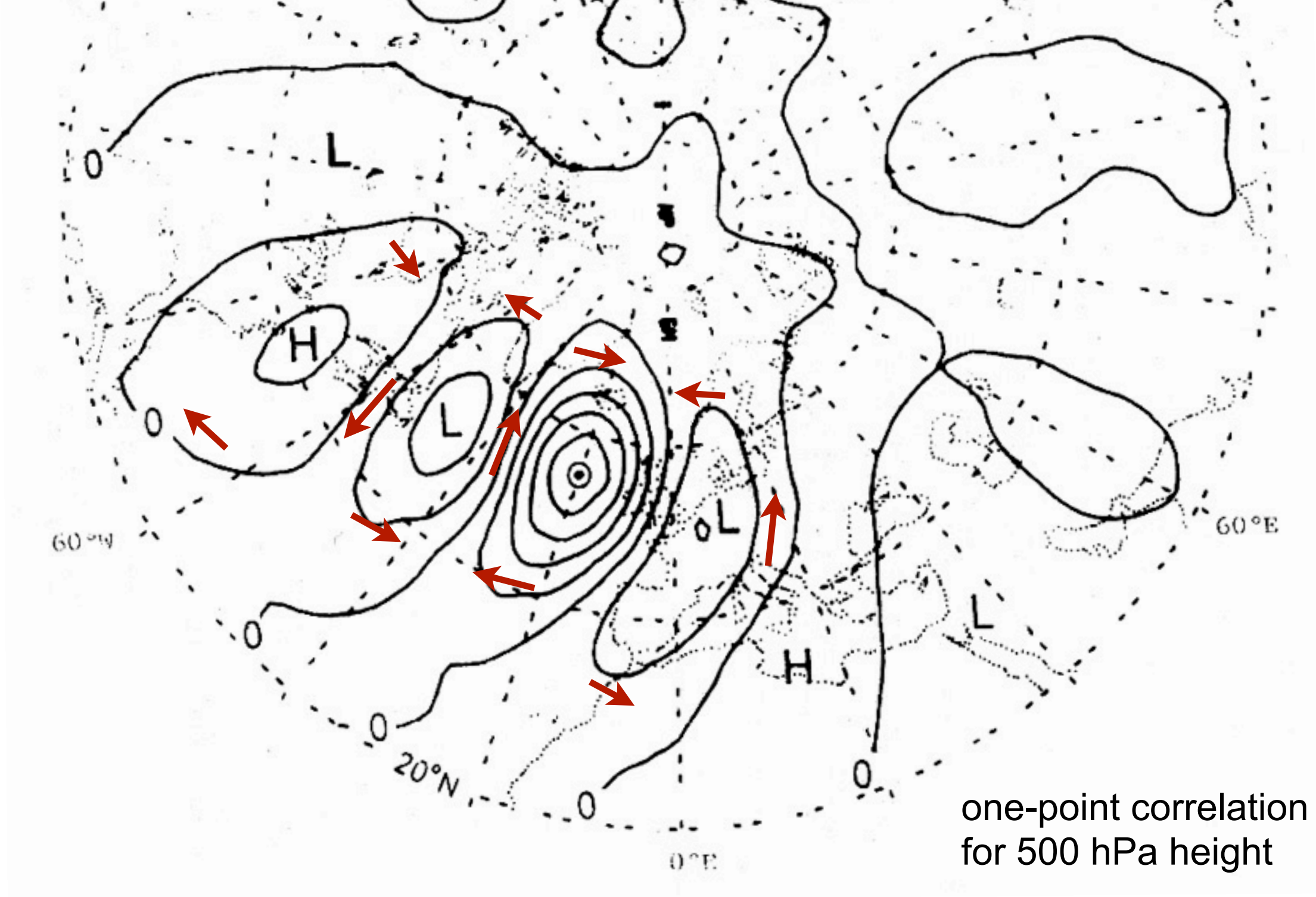


*Blackmon et al. JAS 1977*



2 to 6.5 day  
highpass

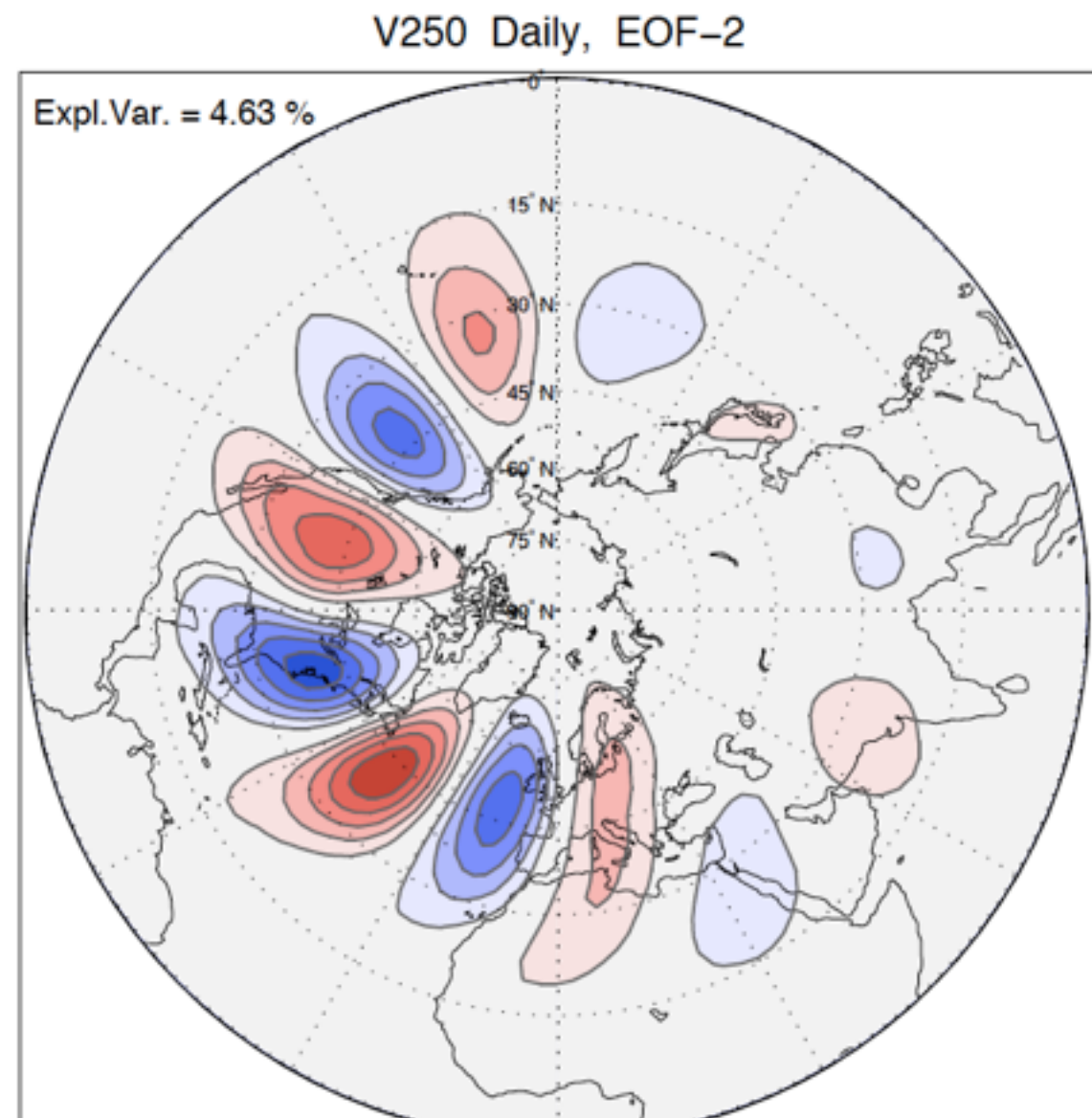
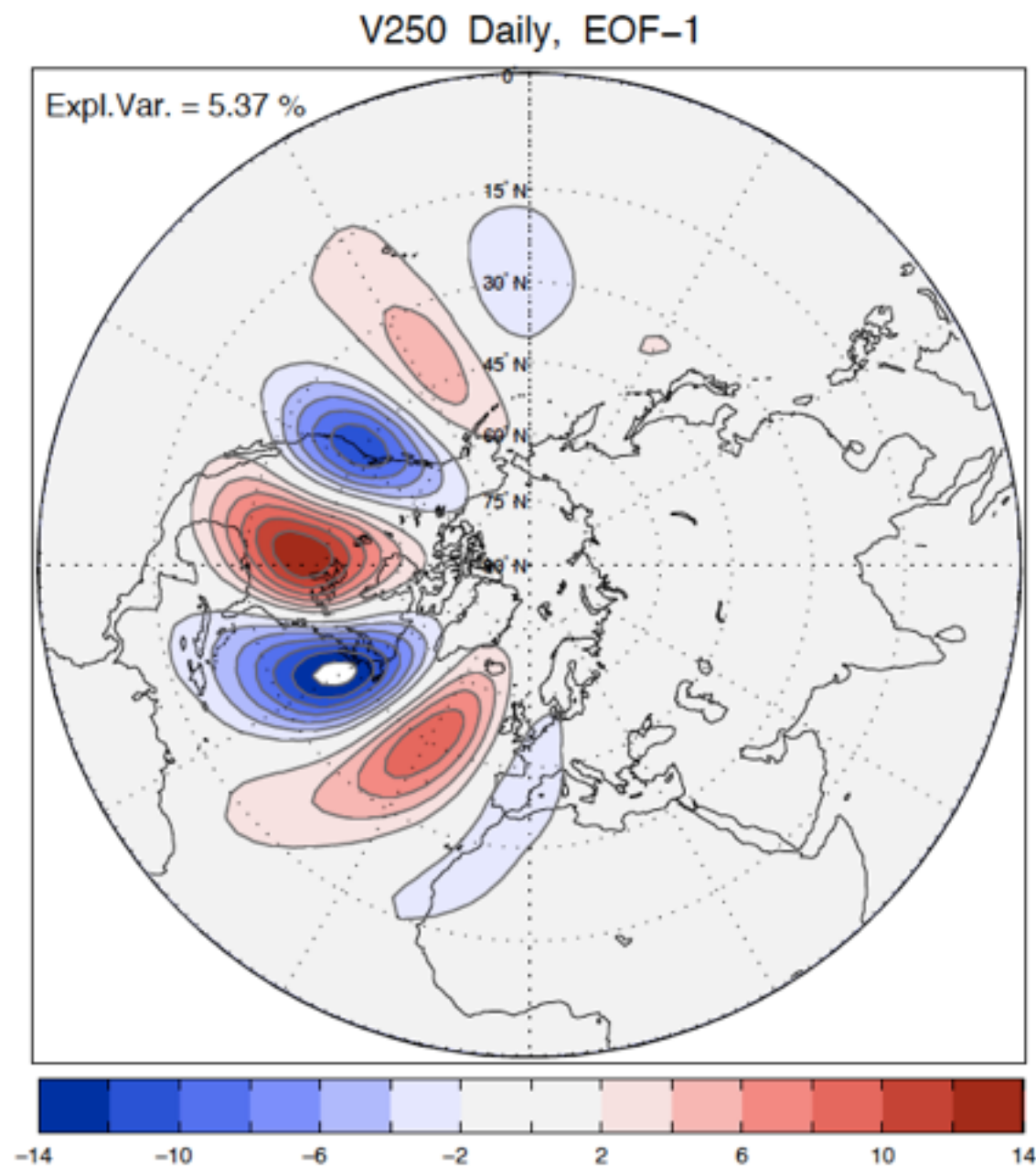




Anisotropy of high frequency transients

$$\overline{v'v'} > \overline{u'u'}$$





*Leading wintertime EOFs, Courtesy of Panos Athanasiadis  
based on daily unfiltered data*

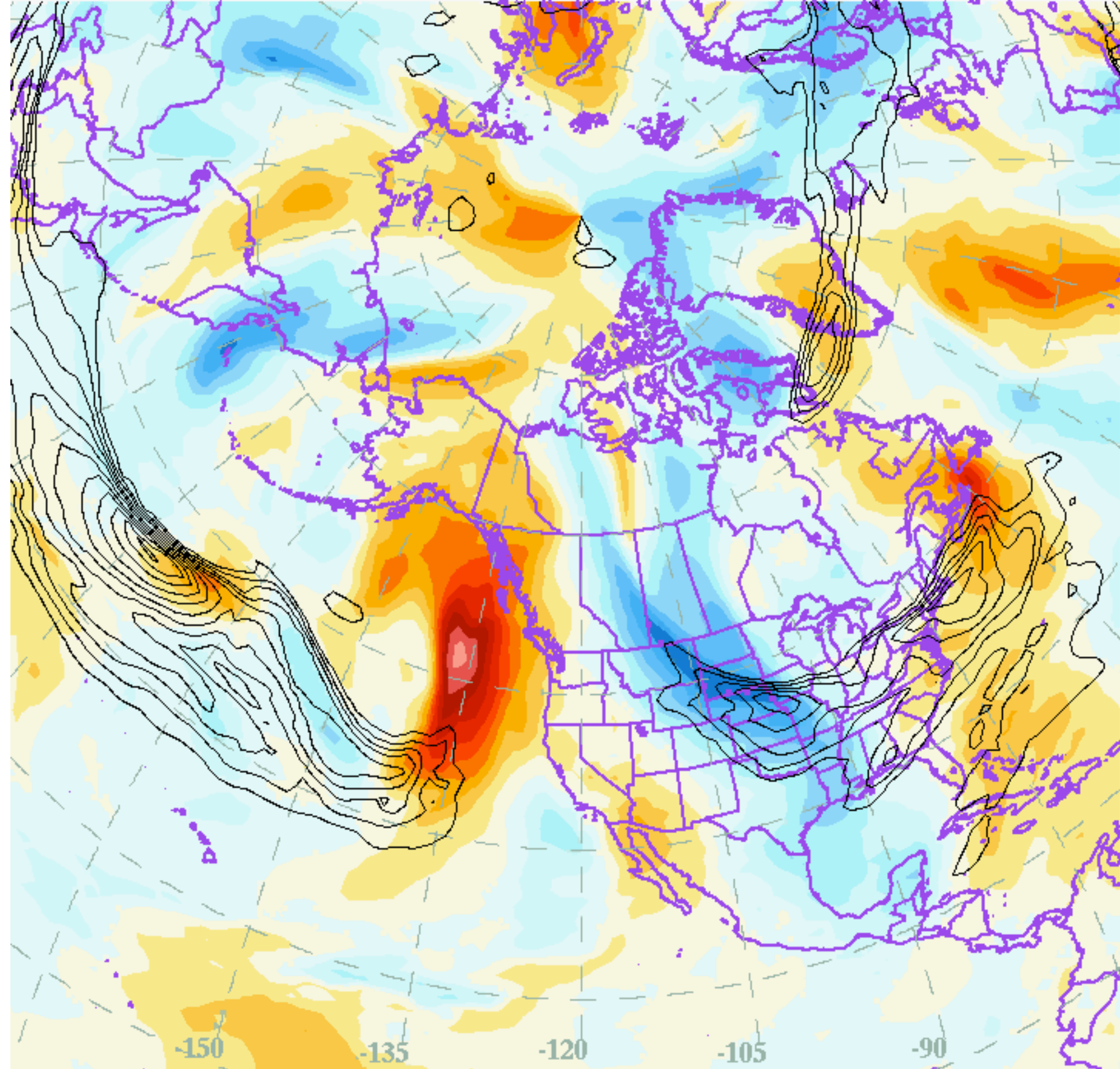
Anisotropy of high frequency transients

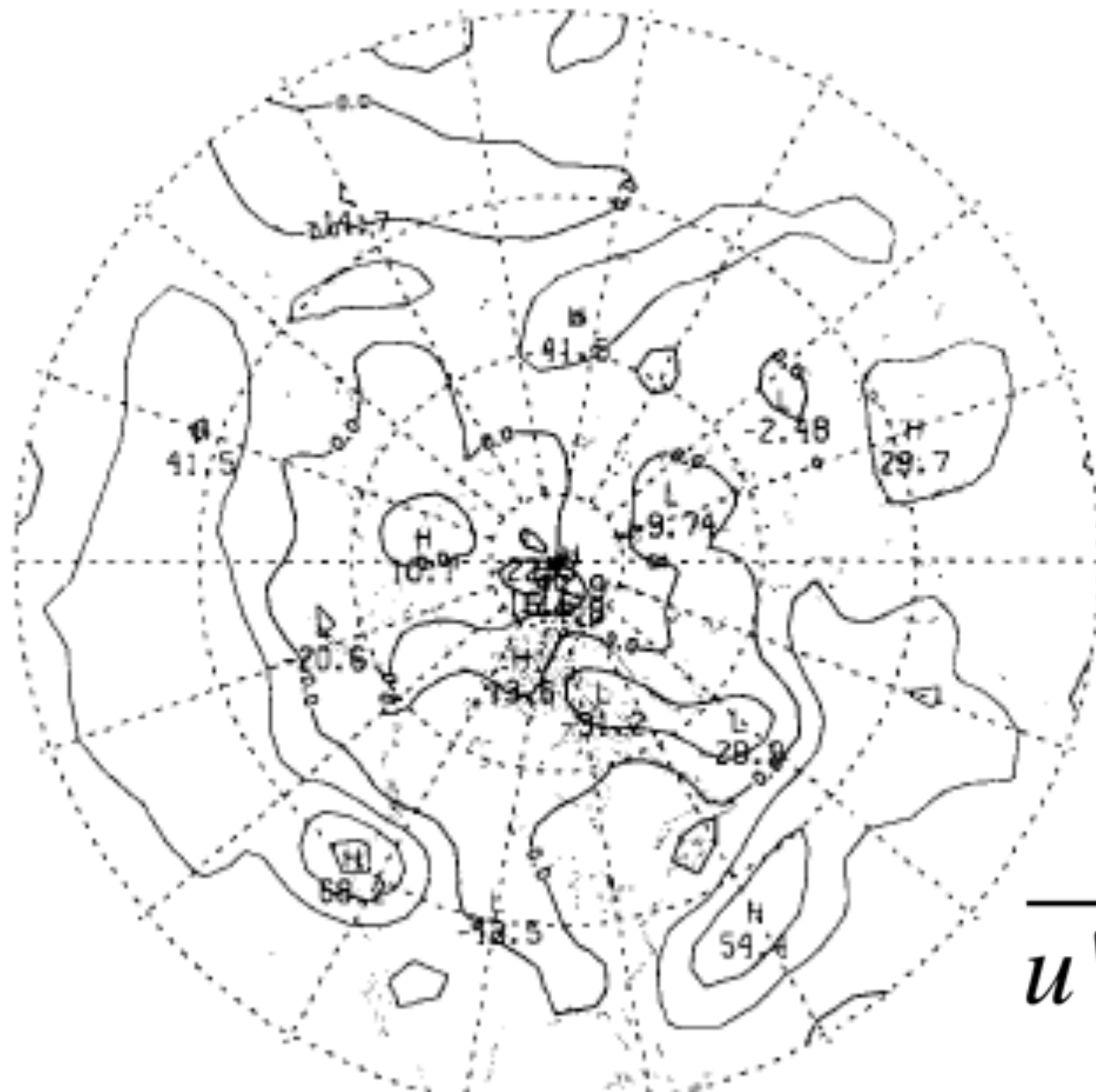
$$\overline{v'v'} > \overline{u'u'}$$

$u'$  contours  
positive anomalies only

$v'$  colored shading  
tan poleward

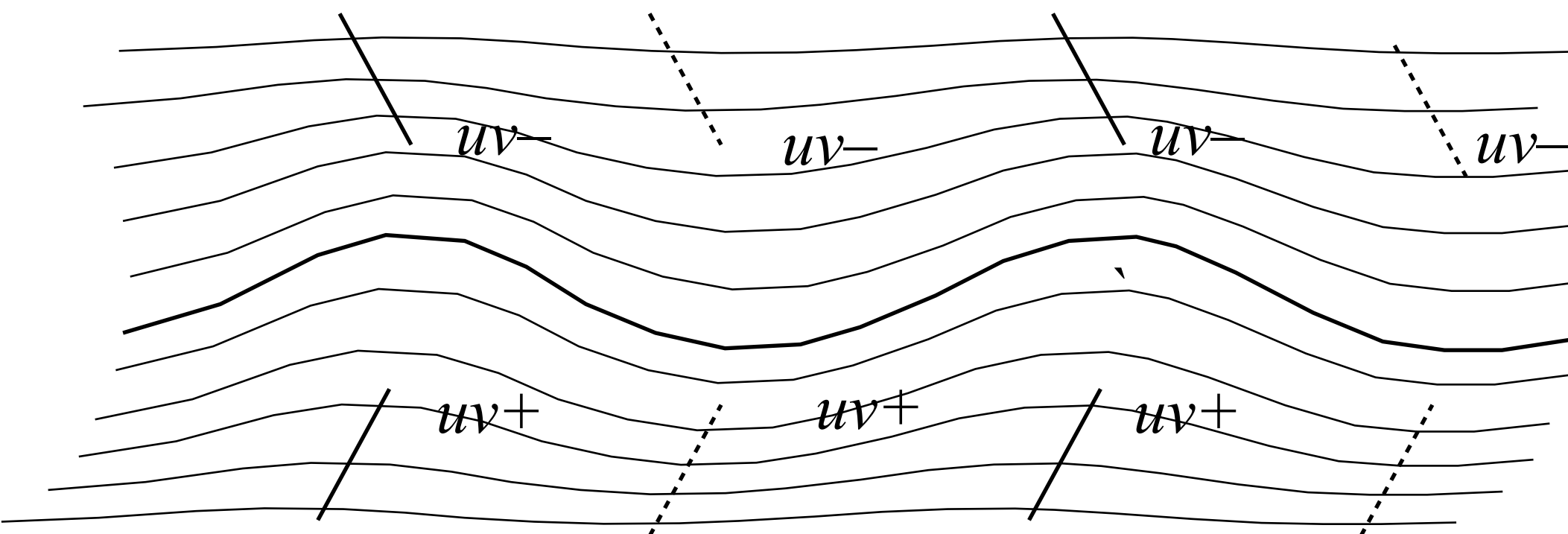
*courtesy of David Ovens*





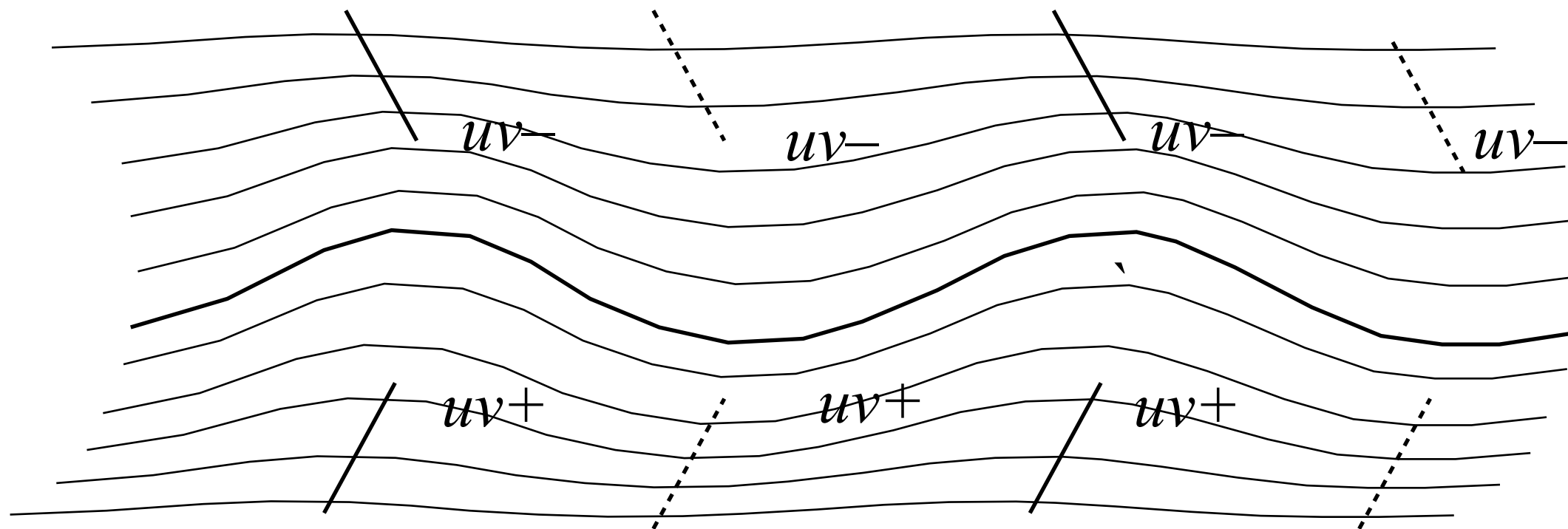
2 to 6.5 day  
highpass

$\overline{u'v'}$



$$-\frac{\partial}{\partial y}[\overline{u'v'}] > 0$$





Using  $\overline{u'v'}$  as an example, consider the distributions of

$$\overline{u_g'\Phi'}, \overline{v_g'\Phi'}, \overline{u_a'\Phi'}, \overline{u_g'\zeta'}$$

*for a full length paper about this, see Lau and Wallace JAS 1979*

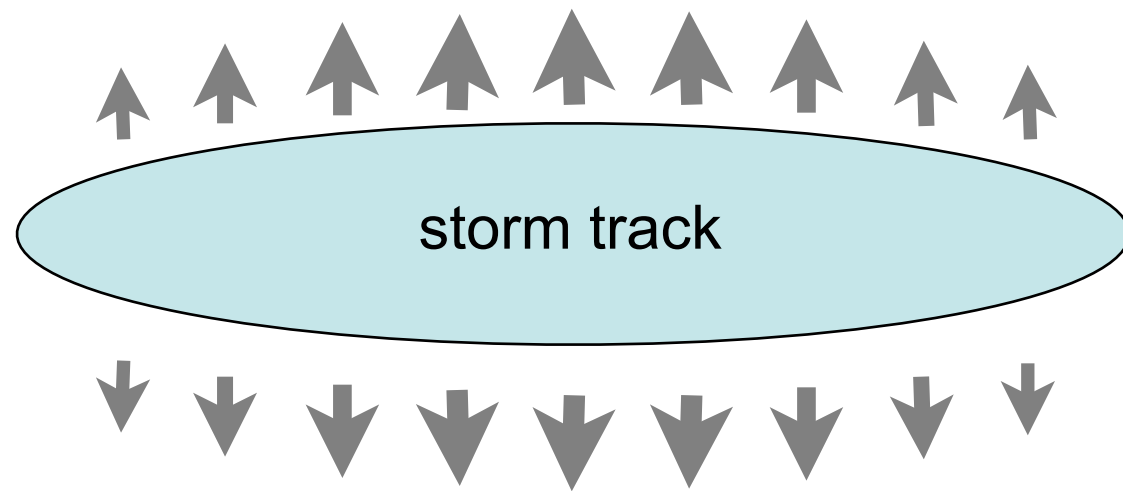
Interaction between the eddies and the time-mean flow in a barotropic flow

$$\frac{\partial \bar{u}}{\partial t} = -\bar{u} \frac{\partial \bar{u}}{\partial x} - \bar{v} \frac{\partial \bar{u}}{\partial y} - \frac{\partial}{\partial x} \overline{u' u'} - \frac{\partial}{\partial y} \overline{u' v'} + f v_a + F_x$$

$$\frac{\partial \bar{v}}{\partial t} = -\bar{u} \frac{\partial \bar{v}}{\partial x} - \bar{v} \frac{\partial \bar{v}}{\partial y} - \frac{\partial}{\partial x} \overline{u' v'} - \frac{\partial}{\partial y} \overline{v' v'} - f u_a + F_y$$

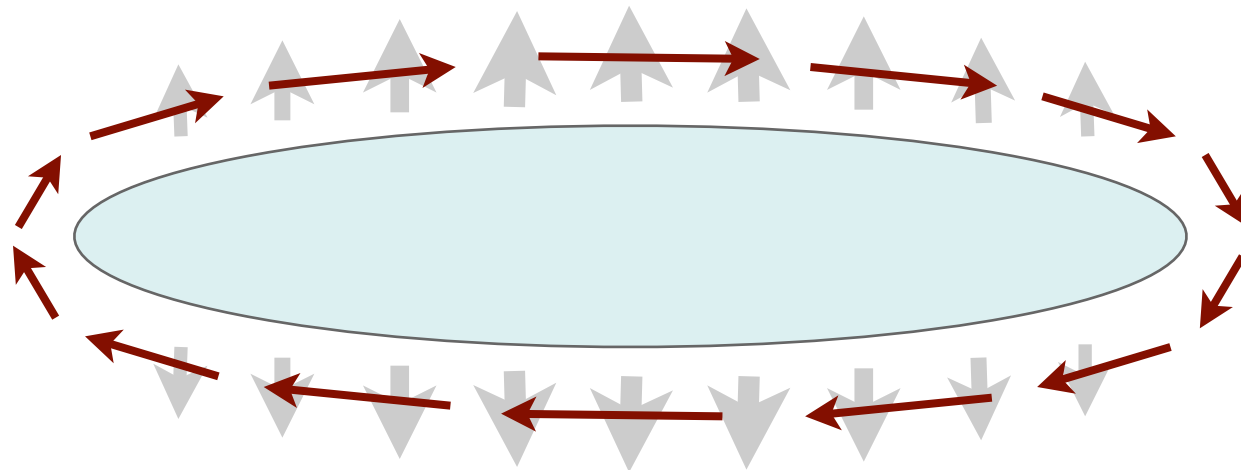
The eddy forcing is difficult to interpret because of the eddy-induced ageostrophic circulation

We can get around this problem by considering the eddy forcing of the vorticity field.

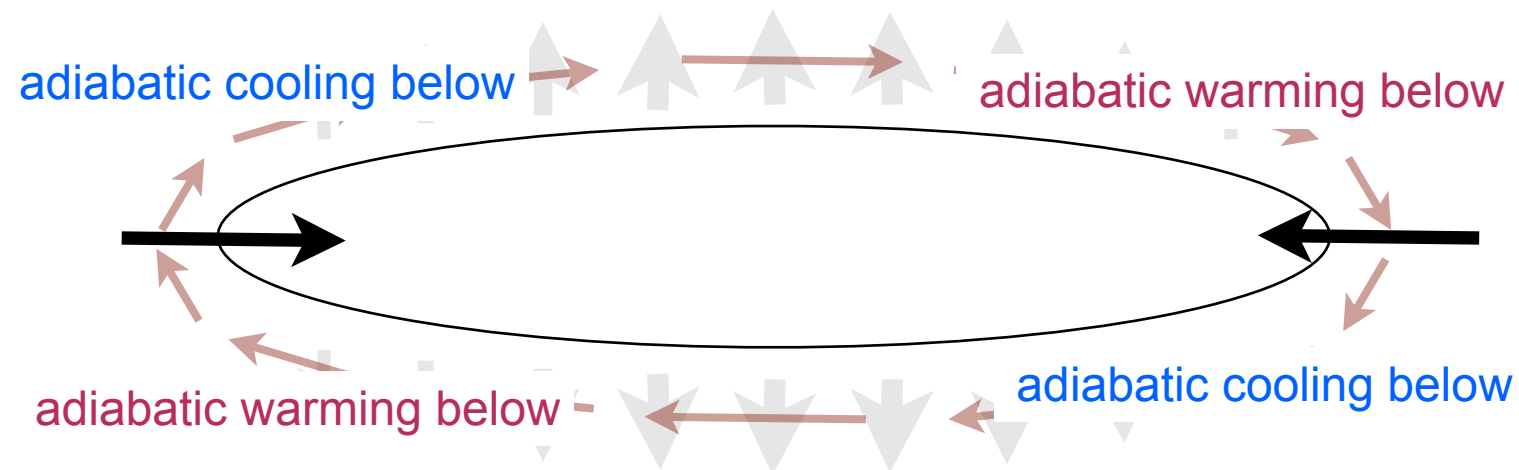


direct effect of eddy forcing

$$-\partial / \partial y (\overline{v'v'}) \text{ only}$$



induced ageostrophic circulation

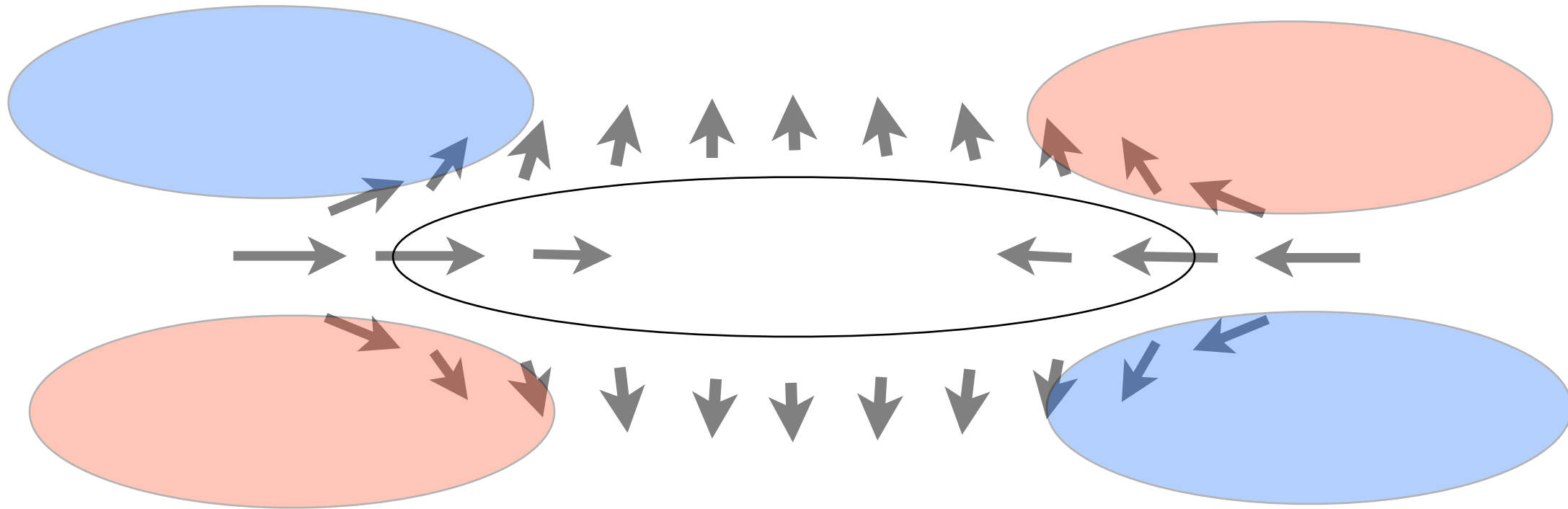


indirect effects of eddy forcing

*after Hoskins, James and White JAS 1983*



# The balanced geostrophic response



Note that the eddies produce an effective *westward transport* of westerly momentum through the storm track

We can see this effect more clearly by considering the vorticity transport by the eddies

*after Hoskins, James and White JAS 1983*

# The eddy covariance tensor

*after Hoskins, James and White JAS 1983*

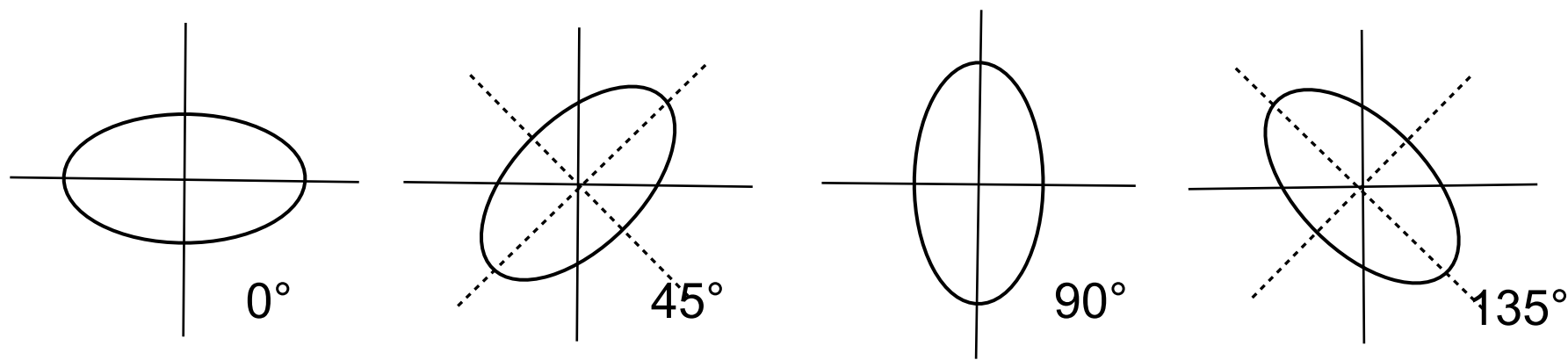
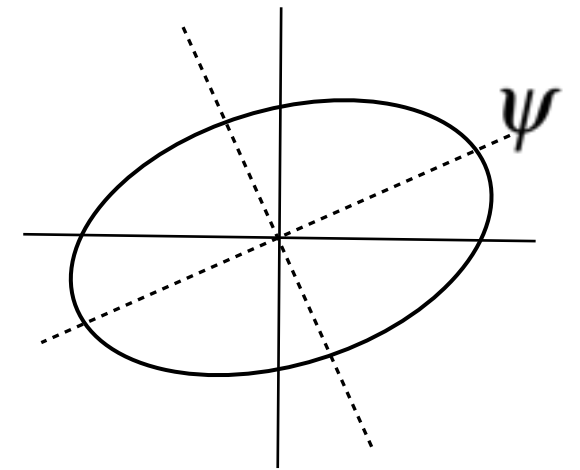
$$\begin{pmatrix} \overline{u'^2} & \overline{u'v'} \\ \overline{u'v'} & \overline{v'^2} \end{pmatrix} = \begin{pmatrix} K & 0 \\ 0 & K \end{pmatrix} + \begin{pmatrix} M & N \\ N & -M \end{pmatrix}$$

$$K = \frac{\overline{u'^2} + \overline{v'^2}}{2} \quad M = \frac{\overline{u'^2} - \overline{v'^2}}{2} \quad N = \overline{u'v'}$$

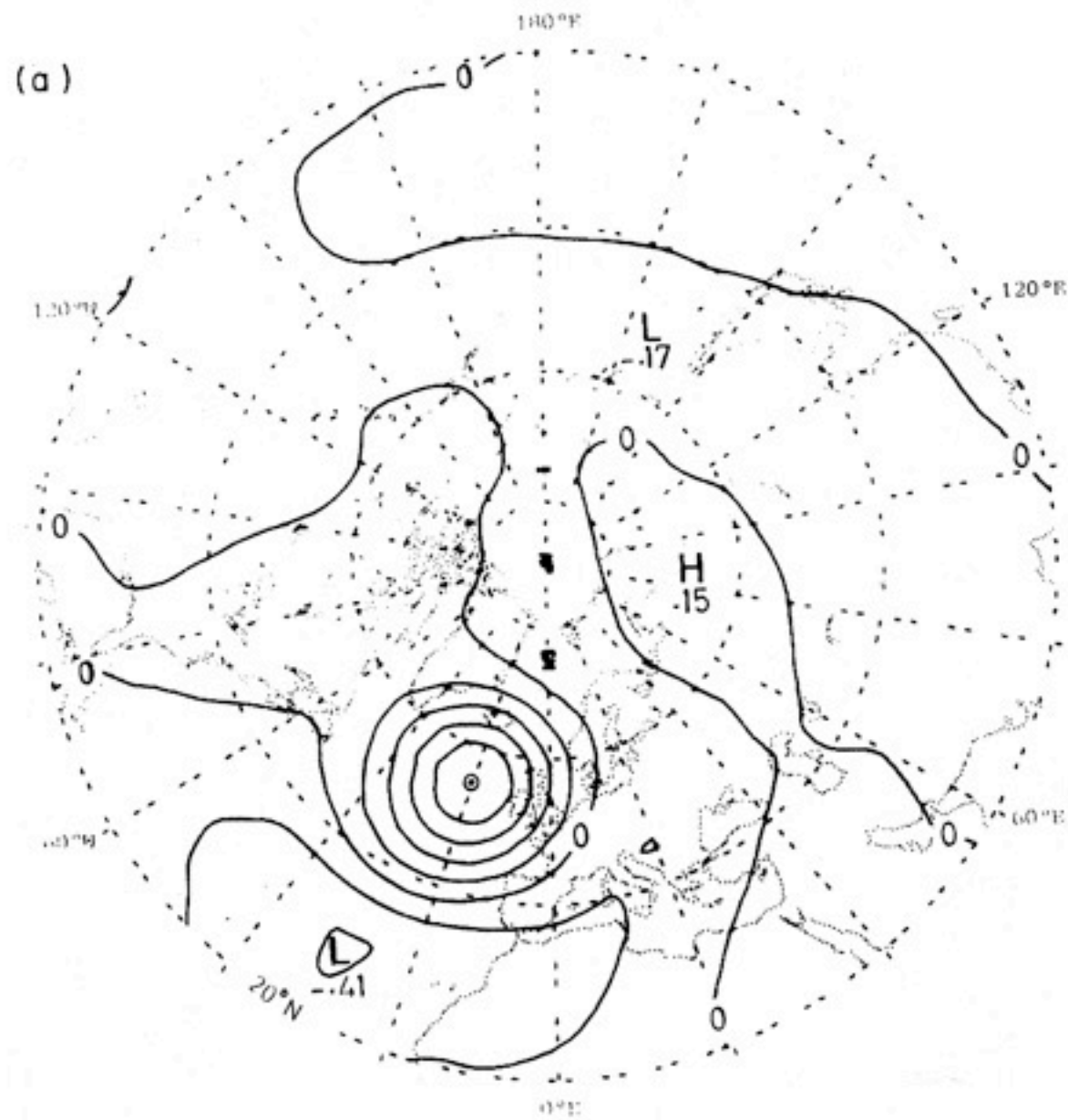
$$\widehat{M} = \sqrt{M^2 + N^2}$$

$\alpha \equiv \widehat{M} / K$  dimensionless coefficient of anisotropy

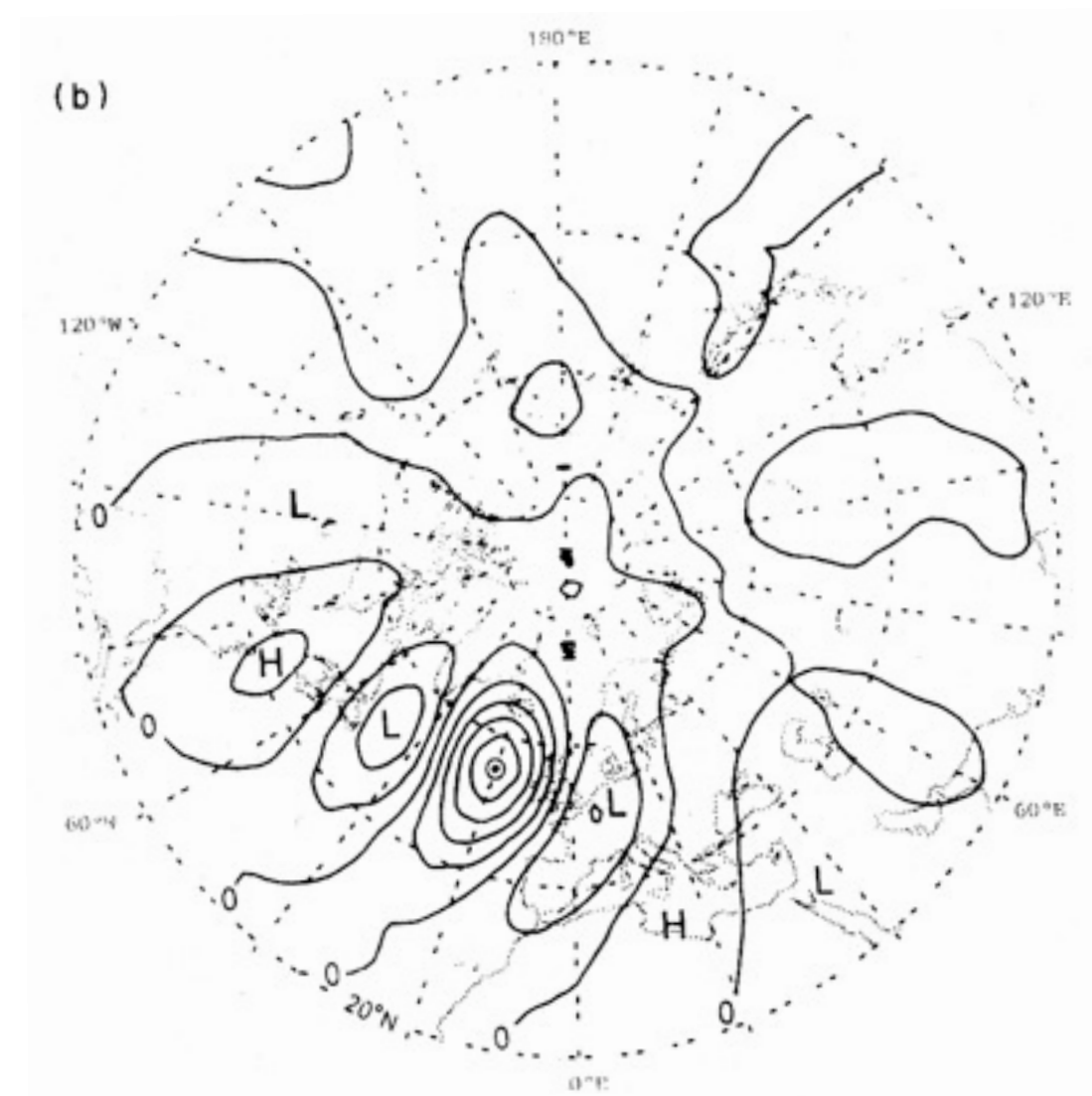
$\psi = \frac{1}{2} \tan^{-1} \frac{N}{M}$  angle of major axis relative to x axis



# One point correlation maps

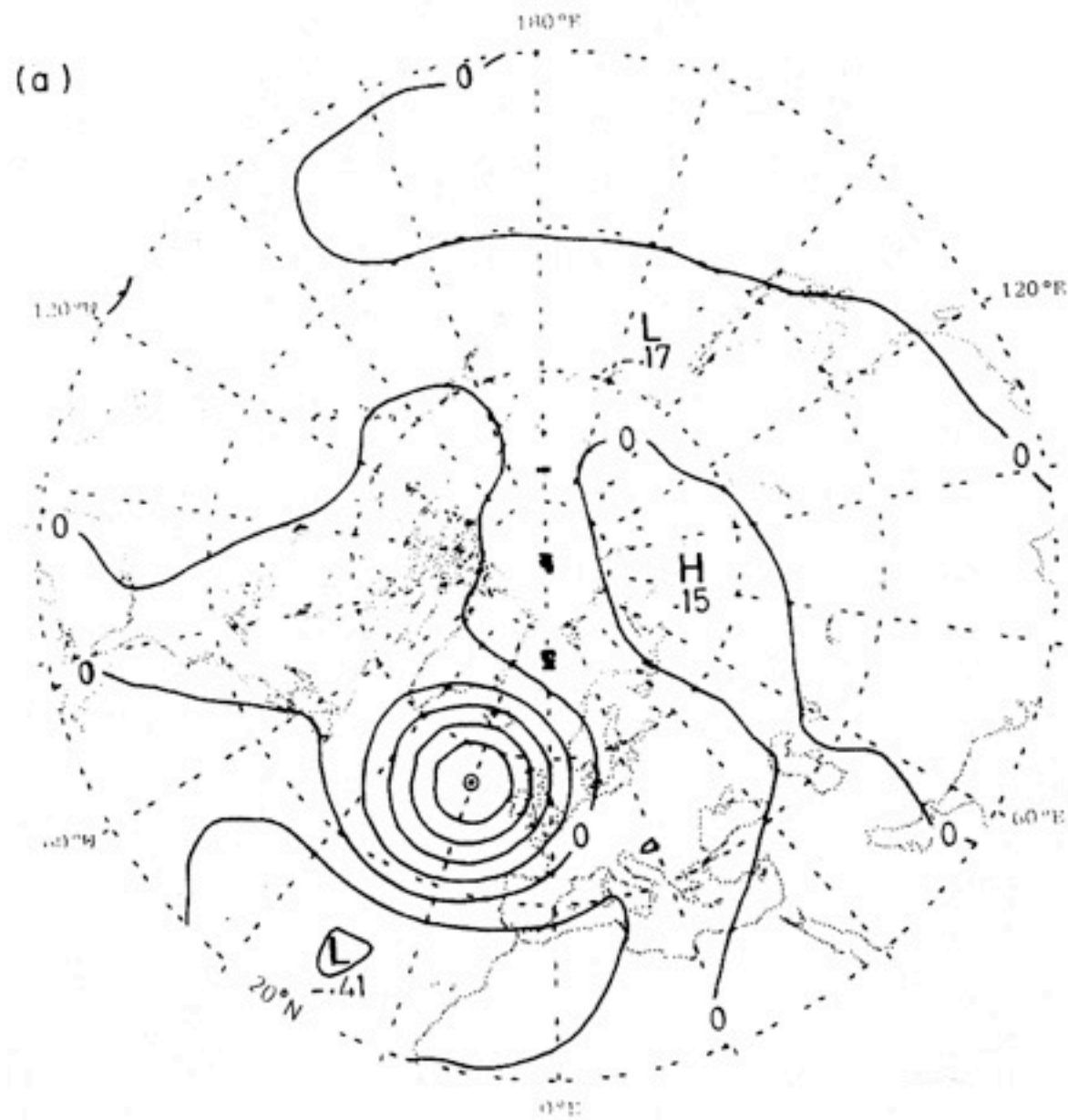


Unfiltered

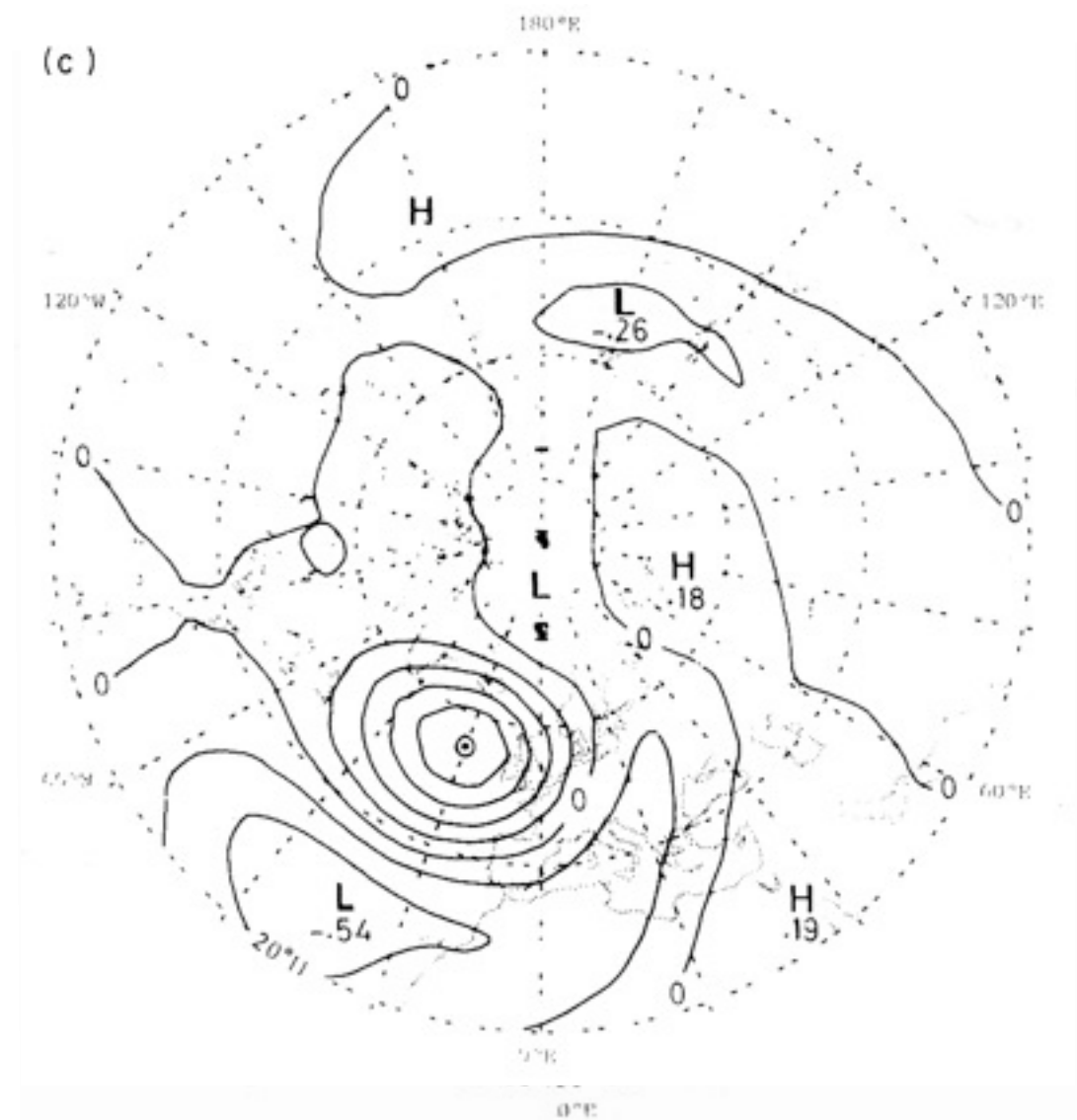


< 6 d

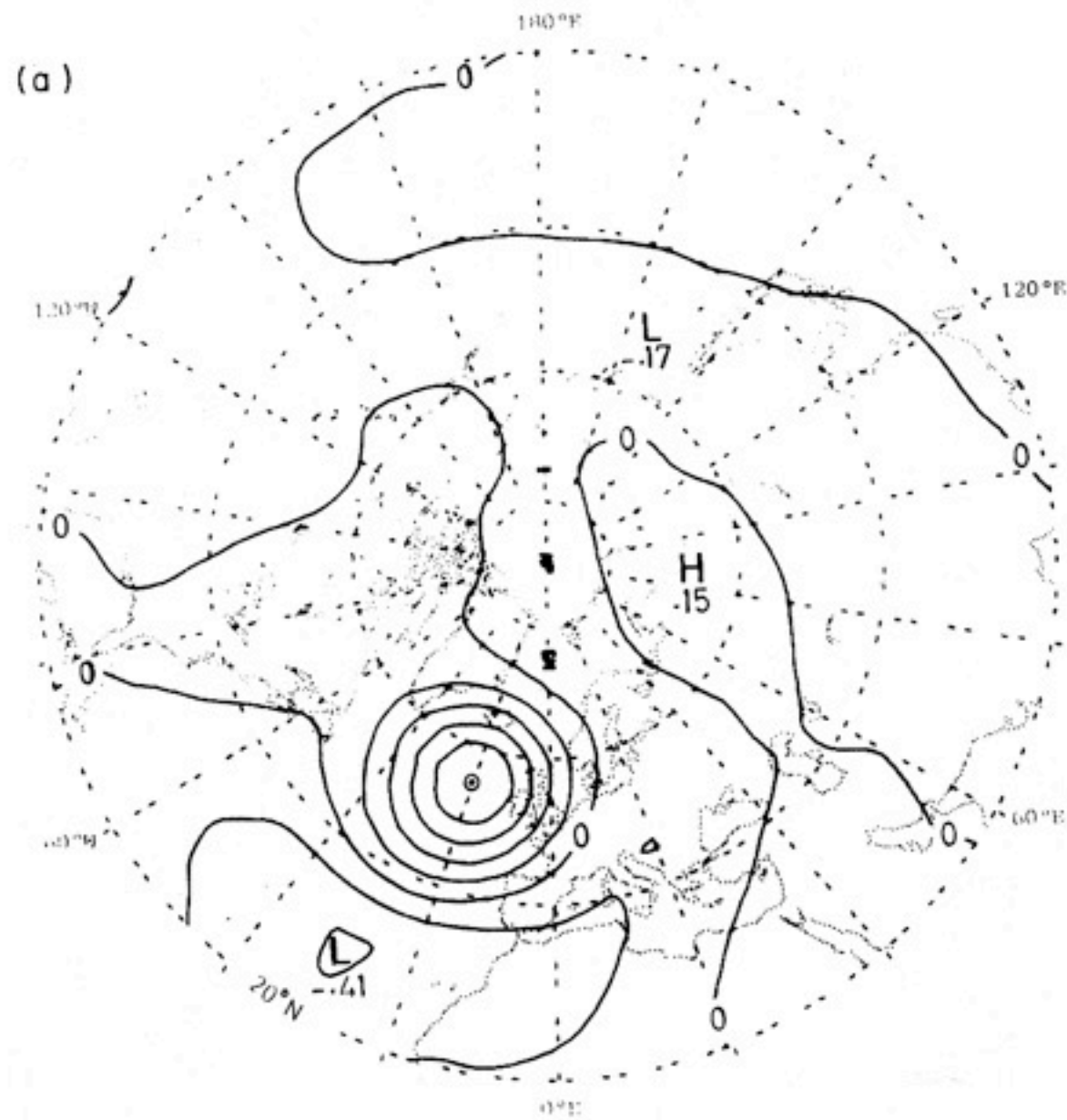




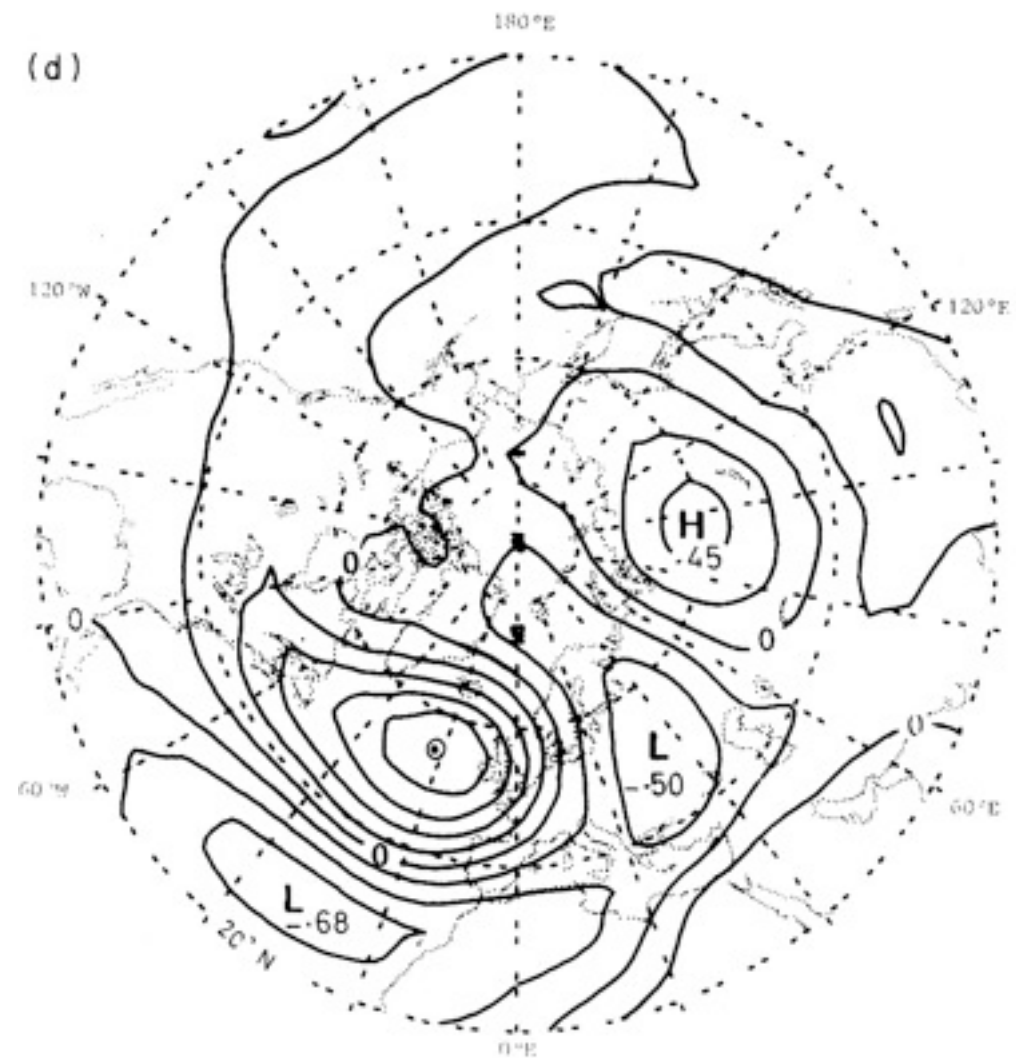
Unfiltered



> 10 d



Unfiltered



>30 d

# Feedback of transients upon the background flow

1. Determining how the eddies change the  $\bar{q}$  field

Estimate 
$$\frac{\partial \bar{q}}{\partial t} = -\nabla \cdot \overline{q' \vec{V}'} = -\frac{\partial}{\partial x} \overline{q' u'} - \frac{\partial}{\partial y} \overline{q' v'}$$

2. Use invertibility principle (solving elliptic equation)

For barotropic flow we can use  $\zeta$  in place of  $q$  in (1) and (2) reduces to solving Poisson's equation

$$\frac{\partial \bar{\Phi}}{\partial t} = \nabla^{-2} \left( \frac{\partial \bar{\zeta}}{\partial t} \right)$$



$$\overline{\zeta' u'} = -M_y + N_x; \quad \overline{\zeta' v'} = -M_x - N_y$$

$$\frac{\partial \bar{\zeta}}{\partial t} = -\nabla \cdot \overline{\vec{V}' \zeta} = 2M_{xy} - N_{xx} + N_{yy} \quad (1)$$

but the features in the  $\overline{u' v'}$  field tend to be zonally elongated along storm tracks. It follows that  $N_{xx} \ll N_{yy}$  so (1) can be rewritten as

$$\frac{\partial \bar{\zeta}}{\partial t} = -\nabla \cdot \overline{\vec{V}' \zeta'} \simeq \frac{\partial}{\partial y} \left( \frac{\partial}{\partial x} 2M + \frac{\partial}{\partial y} N \right) \quad (2)$$

or

$$\frac{\partial \bar{\zeta}}{\partial t} \simeq -\frac{\partial}{\partial y} (\nabla \cdot \vec{E}) \quad \text{where} \quad \vec{E} \equiv (-2M, -N) \quad (3)$$

Features in the mean flow also tend to be zonally oriented, so

$$\frac{\partial \bar{u}}{\partial y} \gg \frac{\partial \bar{v}}{\partial x}$$

Hence, (3) can be written as  $\frac{\partial}{\partial t} \left( -\frac{\partial \bar{u}}{\partial y} \right) = -\frac{\partial}{\partial y} (\nabla \cdot \vec{E})$

or  $\frac{\partial}{\partial y} \left( \frac{\partial \bar{u}}{\partial t} \right) = \frac{\partial}{\partial y} (\nabla \cdot \vec{E})$

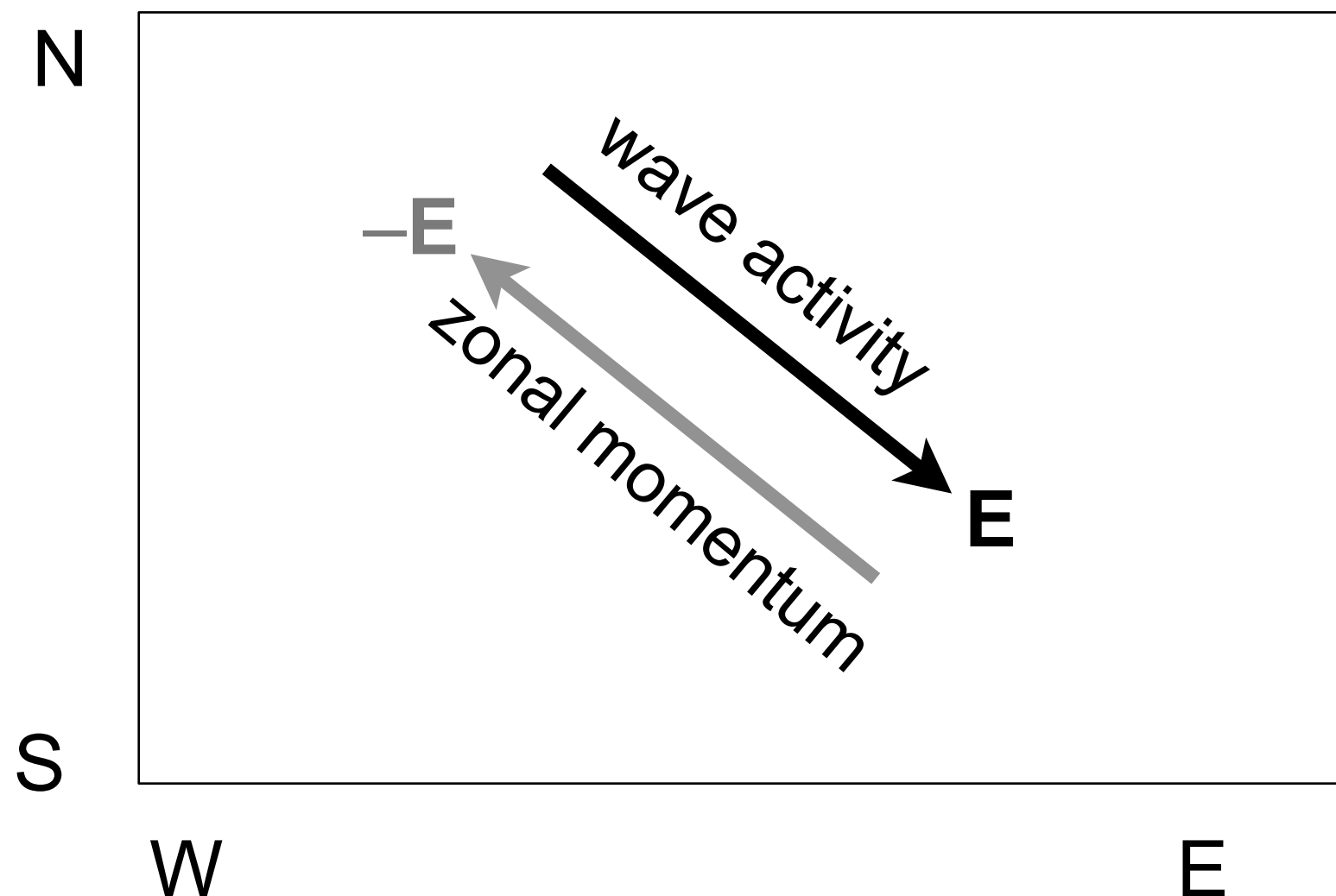
It follows that  $\frac{\partial \bar{u}}{\partial t} = \nabla \cdot \vec{E}$

where  $\vec{E} \equiv -\left( \overline{u'^2} - \overline{v'^2}, \overline{u'v'} \right)$

The “ $E$  vector” in this horizontal (barotropic) flow, with sign reversed, traces the 2-dimensional flux of  $u$  by the transient eddies.

Analogous to the Eliassen-Palm flux vector in the meridional plane.

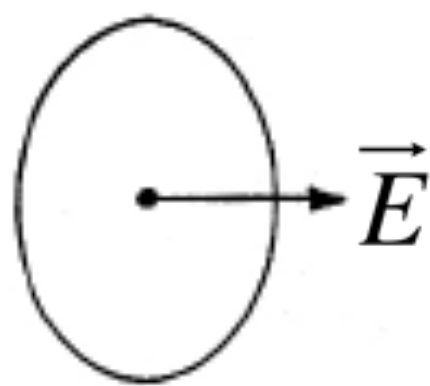
Like the EP flux, it can also be identified with the *group velocity*



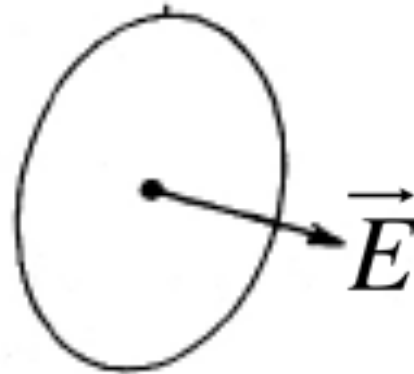


$$\vec{E} \equiv -\left(\overline{u'^2} - \overline{v'^2}, \overline{u'v'}\right)$$

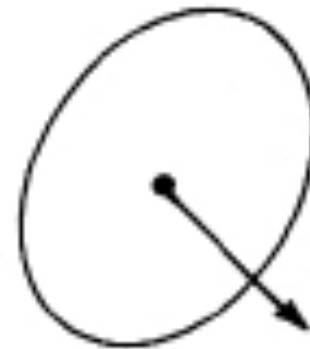
involves the anisotropy of the eddies.



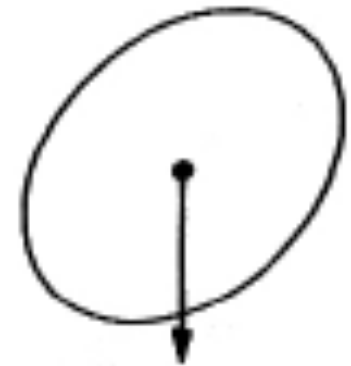
$$M -; N = 0$$



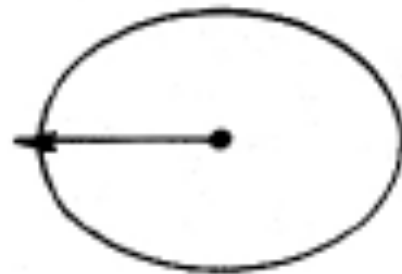
$$M -, N = -M/2$$



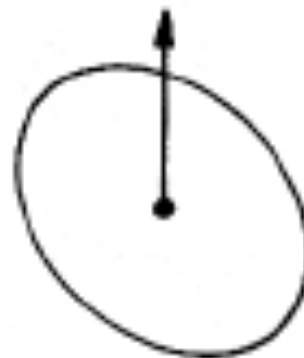
$$M -; N = -2M$$



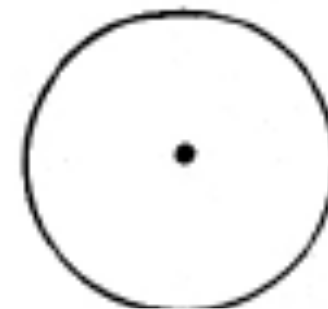
$$M = 0, N +$$



$$M +; N = 0$$



$$M = 0; N -$$

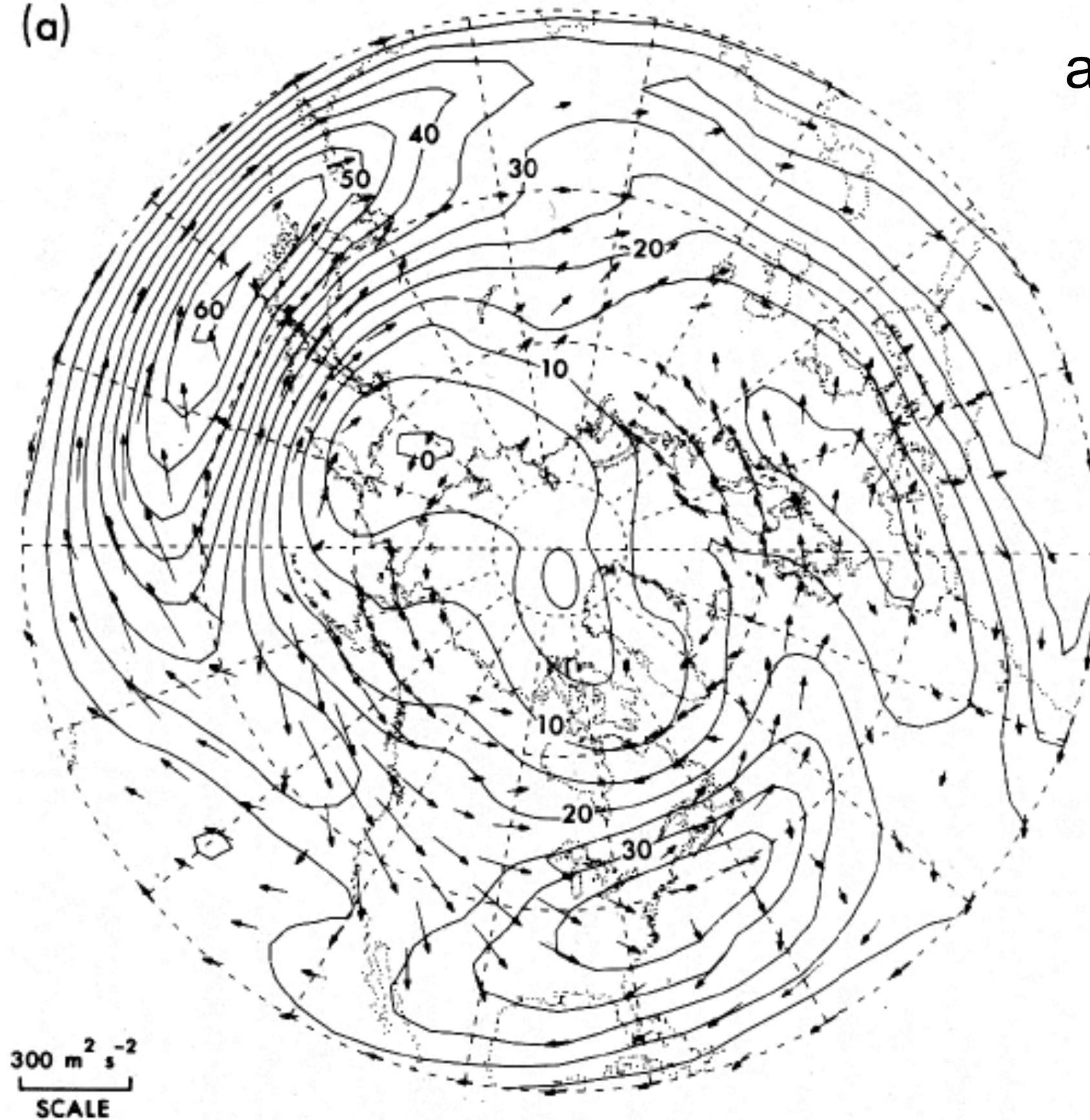


$$M = 0, N = 0$$

$$M = 0, N = 0$$

(a)

all frequencies

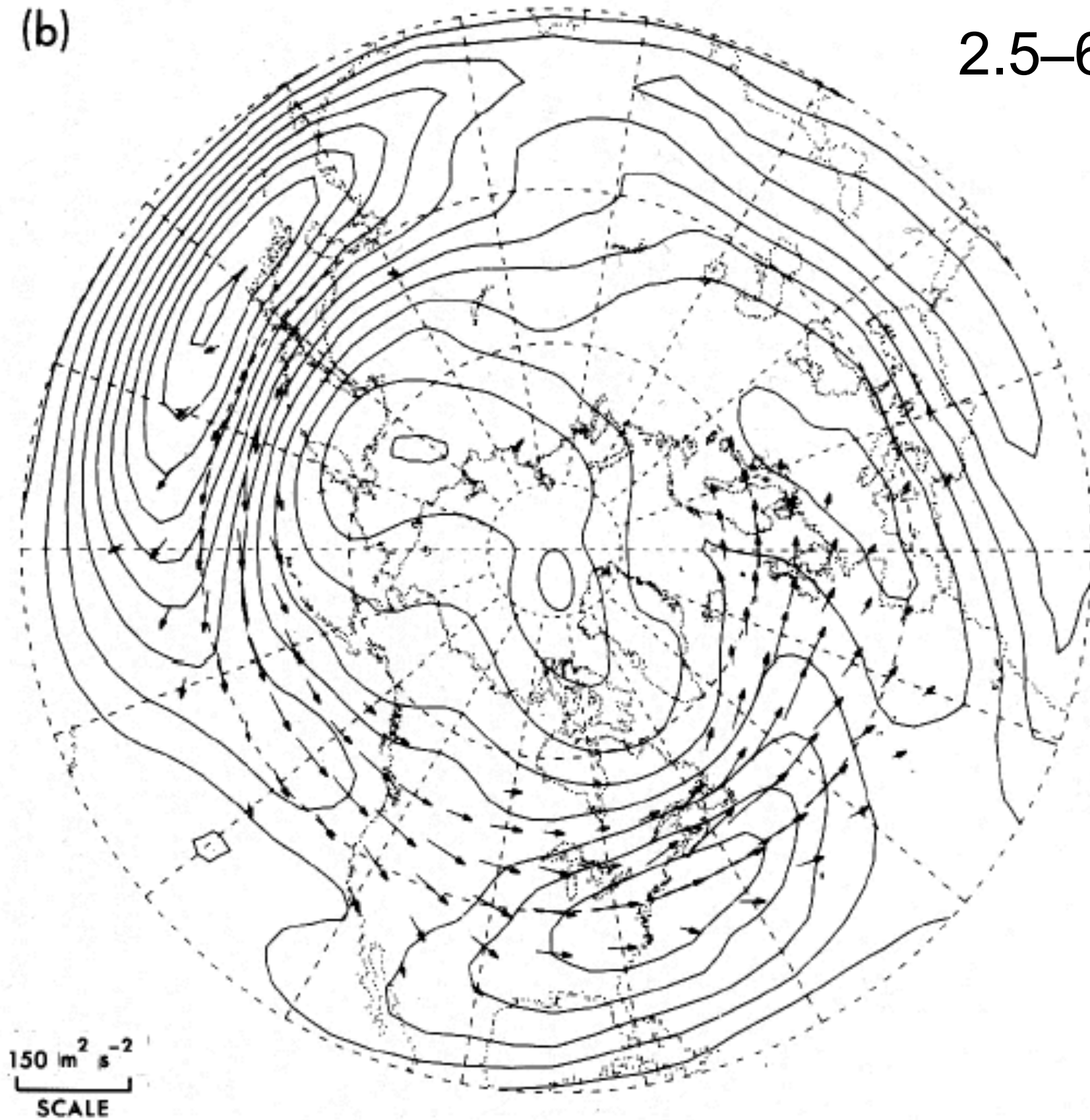


DJF 300 hPa.  $u$  contours; E vectors

*after Wallace and Lau, Issues in Atmospheric and Oceanic Modeling 1985*

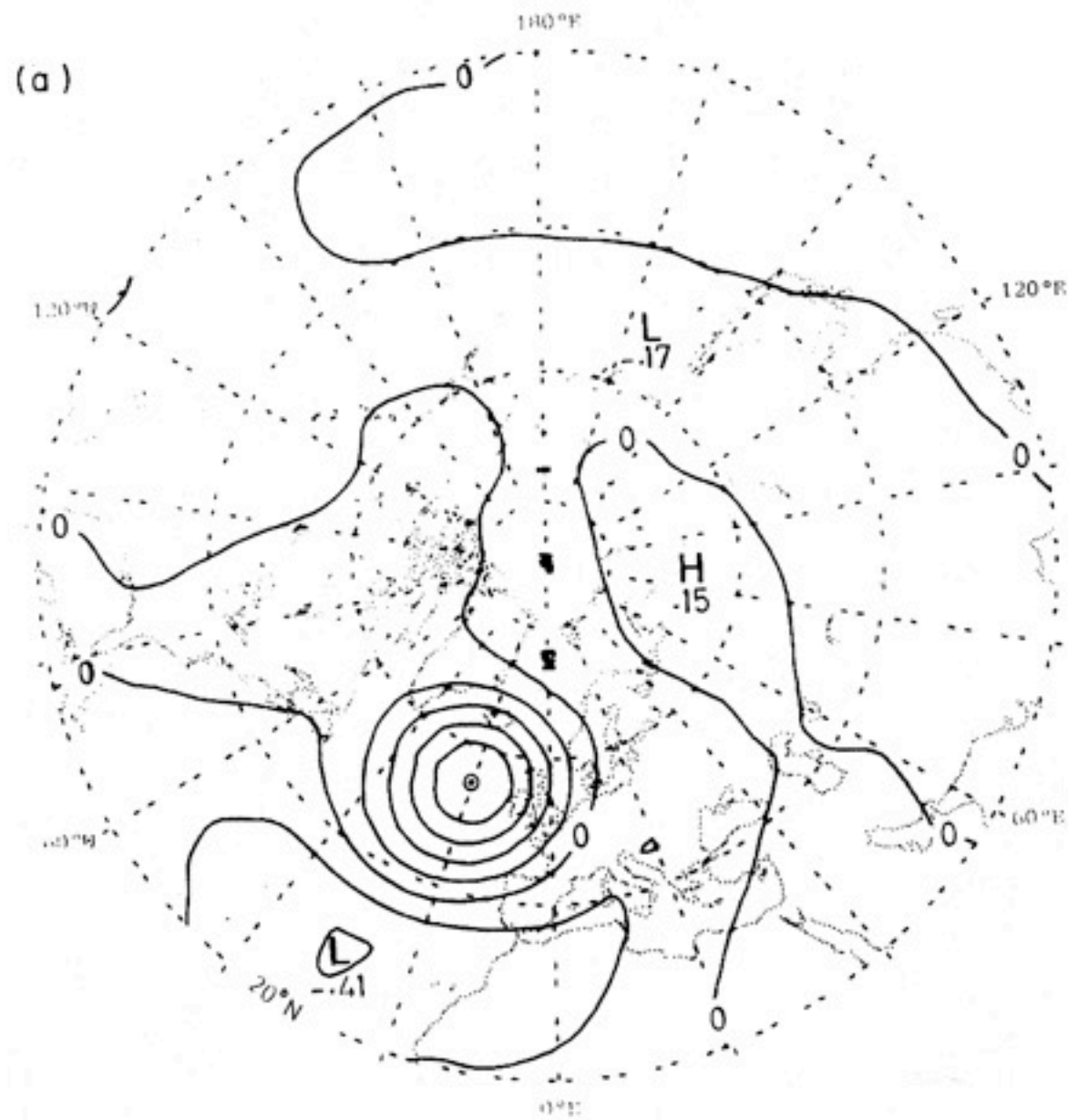
(b)

2.5–6 d highpass

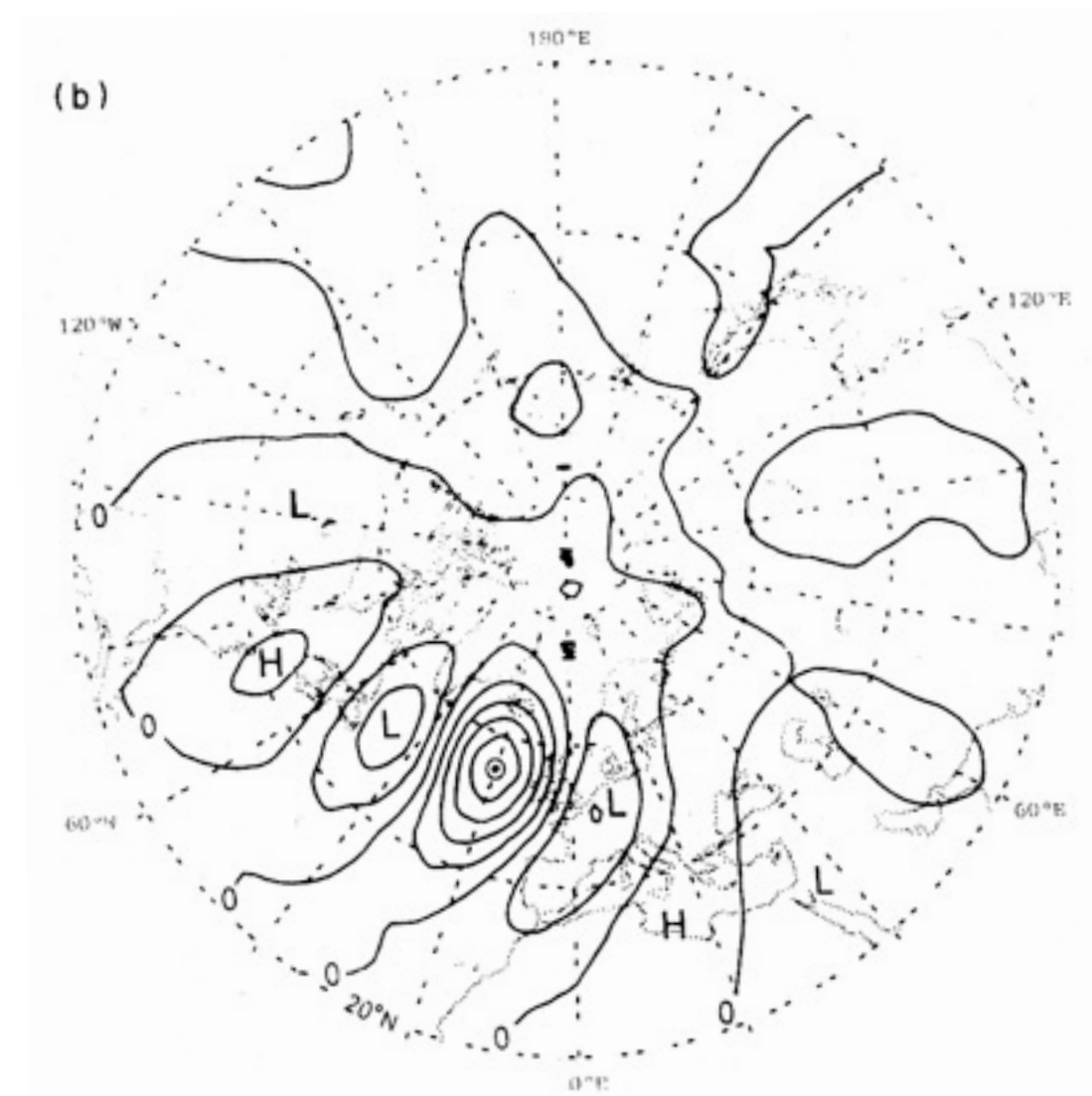


*after Wallace and Lau, Issues in Atmospheric and Oceanic Modeling 1985*

# One point correlation maps



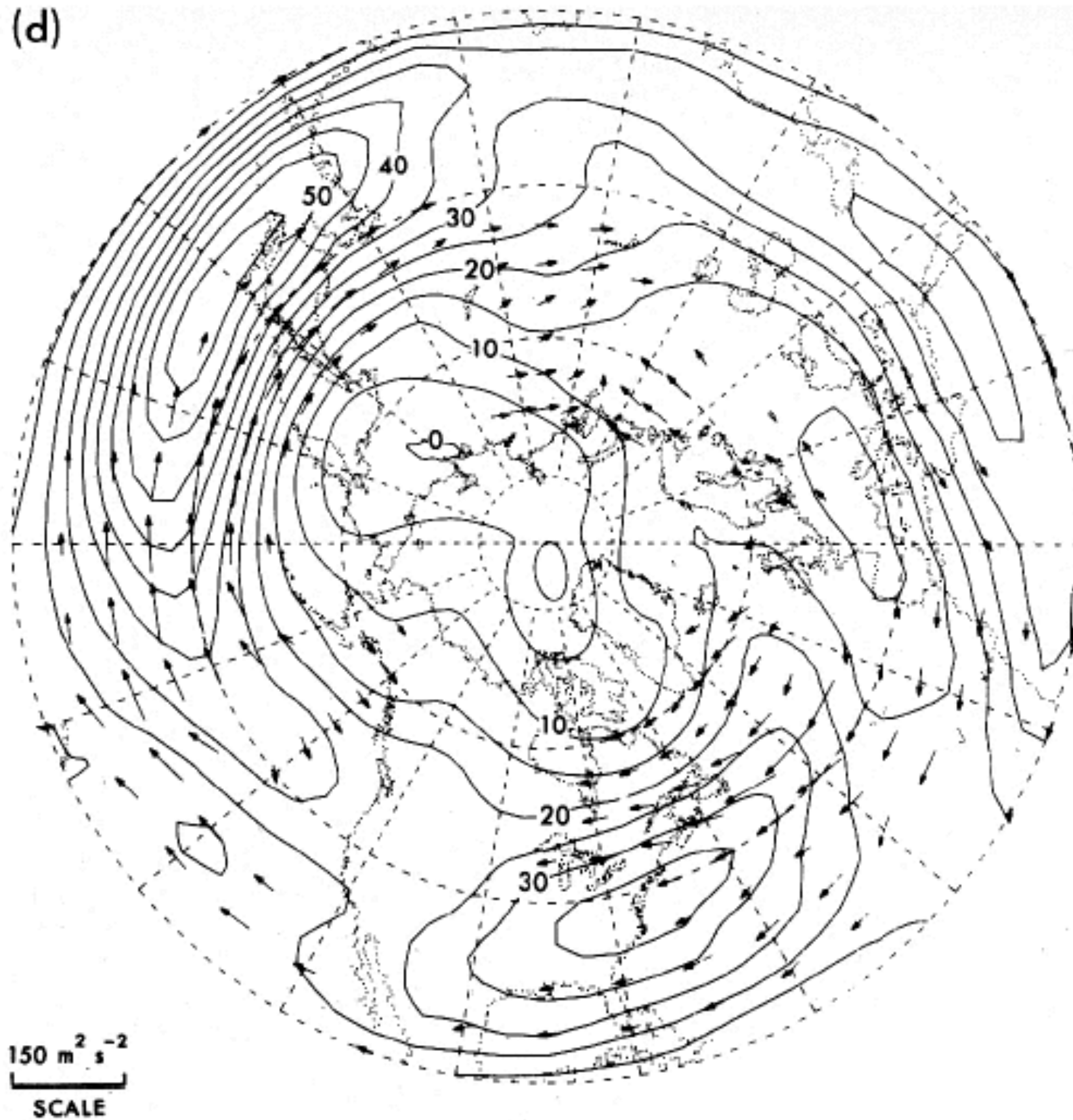
Unfiltered



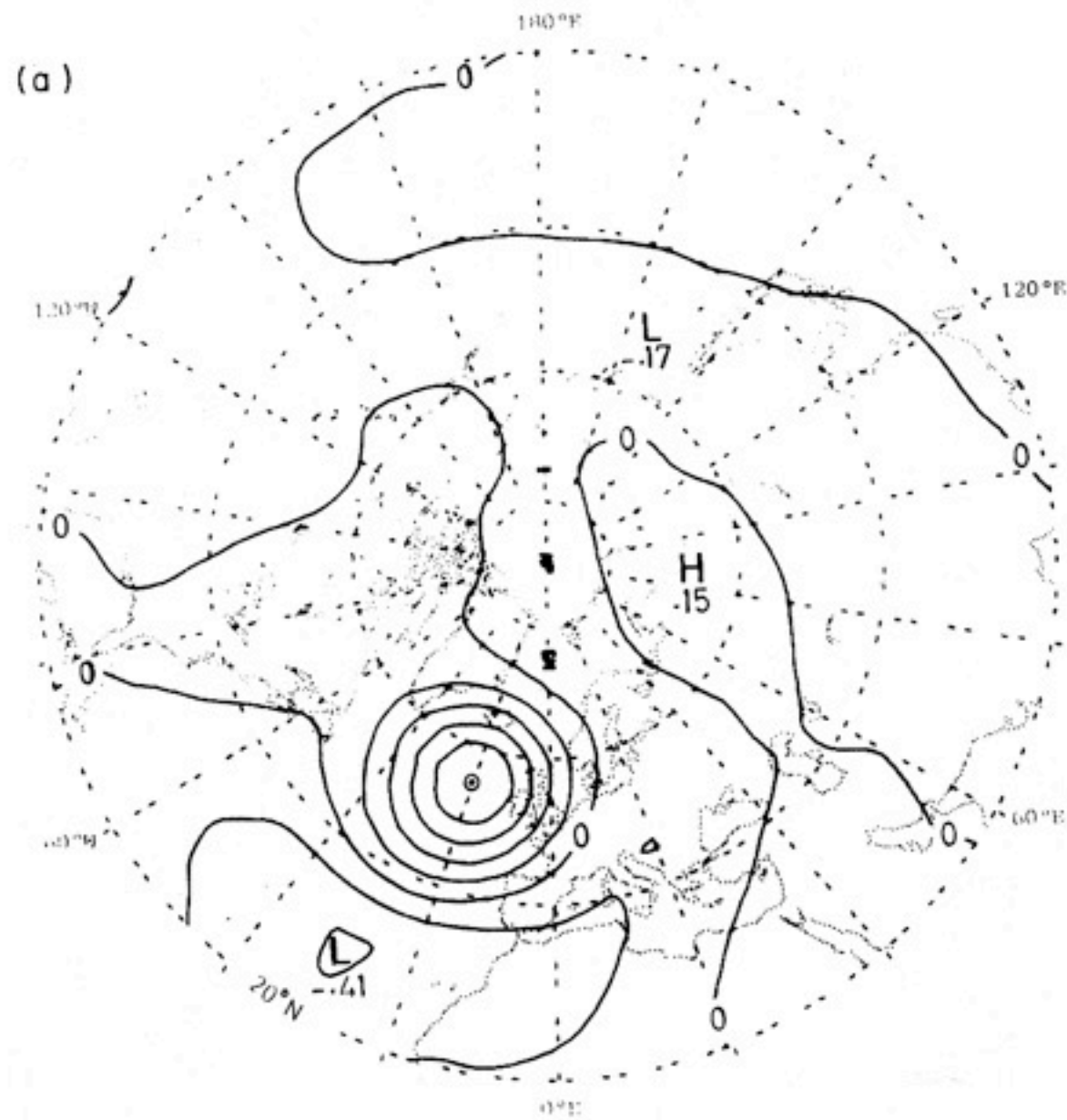
< 6 d



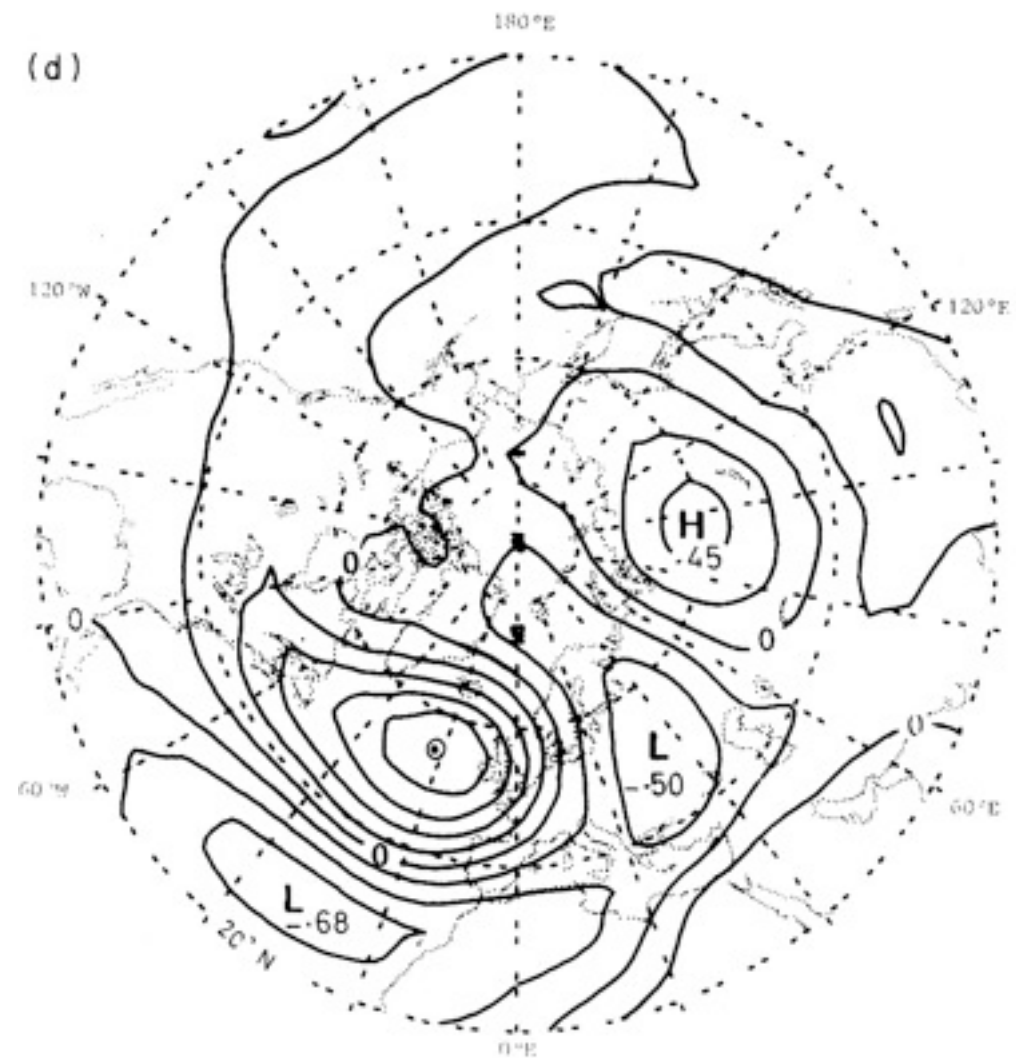
30 d lowpass



*after Wallace and Lau, Issues in Atmospheric and Oceanic Modeling 1985*

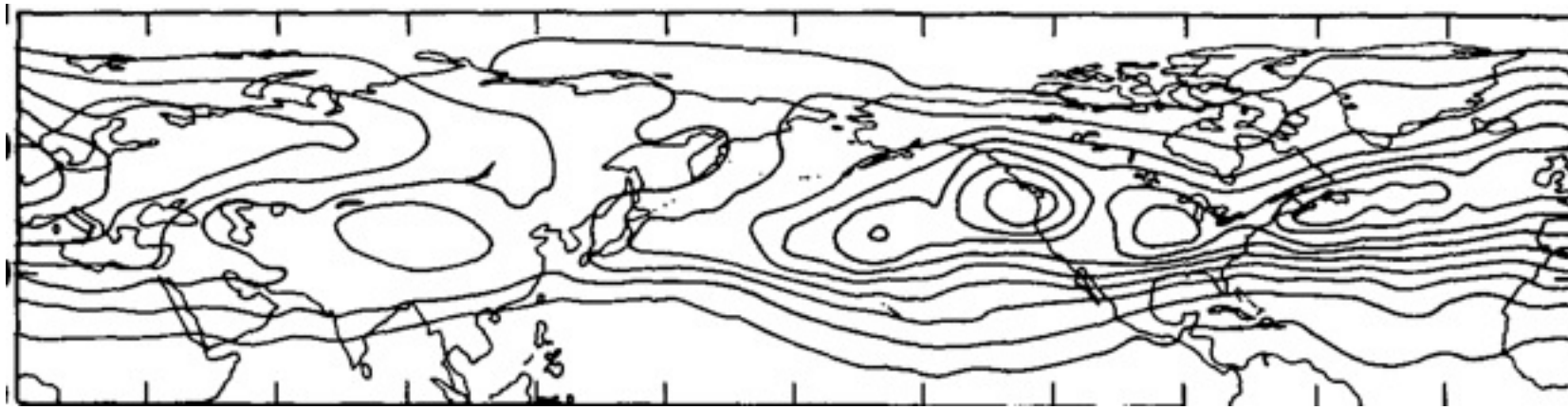


Unfiltered

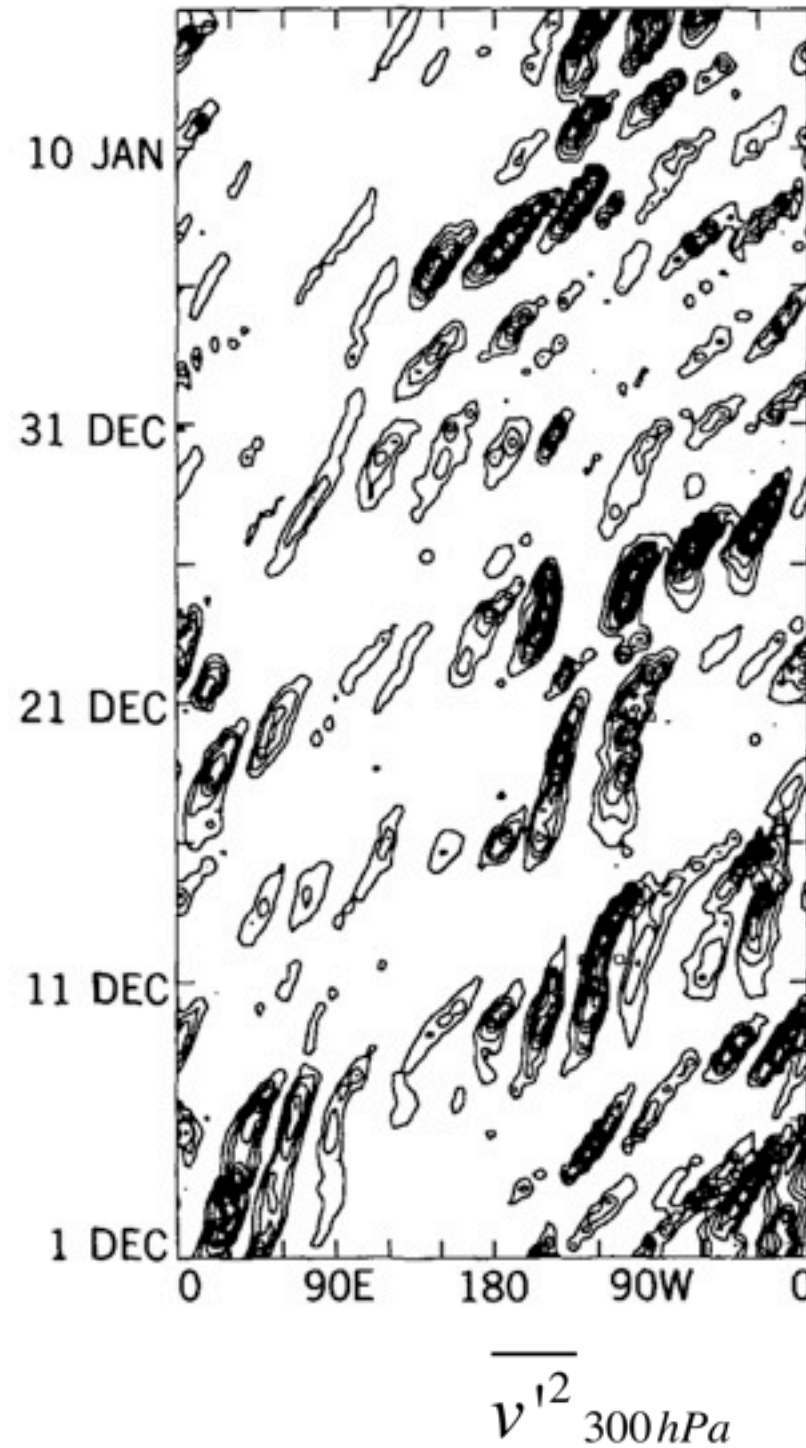
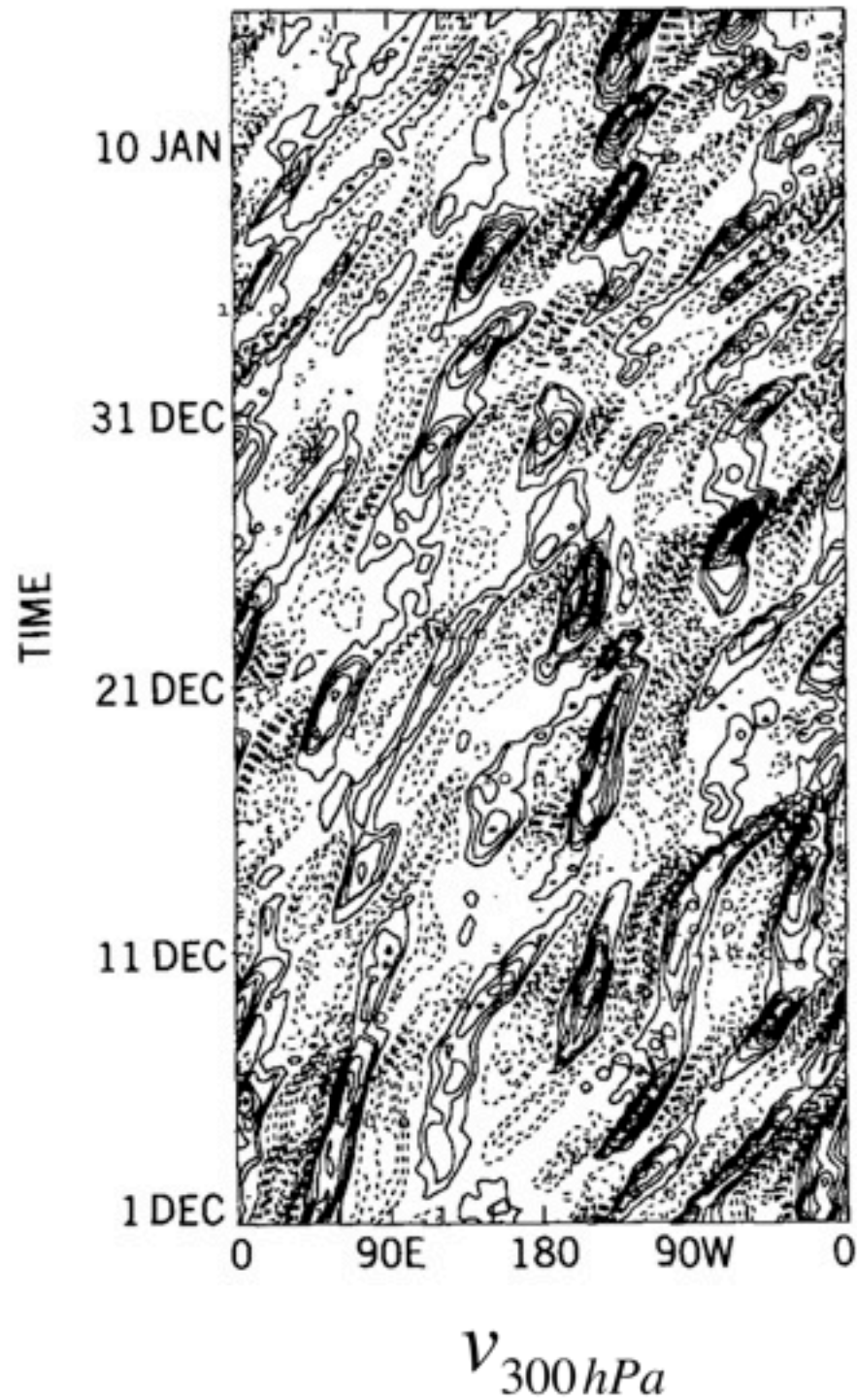


>30 d





$\overline{v'^2}$  300hPa



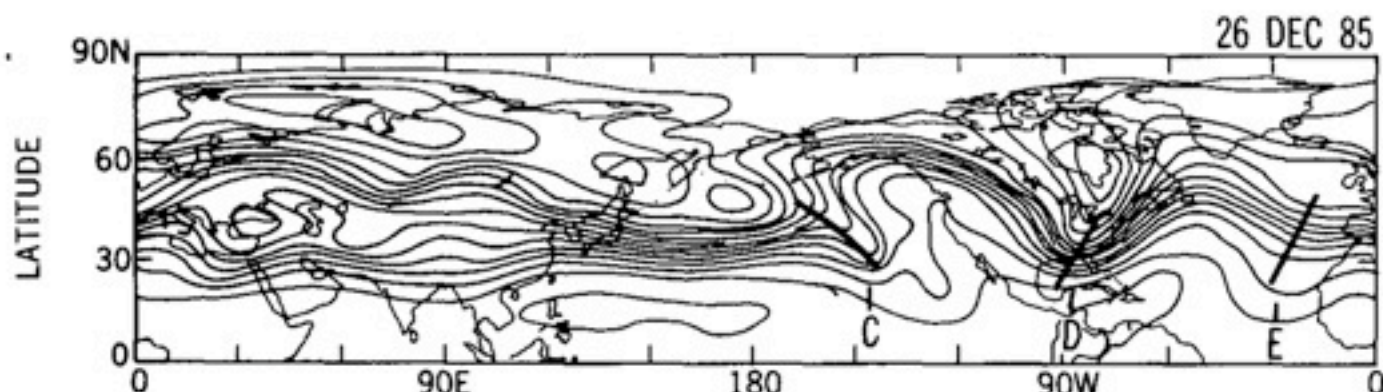
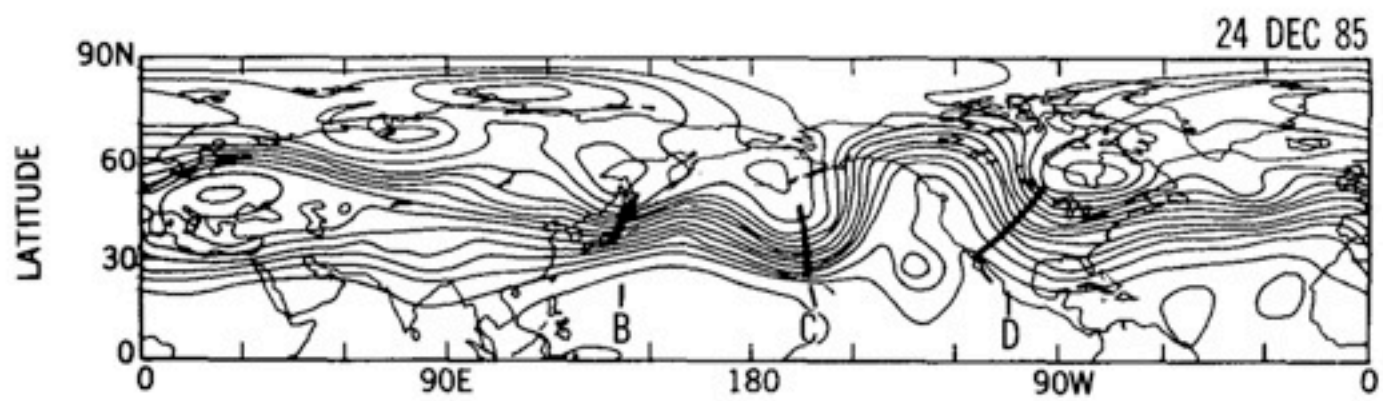
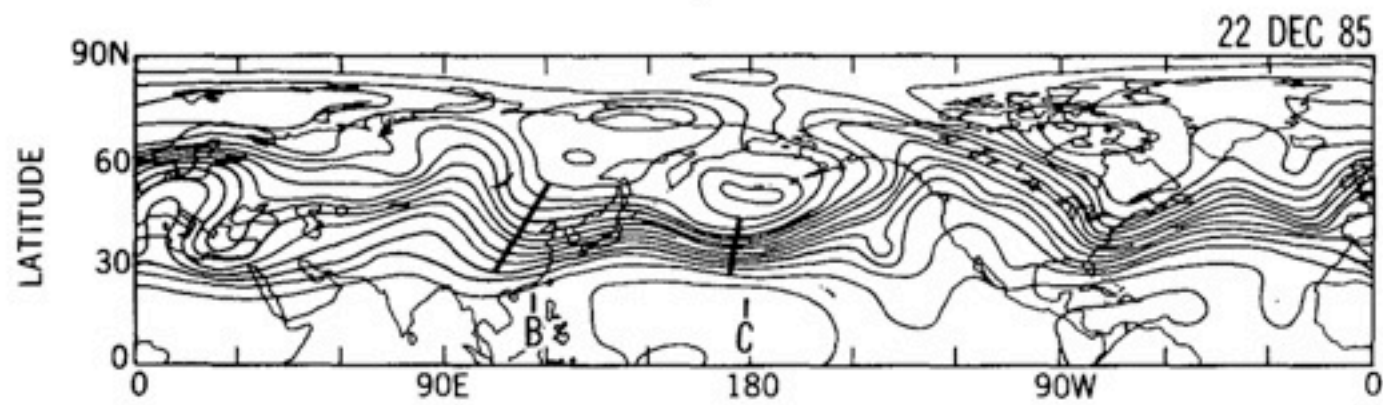
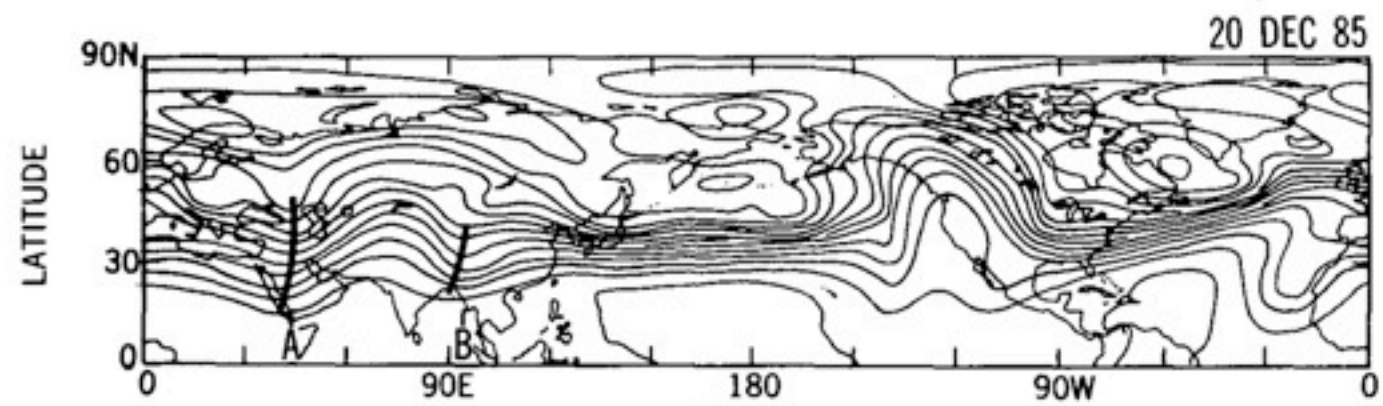
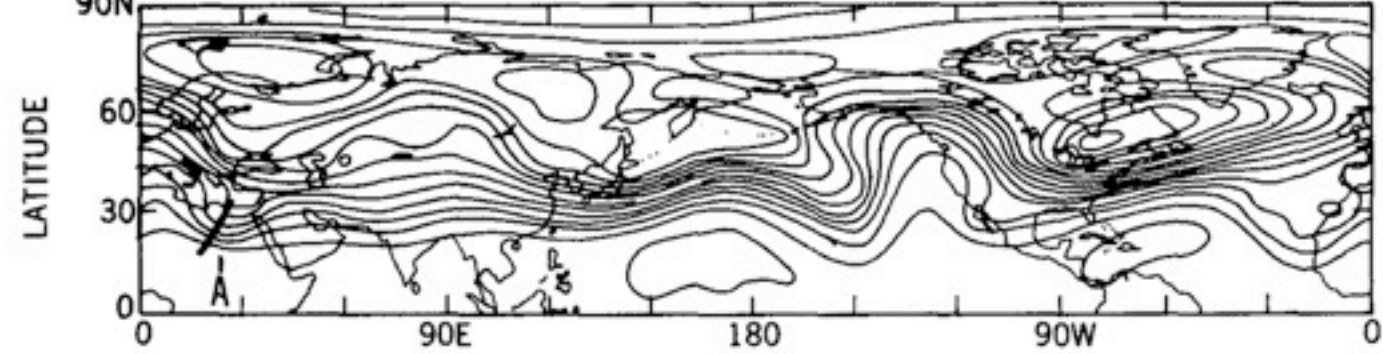
time-longitude sections

$30^\circ$  to  $60^\circ$ N

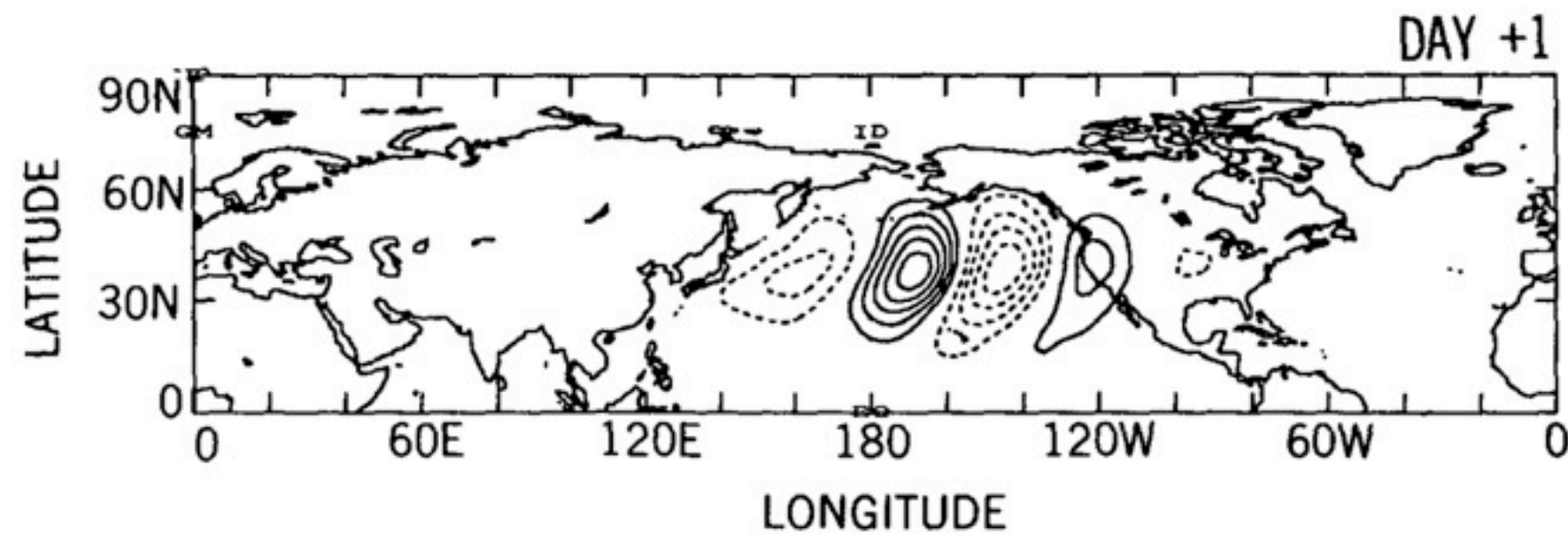
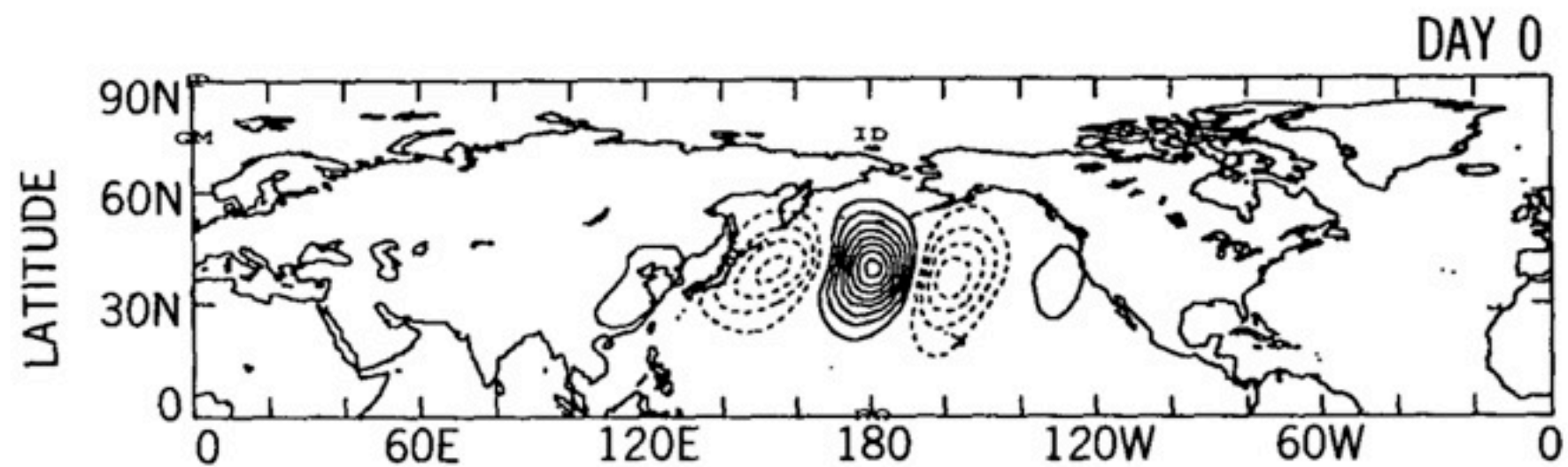
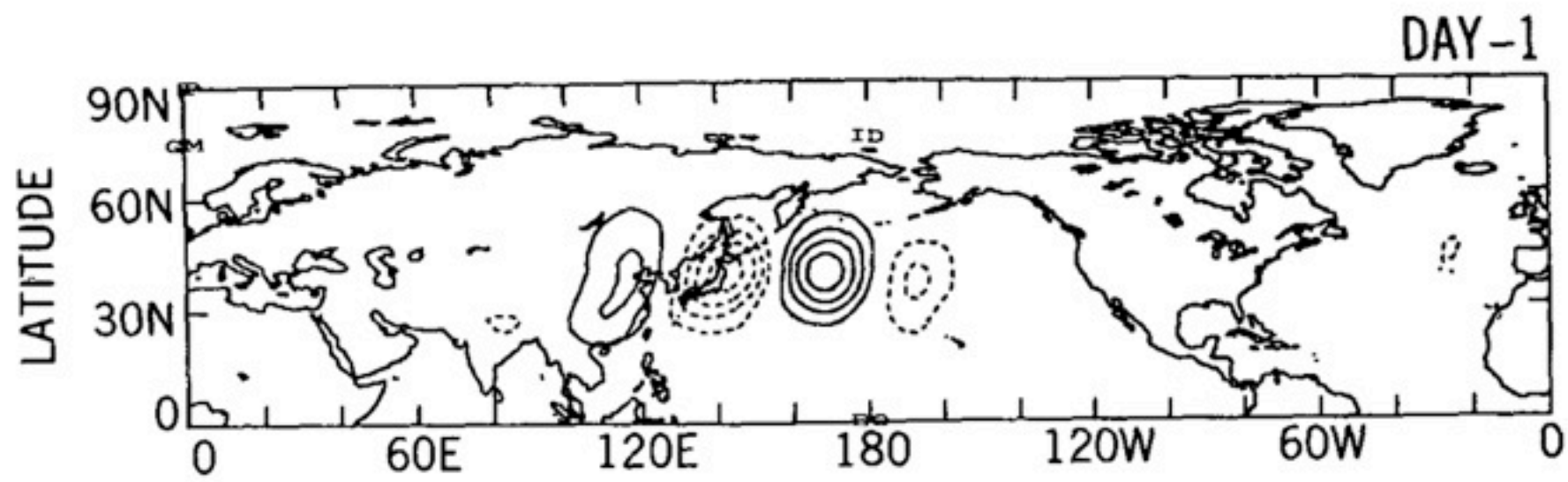
1985-86

Edmund K.M. Chang,  
*J. Atmos. Sci.* 1993

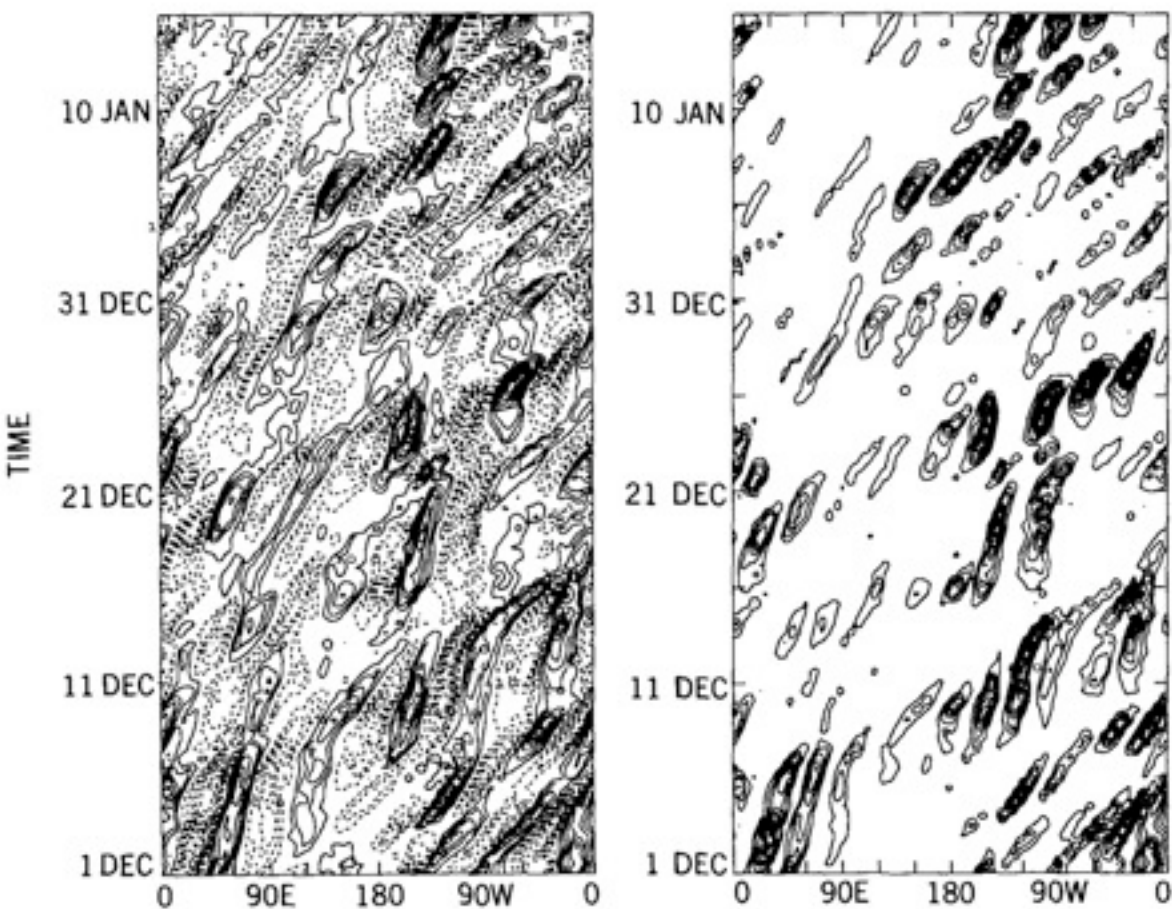






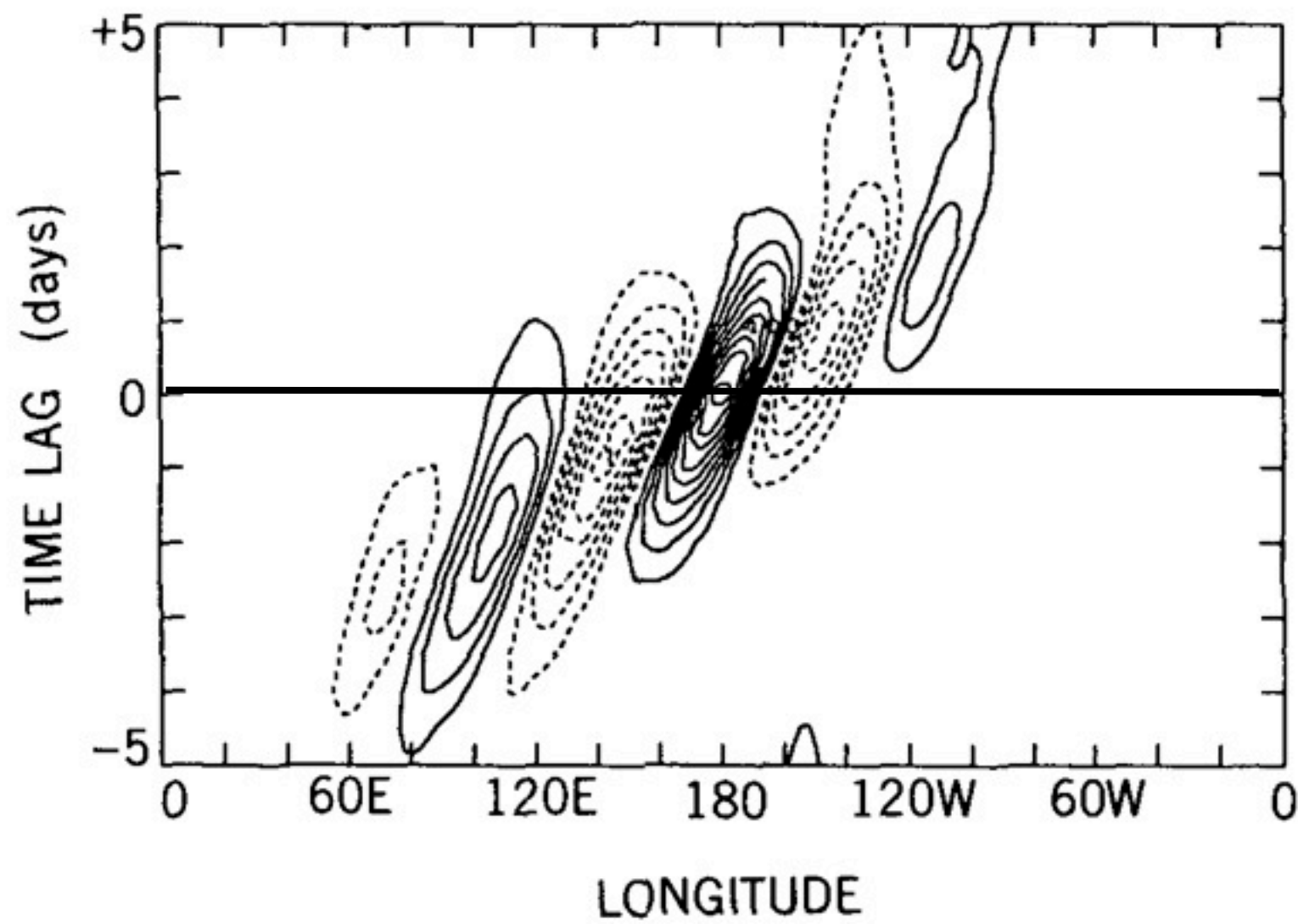


Edmund K.M. Chang,  
*J. Atmos. Sci.* 1993



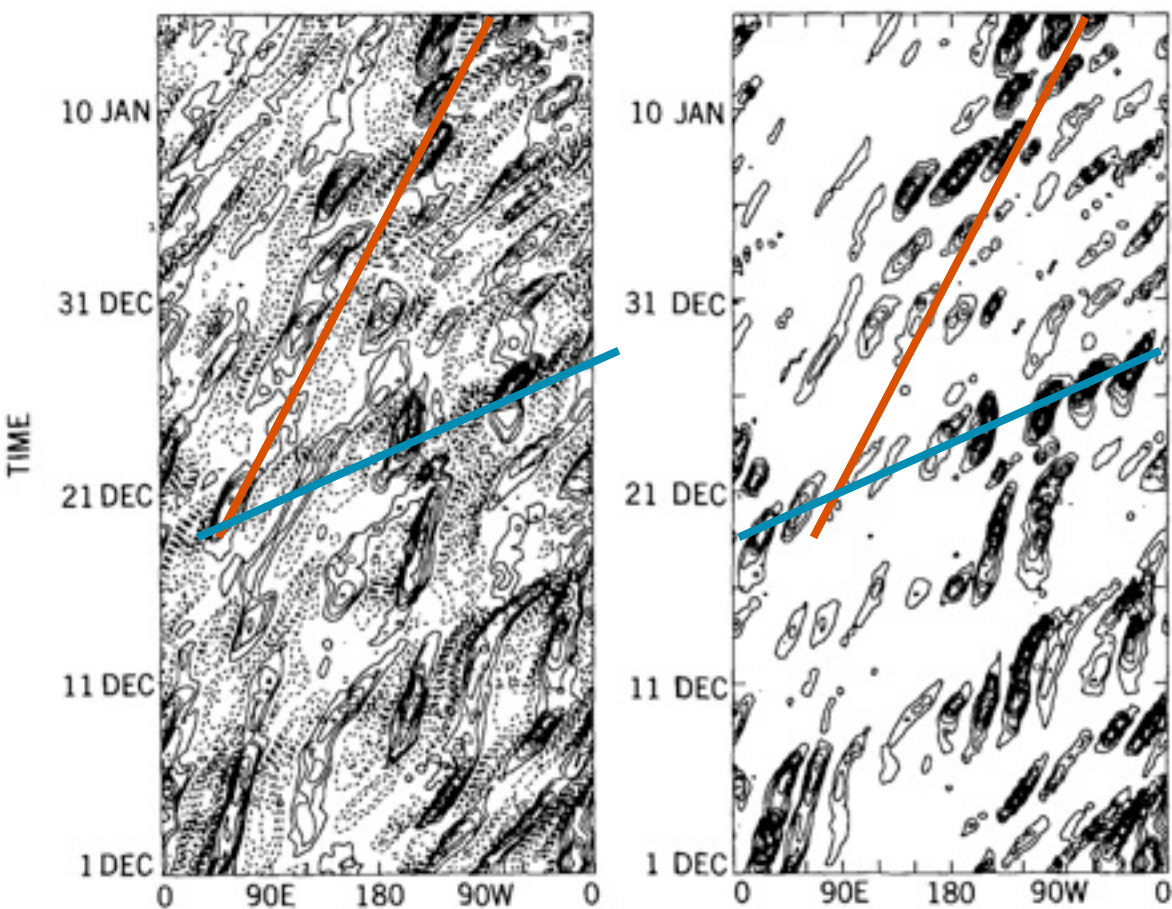
30° to 60°N

lag correlations



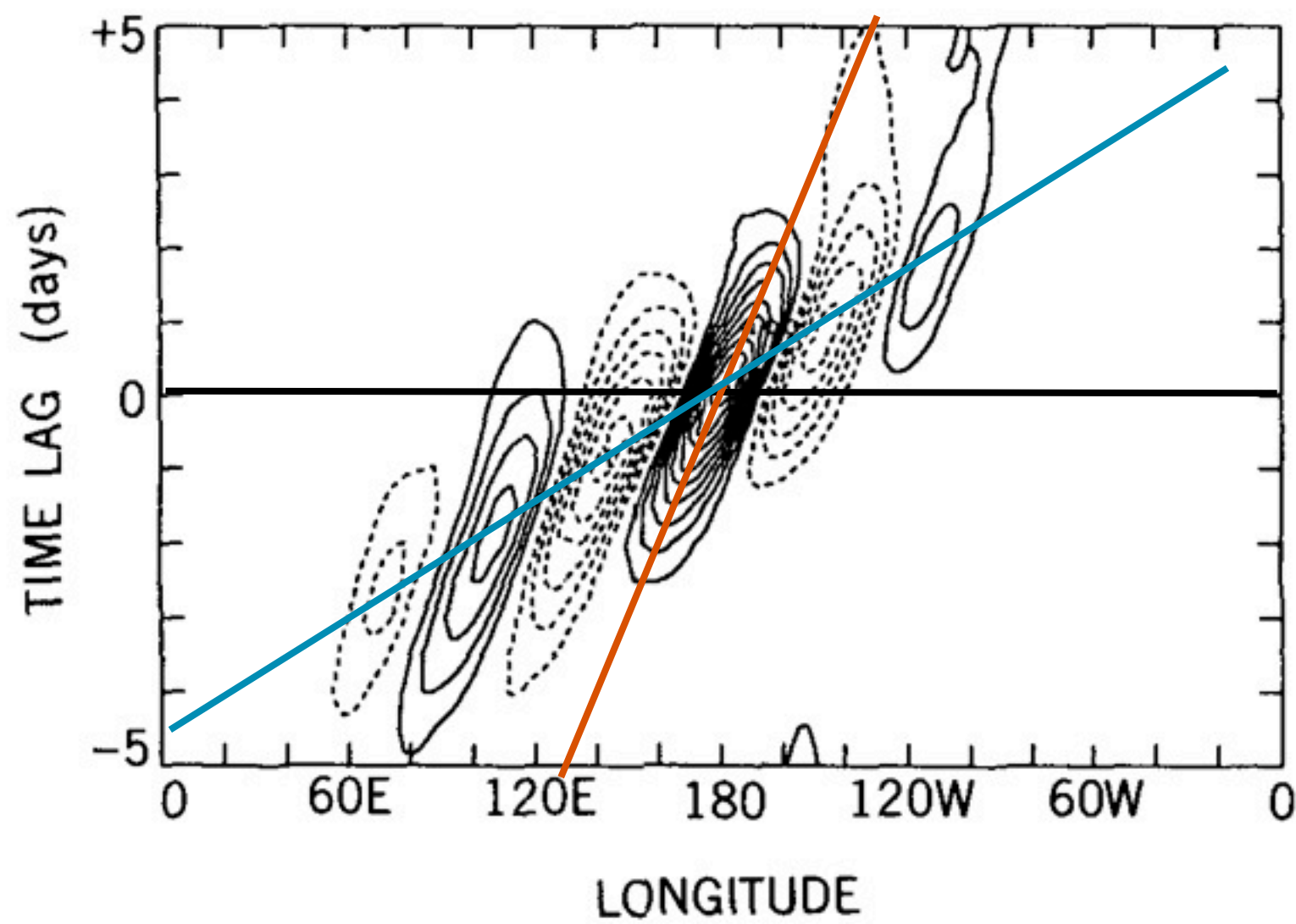
Edmund K.M. Chang,  
*J. Atmos. Sci.* 1993





rate of downstream phase propagation

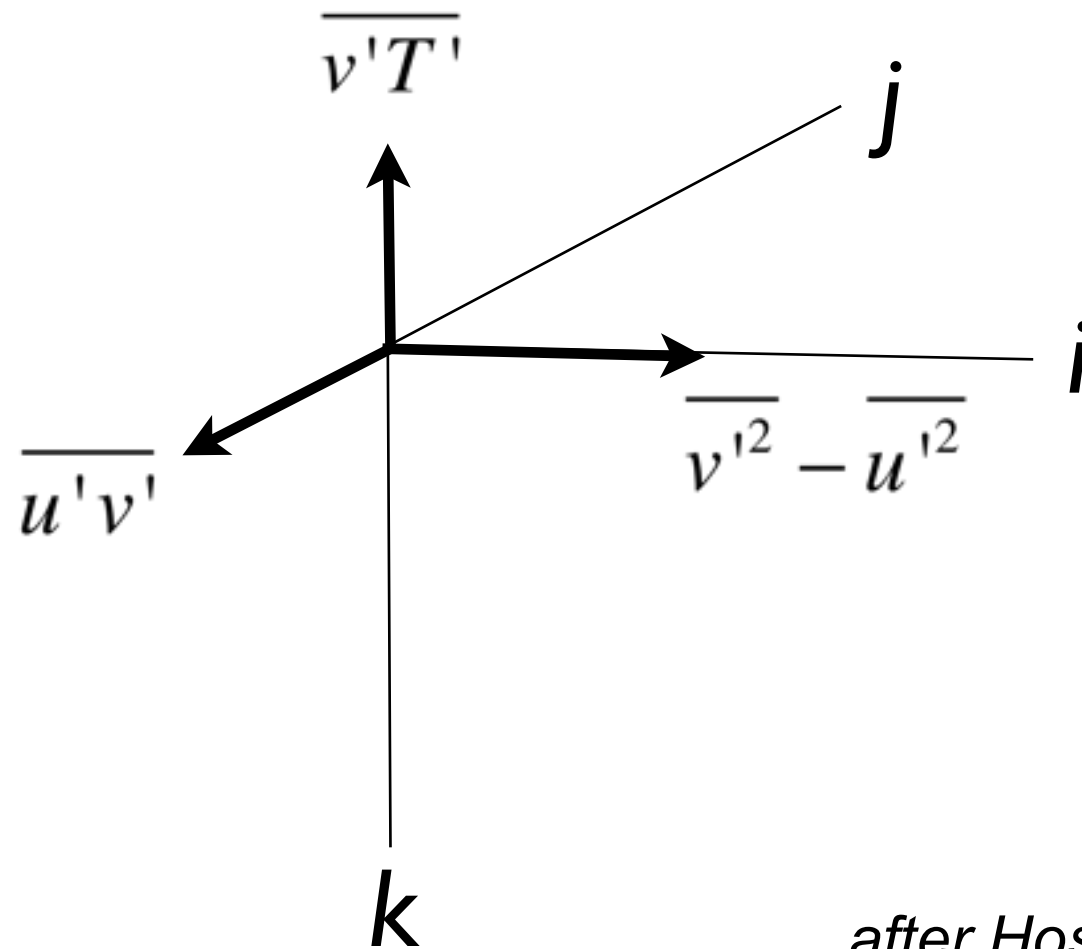
rate of downstream dispersion  
(group velocity)



# Generalization of $E$ -vector formalism to three dimensions

In analogy with  $\frac{\partial \bar{\zeta}}{\partial t} \simeq -\frac{\partial}{\partial y}(\nabla \cdot \vec{E})$  where  $\vec{E} \equiv (\overline{v'^2} - \overline{u'^2}, -\overline{u'v'})$

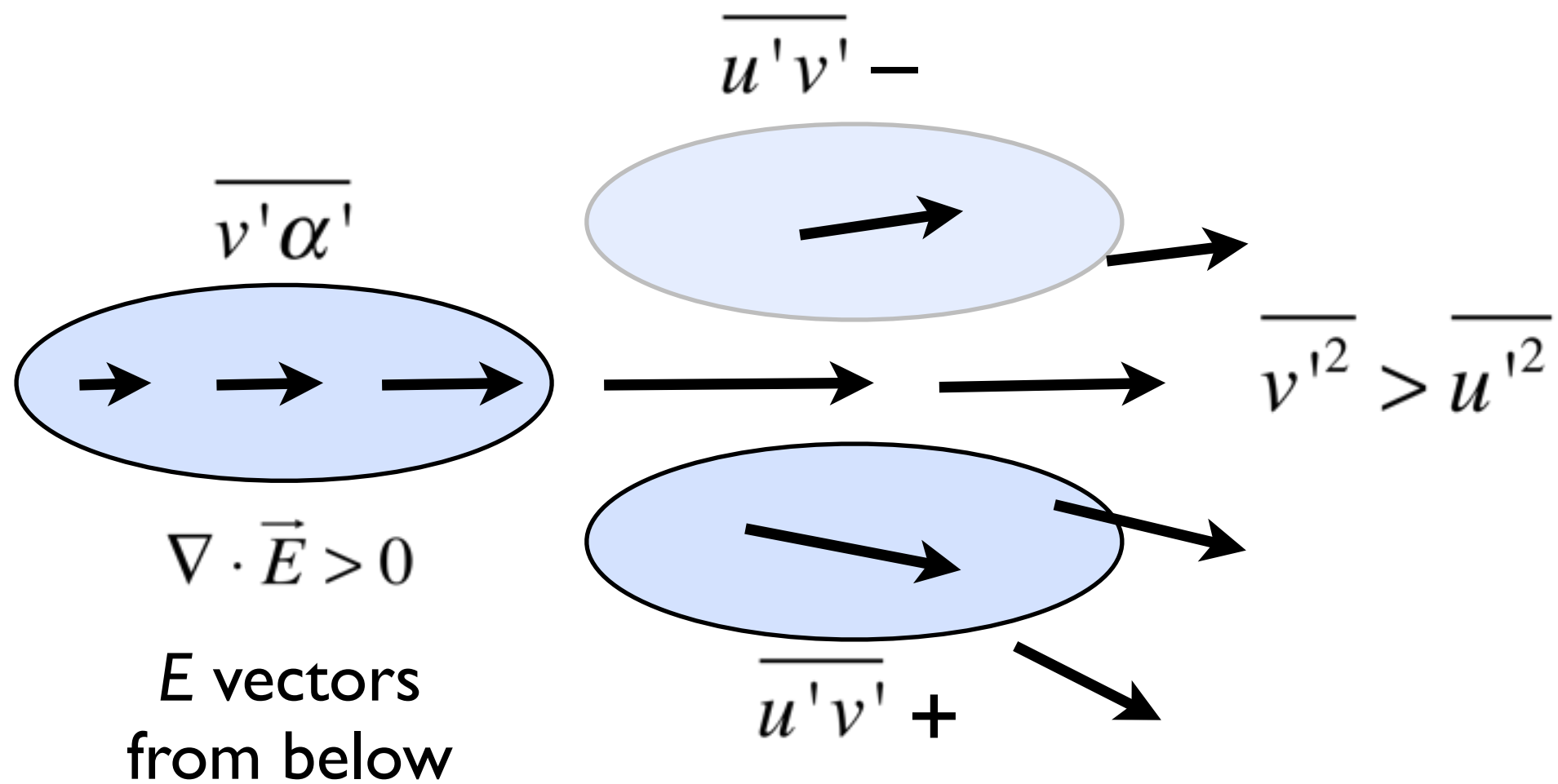
we can write  $\frac{\partial \bar{q}}{\partial t} \simeq -\frac{\partial}{\partial y}(\nabla_3 \cdot \vec{E})$  where  $\vec{E} \equiv \left( \overline{v'^2} - \overline{u'^2}, -\overline{u'v'}, -\frac{\overline{v'\alpha'}}{\sigma} \right)$



*after Hoskins, James and White JAS 1983*



# Idealized storm track showing $E$ vectors



After Lau and Holopainen *JAS* 1984

Transient eddy forcing of the climatological-mean flow

$$\left\{ \frac{1}{f} \nabla^2 + f \frac{\partial}{\partial p} \left( \frac{1}{\sigma} \frac{\partial}{\partial p} \right) \right\} \frac{\partial \Phi}{\partial t} = D + R_1,$$

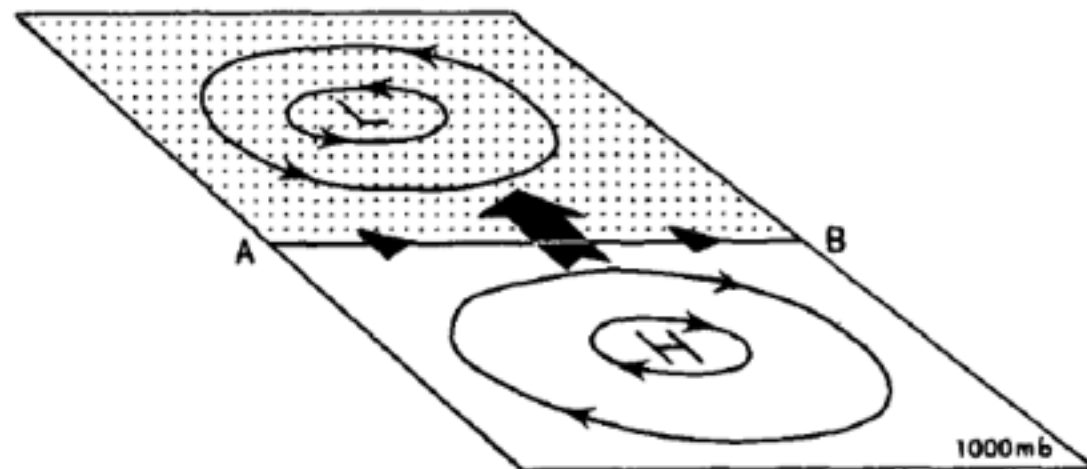
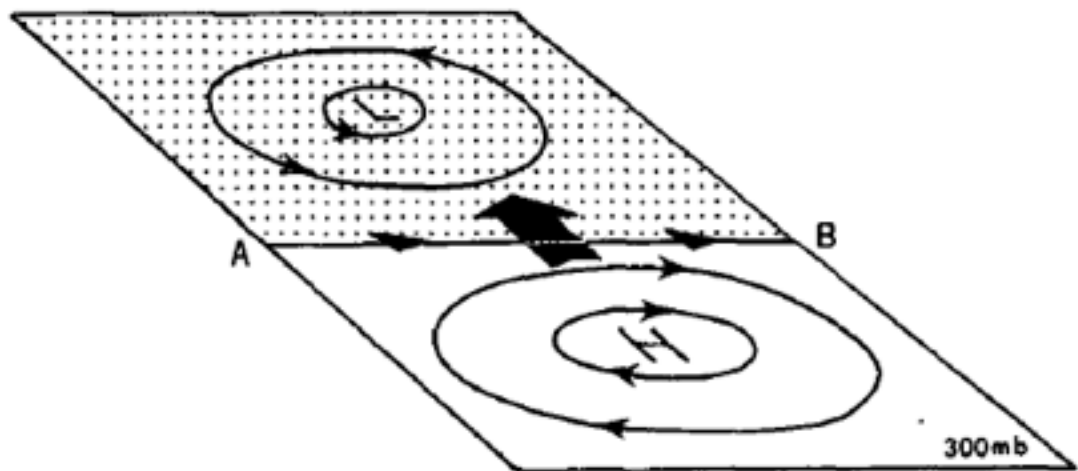
Time-averaged geopotential tendency equation

$$D = D^{\text{HEAT}} + D^{\text{VORT}},$$

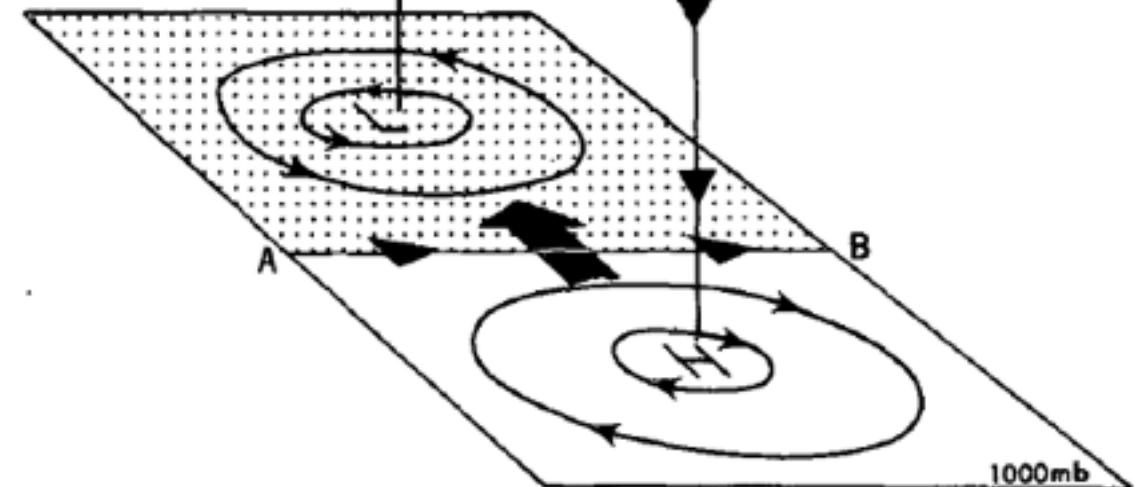
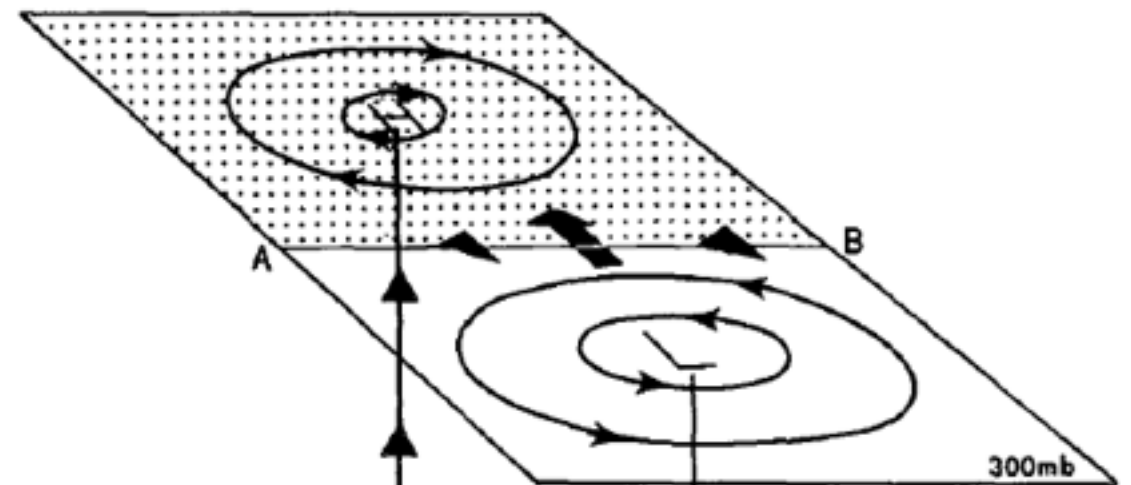
$$D^{\text{HEAT}} = f \frac{\partial}{\partial p} \left( \frac{\nabla \cdot \overline{\mathbf{V}'\theta'}}{\bar{S}} \right)$$

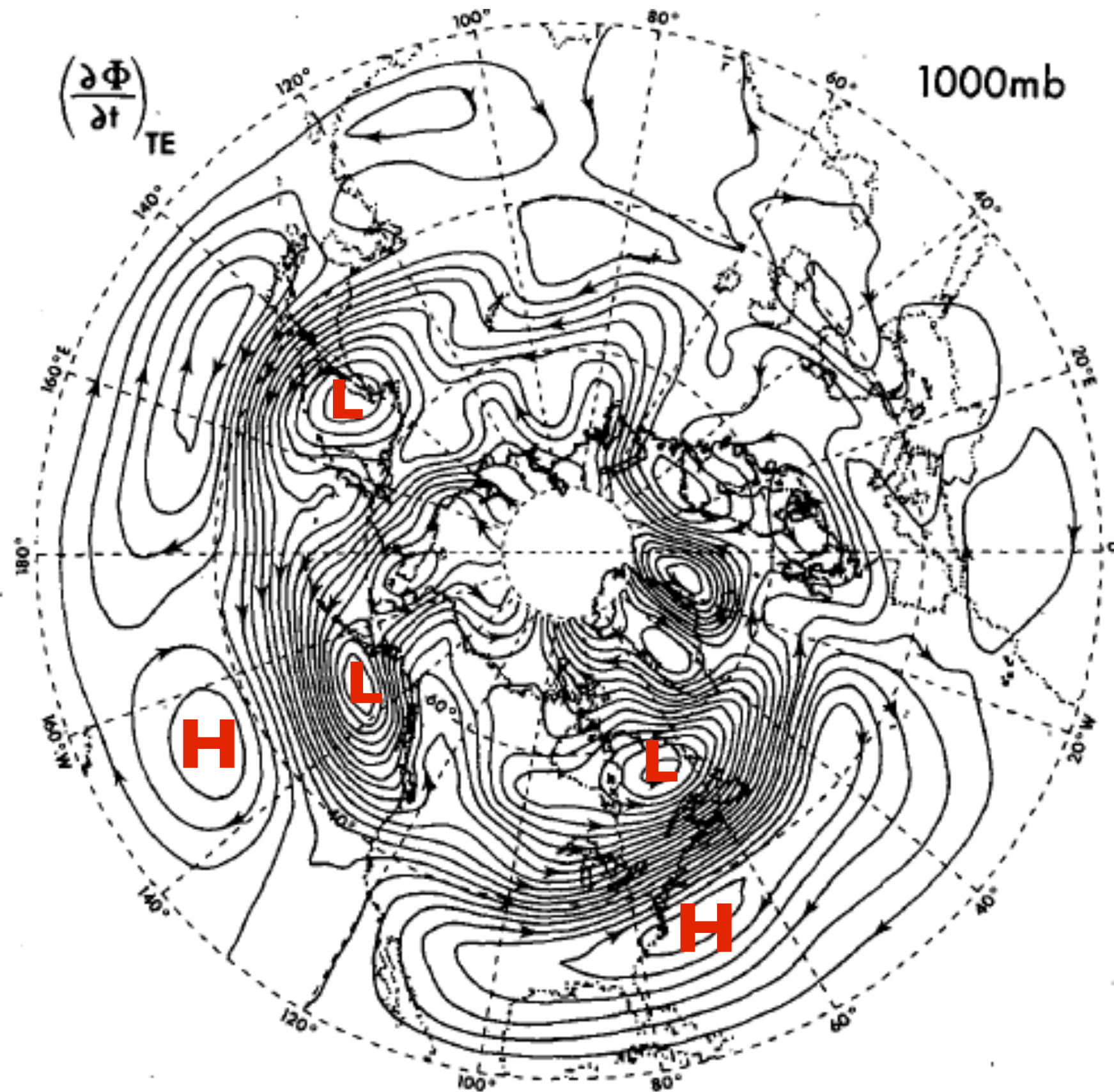
$$D^{\text{VORT}} = -\nabla \cdot \overline{\mathbf{V}'\zeta'}.$$

VORTICITY FLUXES

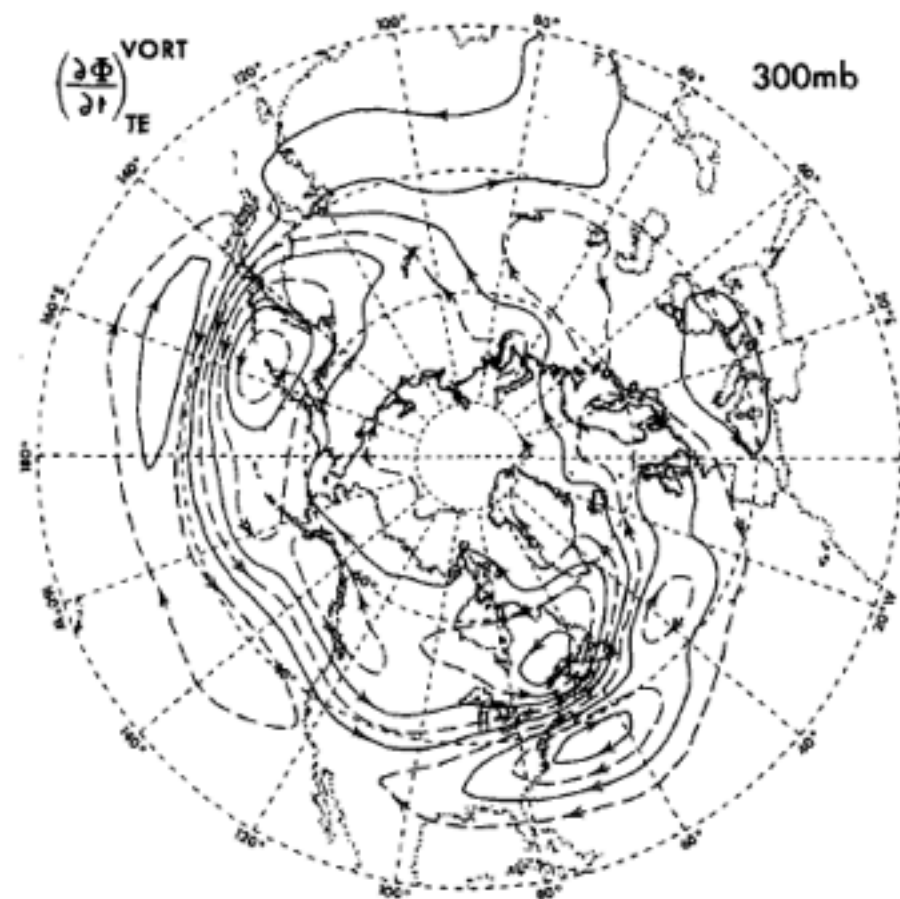
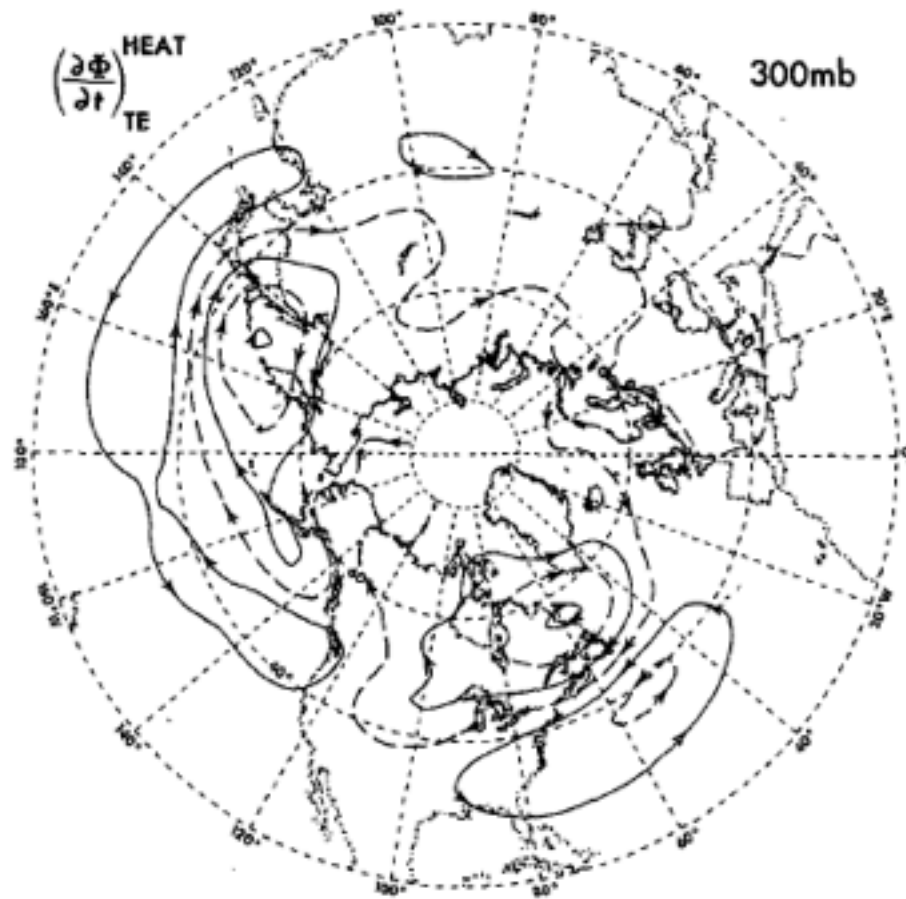


HEAT FLUXES



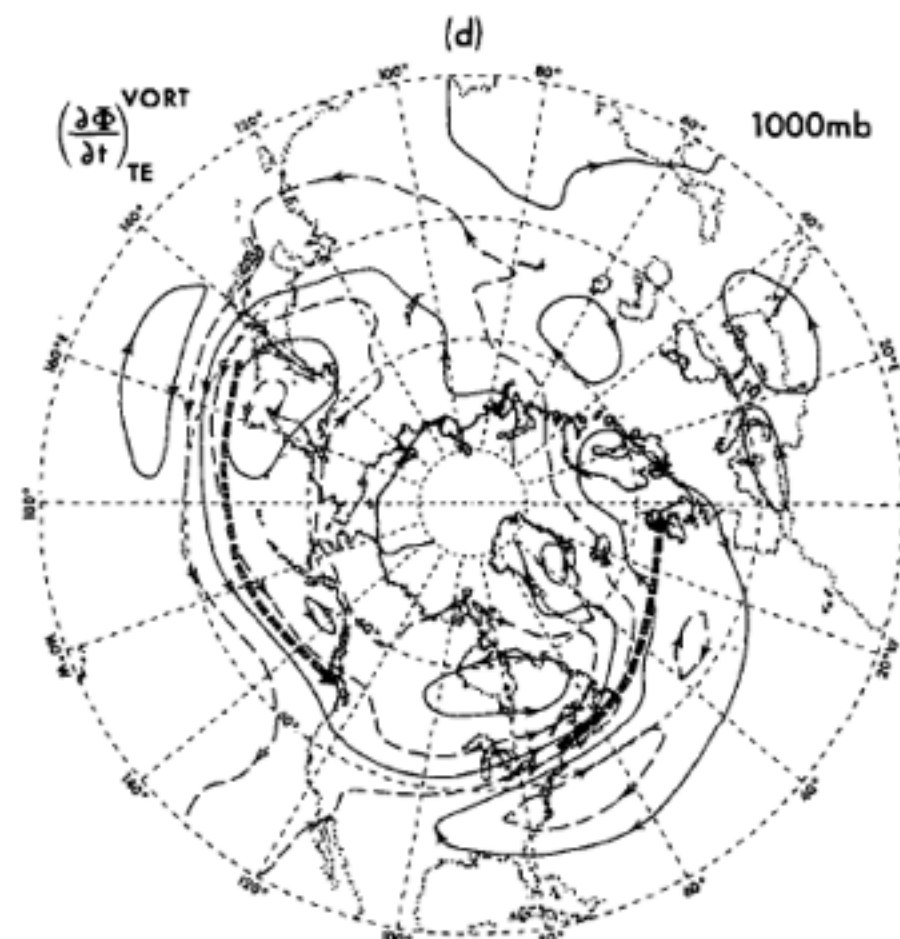
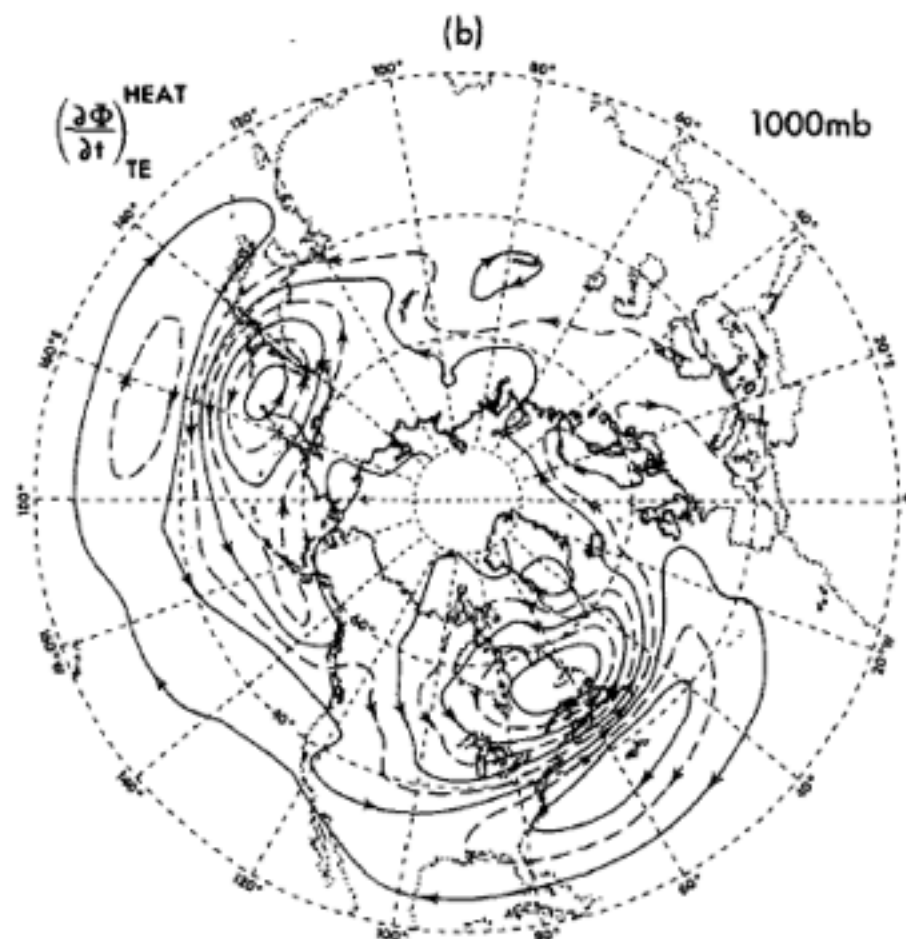


Tendency induced by all transients: high frequencies dominate  
*After Lau and Holopainen JAS 1984*



tendency induced  
by high frequency  
transients

at 300 hPa the tendencies  
induced by the heat and  
vorticity fluxes by the  
transient eddies oppose one  
another so the net forcing is  
rather small.

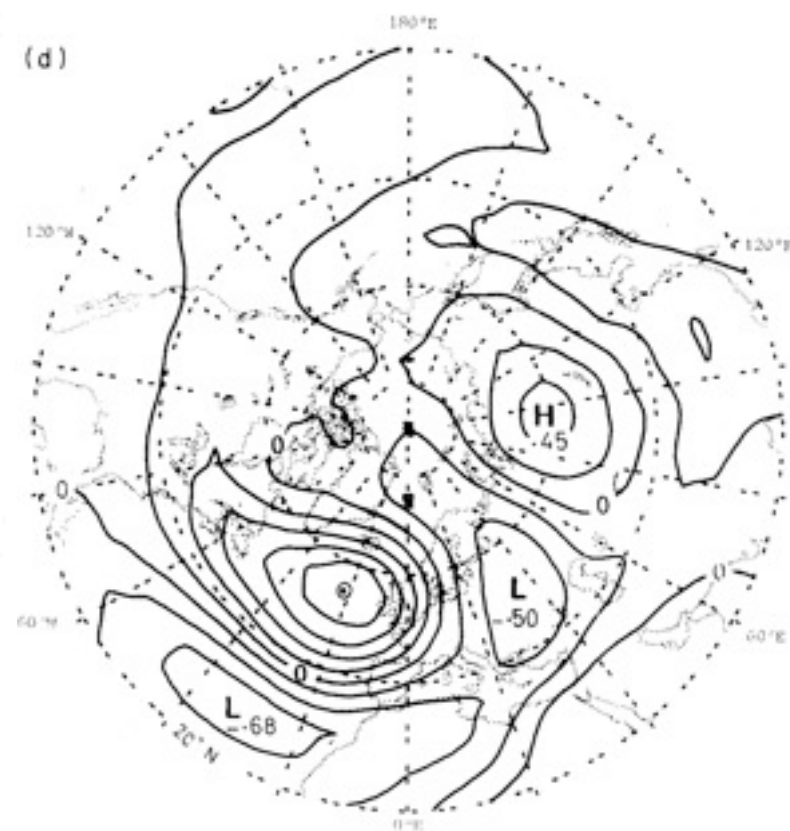
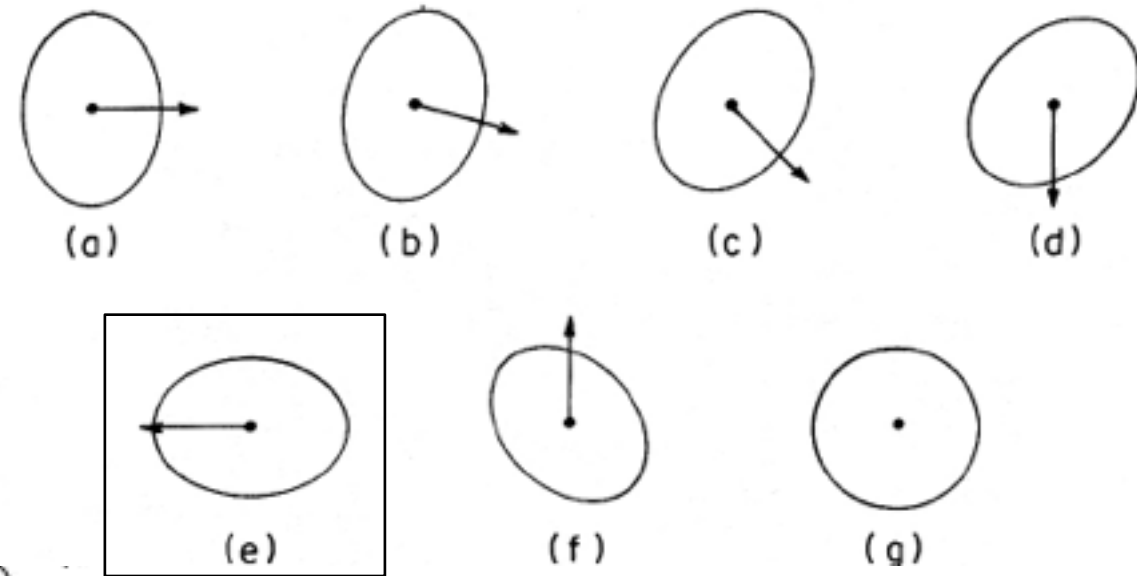
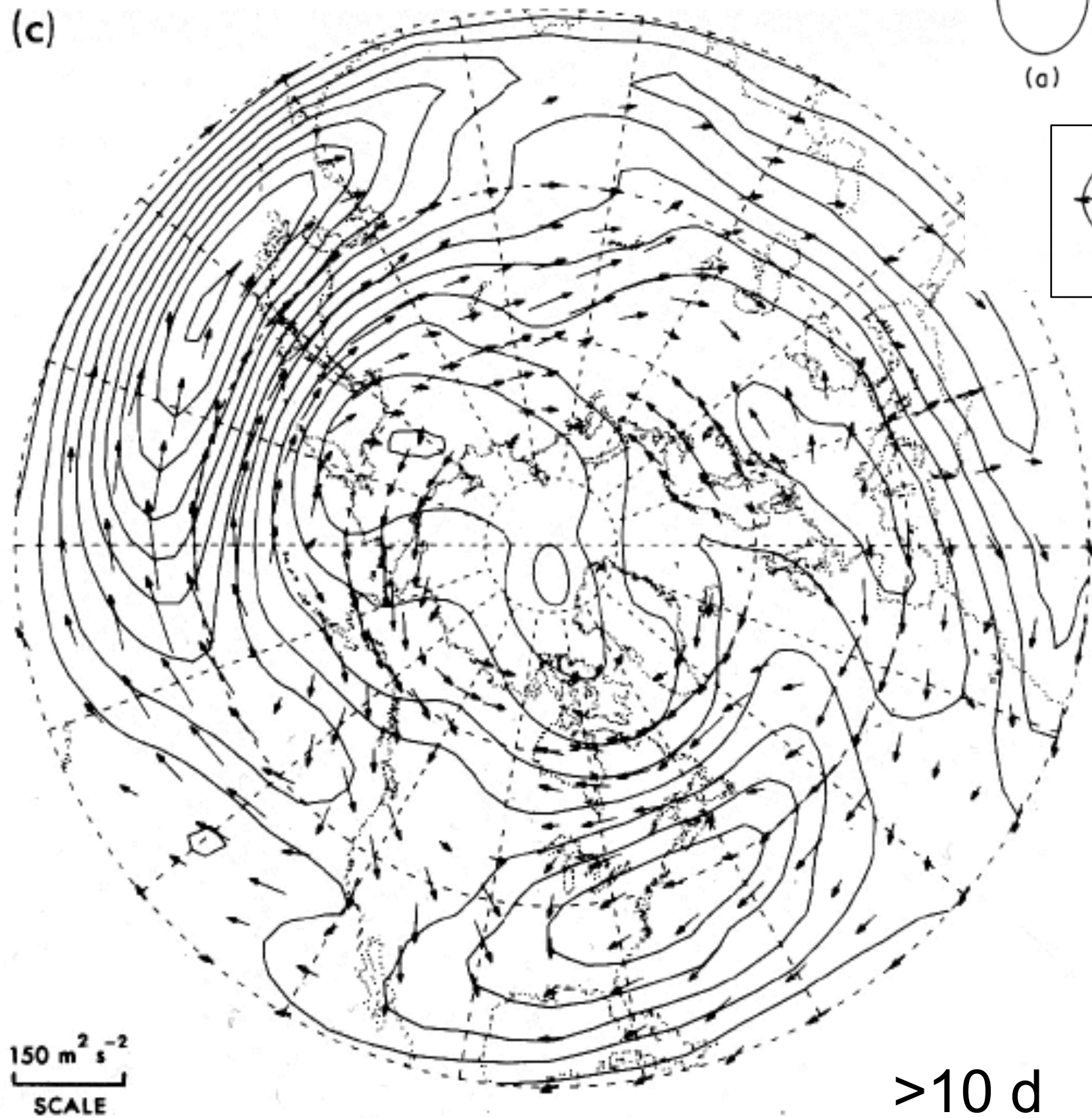


at 1000 hPa they reenforce  
one another so the eddy-  
induced tendency is much  
larger. Note how the heat  
fluxes dominate.

*After Lau and Holopainen JAS 1984*



More about the evolution of the transients



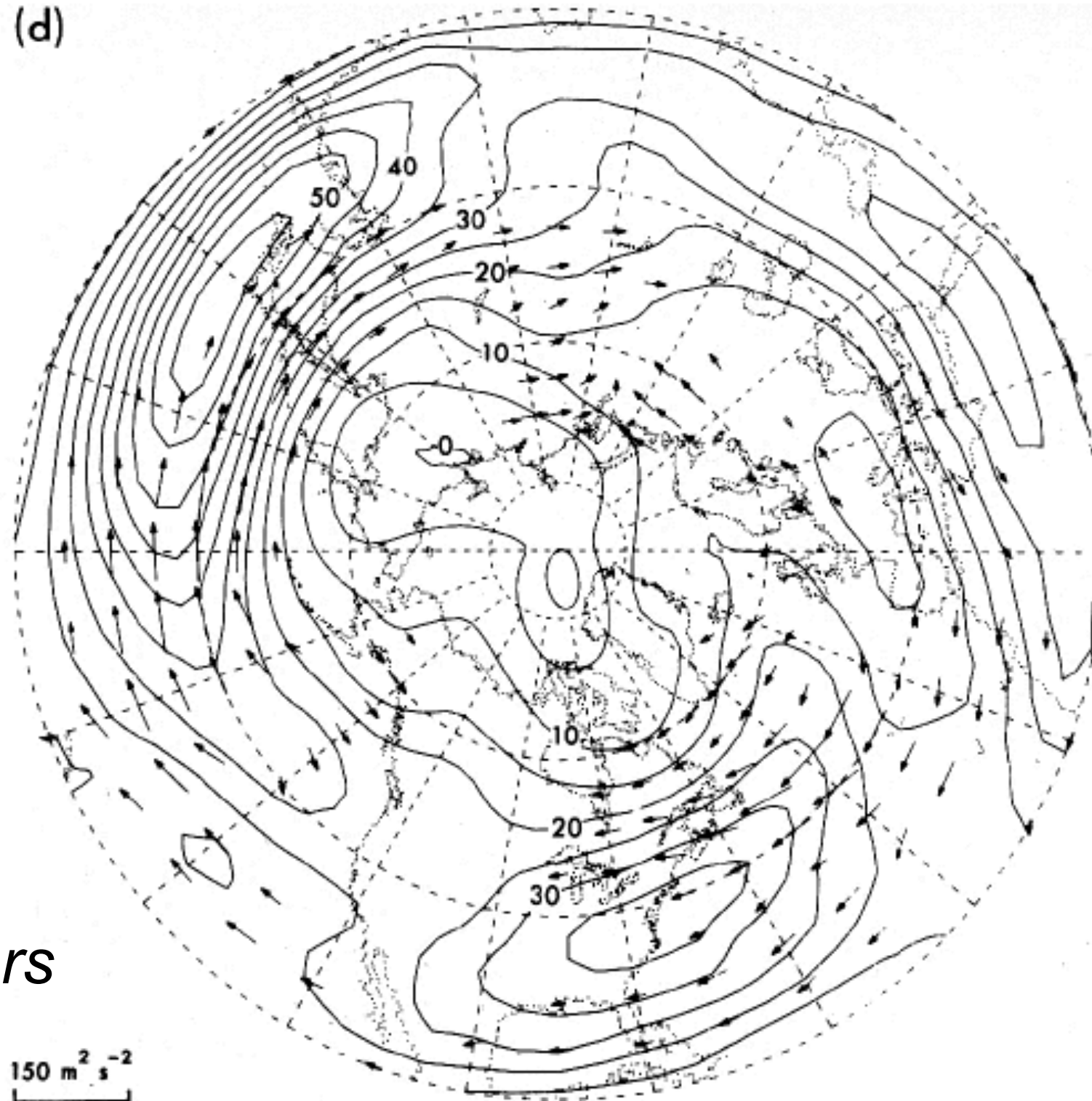
after Wallace and Lau, *Issues in Atmospheric and Oceanic Modeling* 1985

>30 d

(d)

*E* vectors

150  $\text{m}^2 \text{s}^{-2}$   
SCALE



Note dominance of westward arrows: zonally elongated perturbations localized in climatological-mean jet exit regions

*after Wallace and Lau, Issues in Atmospheric and Oceanic Modeling 1985*

# Why do transient disturbances become more elongated as we go toward lower frequency?

First consider the situation in the ocean in which there is no zonal background flow. All transient perturbations propagate westward for to the beta effect. The phase speed is given by

$$c = \frac{\beta}{k^2 + l^2}$$

and the frequency apparent to a fixed observer by

$$\omega = \frac{k\beta}{k^2 + l^2}$$

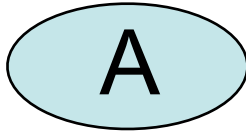
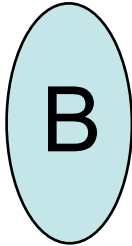
where  $k$  is the zonal wavenumber (the inverse of wavelength) and  $l$  is the meridional wavenumber.

Longer waves (i.e., waves with smaller two-dimensional wavenumber  $k^2 + l^2$ ) propagate westward faster than shorter waves.

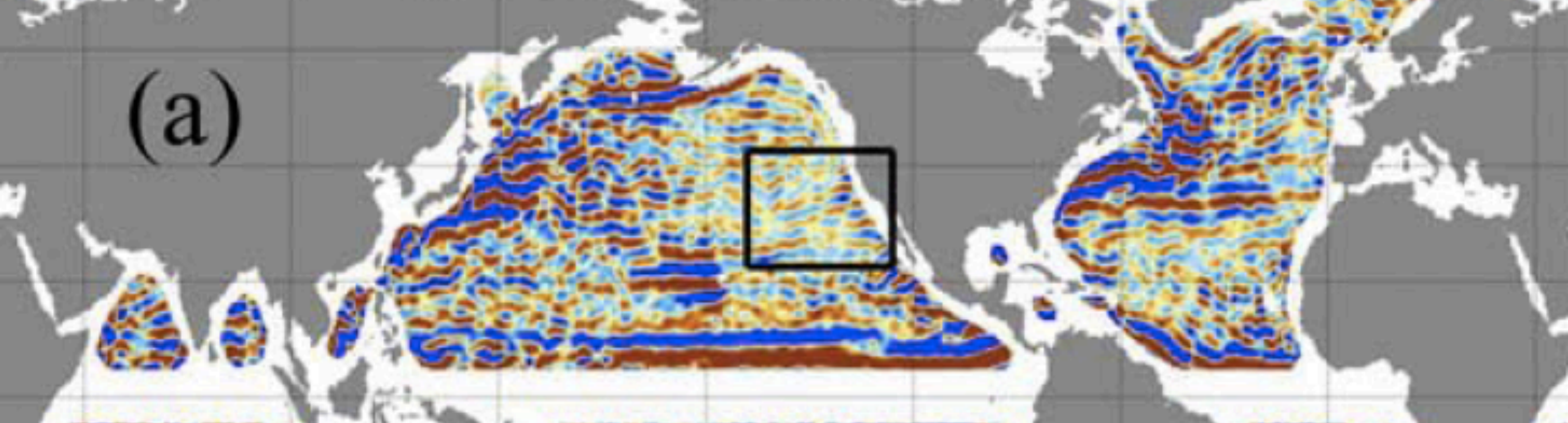


Why do transient disturbances become more elongated as we go toward lower frequency?

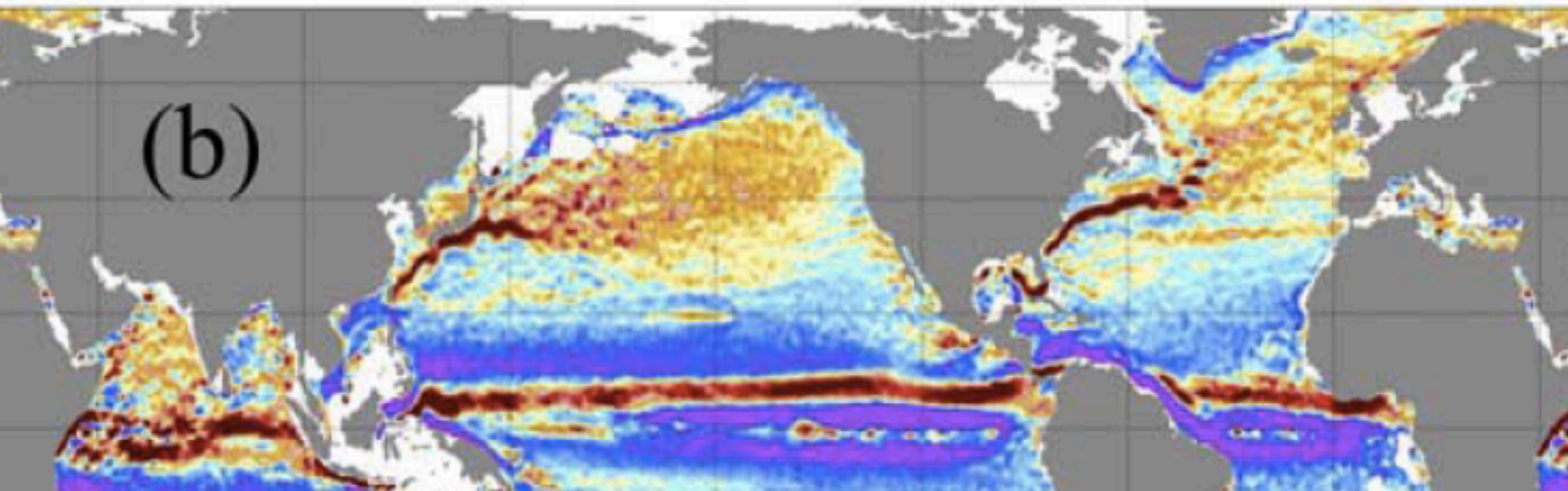
Now consider two disturbances with the same two-dimensional wavenumber  $k^2 + l^2$

One shaped like  A and the other shaped like  B

A takes longer to pass the fixed observer, so it has a lower frequency. If a spectrum of waves is present, with some being zonally elongated, like A and some meridionally oriented like B. The more strongly we lowpass filter the data, the more the zonally elongated disturbances will be favored. In the limit of zero frequency, the wind perturbations will be nearly purely zonal.



zonal geostrophic surface velocity based on satellite altimetry



zonal geostrophic surface velocity based drifters



# Why do transient disturbances become more elongated as we go toward lower frequency?

Now consider two disturbances with the same two-dimensional wavenumber  $k^2 + l^2$

One shaped like  and the other shaped like 

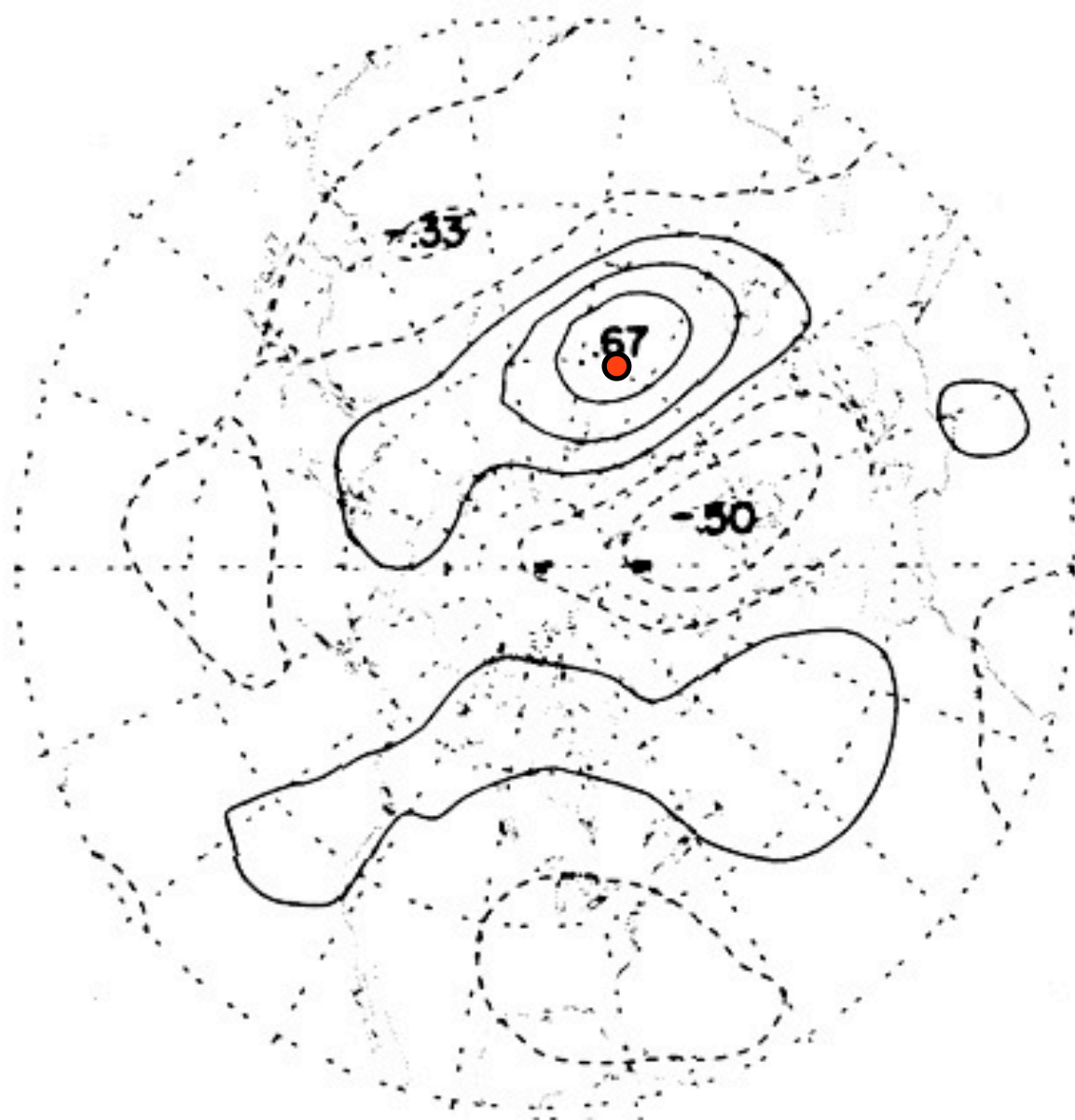
A takes longer to pass the fixed observer, so it has a lower frequency. If a spectrum of waves is present, with some being zonally elongated, like A and some meridionally oriented like B. The more strongly we lowpass filter the data, the more the zonally elongated disturbances will be favored. In the limit of zero frequency, the wind perturbations will be nearly purely zonal.

The argument carries over to the atmosphere where  $\omega = k \left( u - \frac{\beta}{k^2 + l^2} \right)$ . In this case, the most slowly propagating disturbances will be the ones for which the term in parentheses is smallest, but for a prescribed spectrum of  $k^2 + l^2$ , the zonally elongated ones will still have the lowest frequencies.

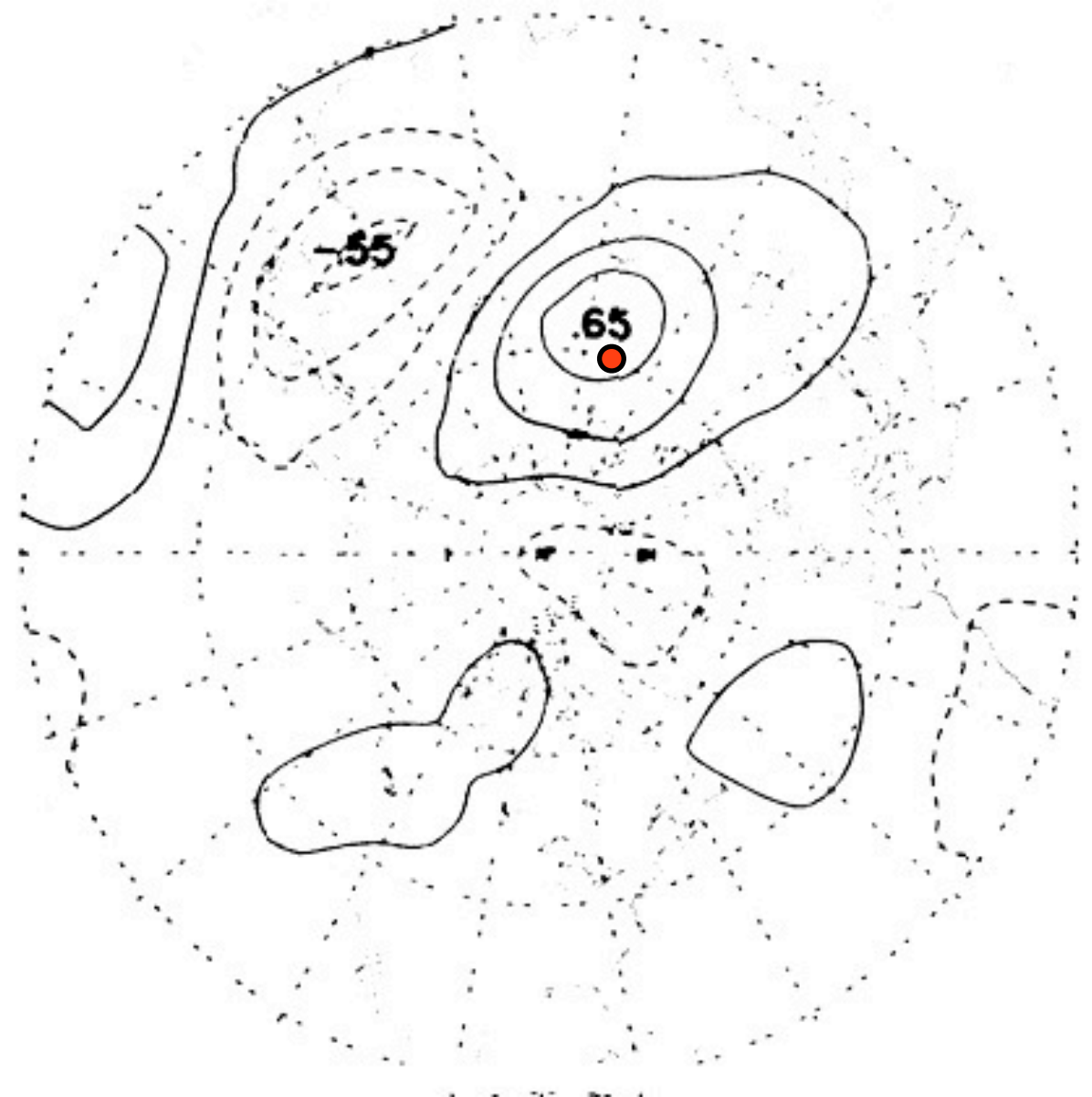
More about the evolution of the transients



# 10-30d bandpass DJF 500 hPa height



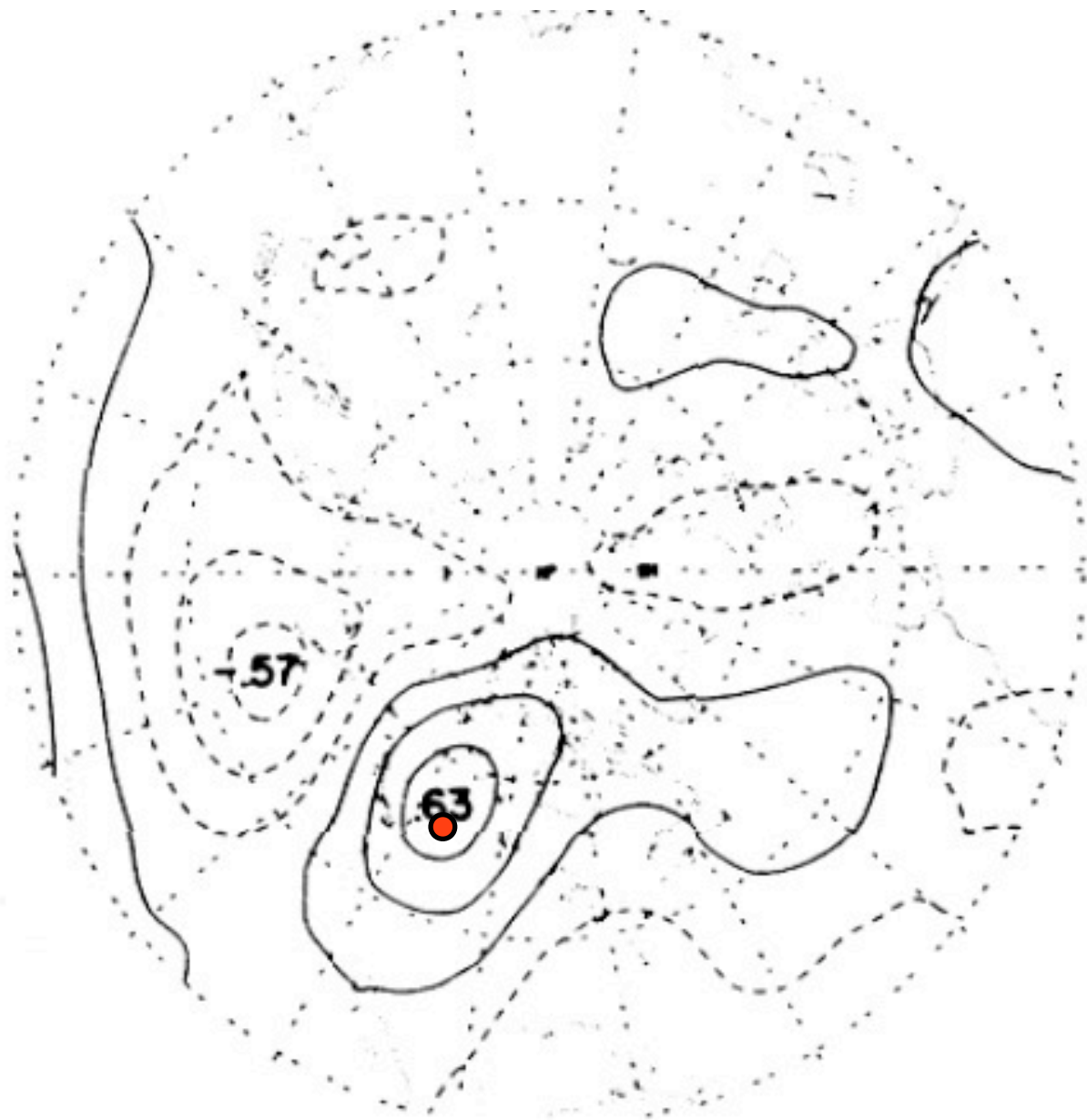
5 days earlier



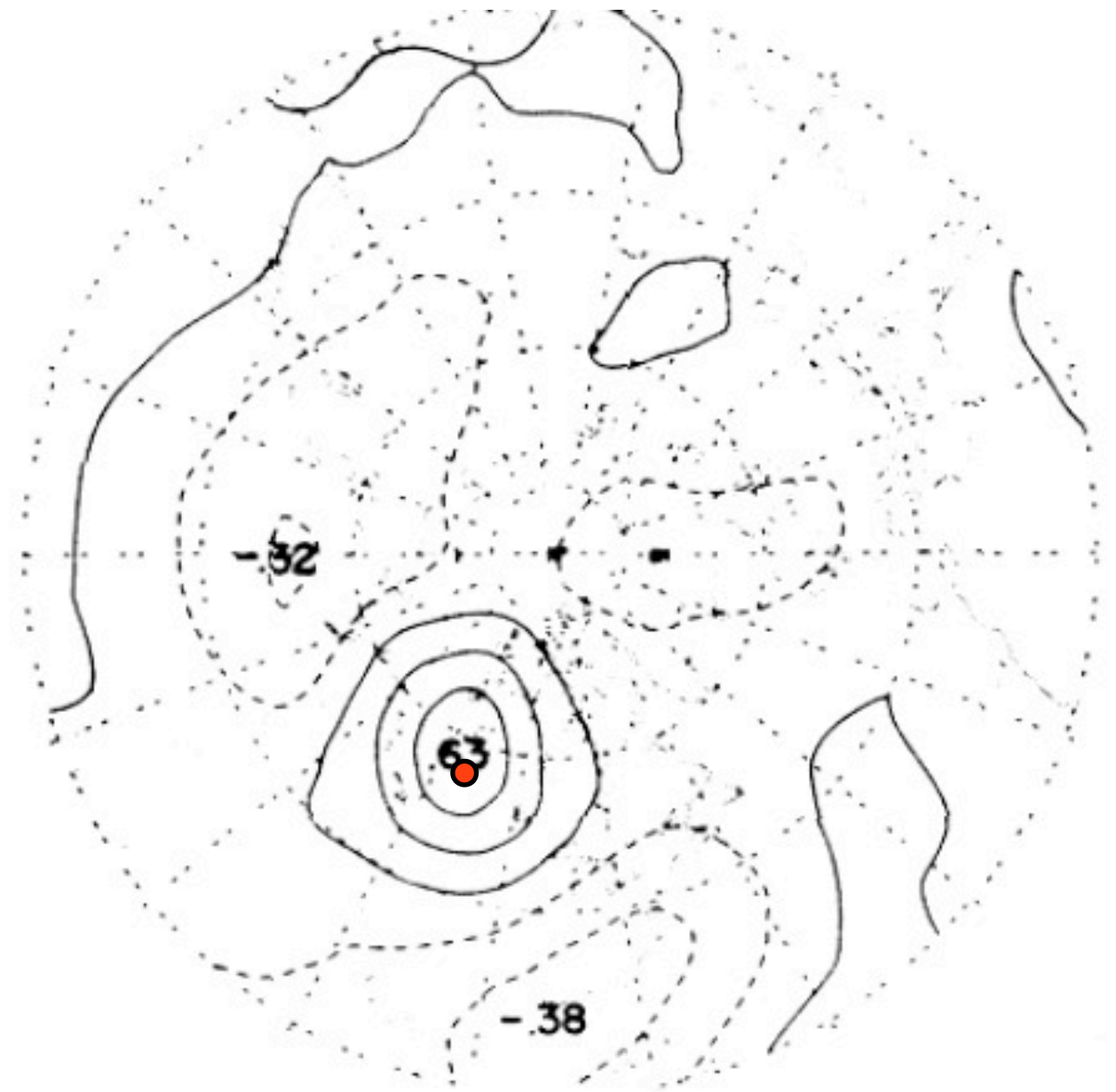
5 days later

lag correlations with 500 hPa height at reference grid point

*after Blackmon et al. JAS (1984b)*

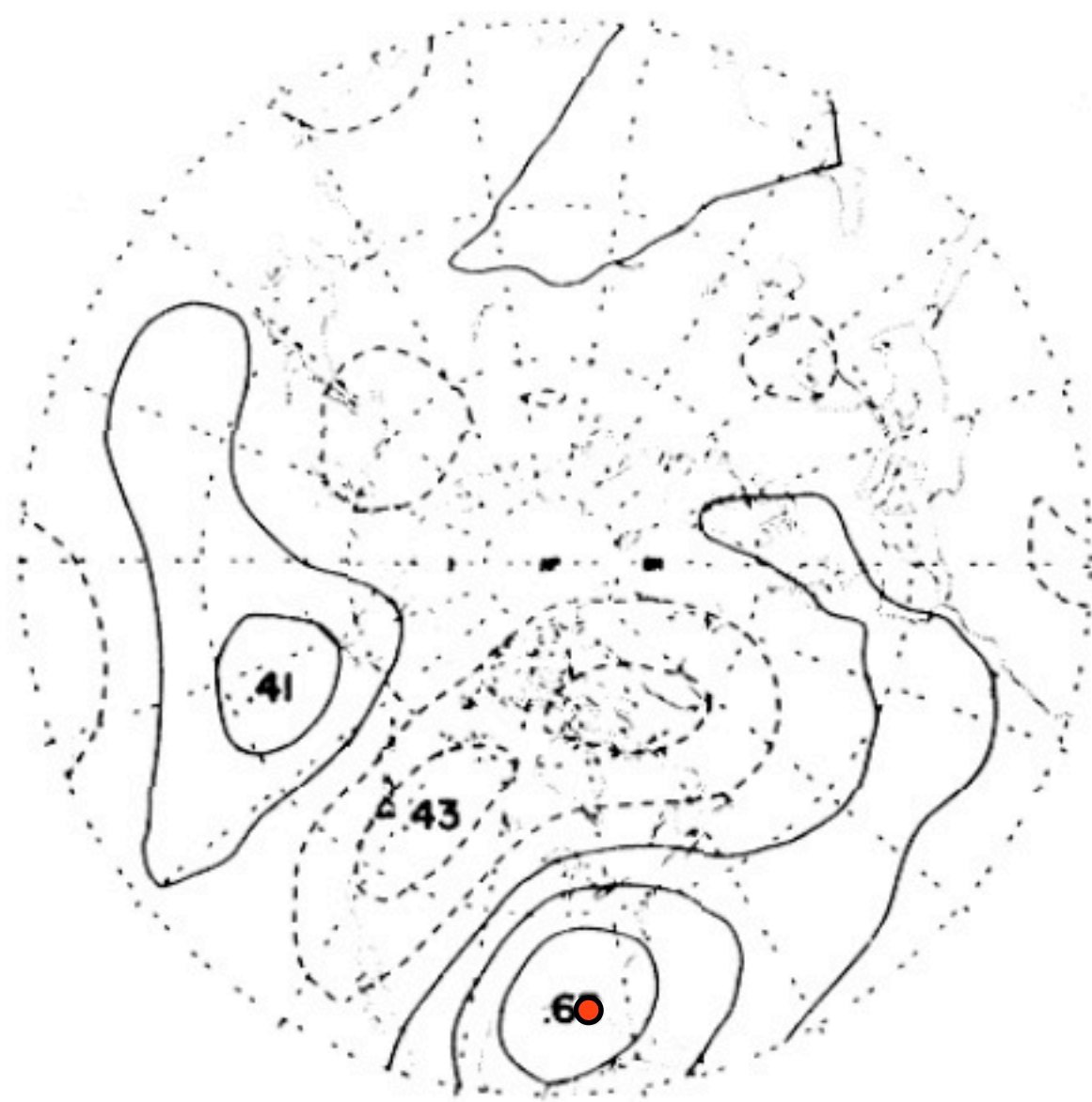


5 days earlier

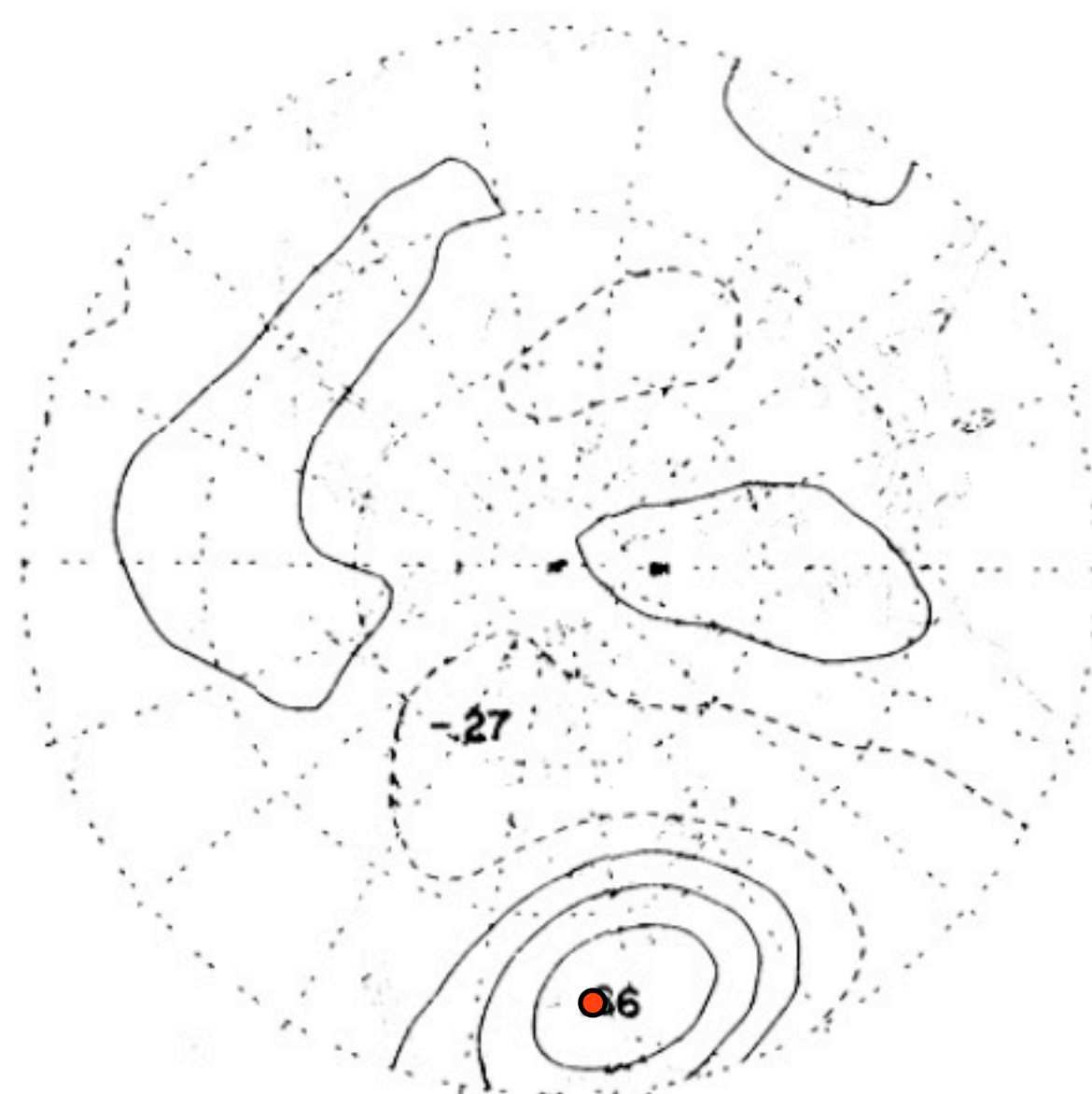


5 days later

*after Blackmon et al. JAS (1984b)*



5 days earlier

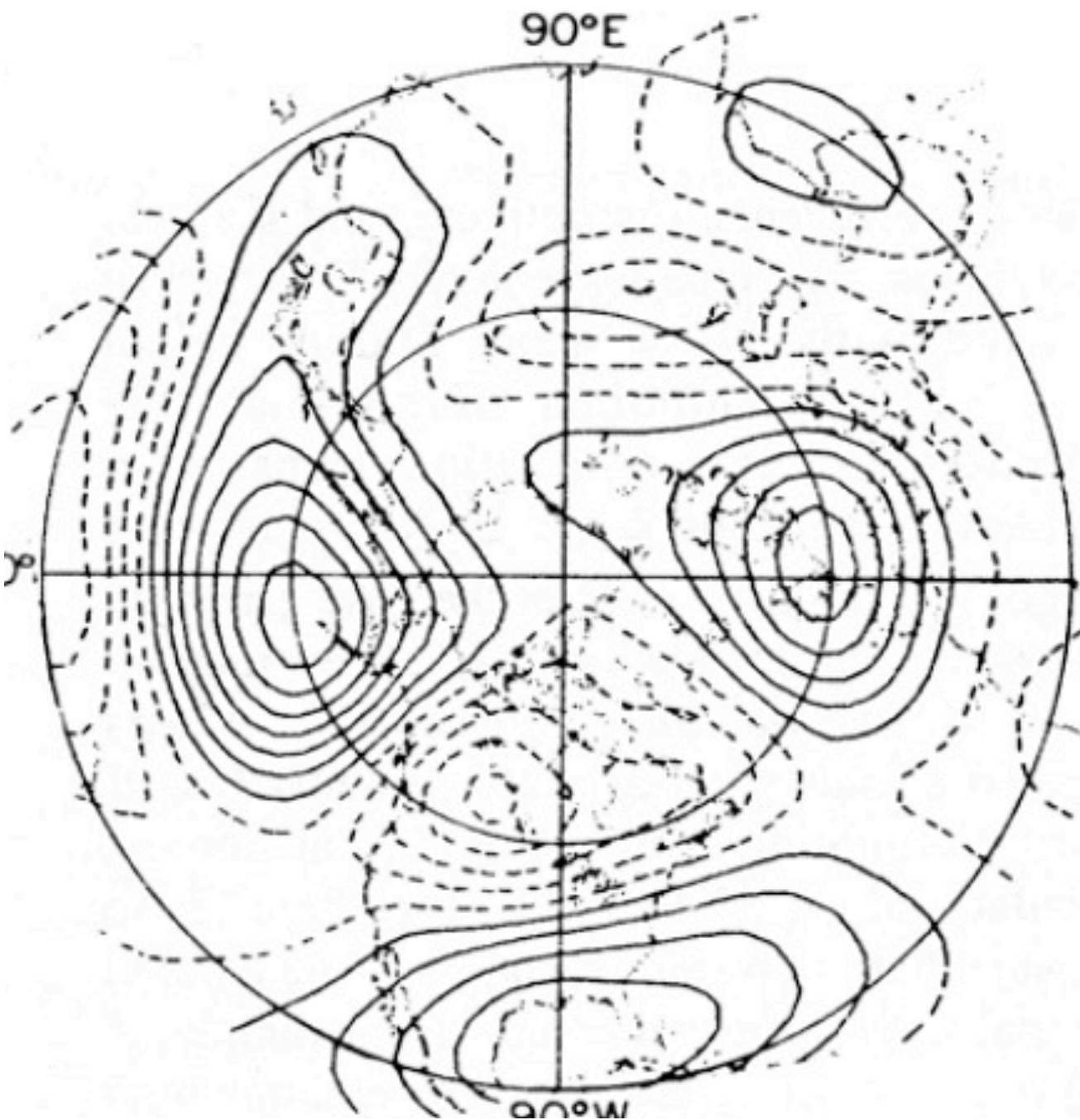


5 days later

*after Blackmon et al. JAS (1984b)*



Regressed onto the cosine coefficient  
of zonal wavenumber 2 on 50°N



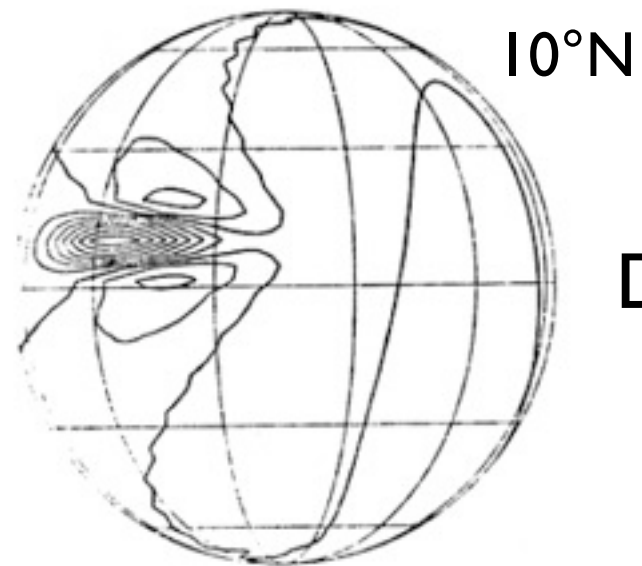
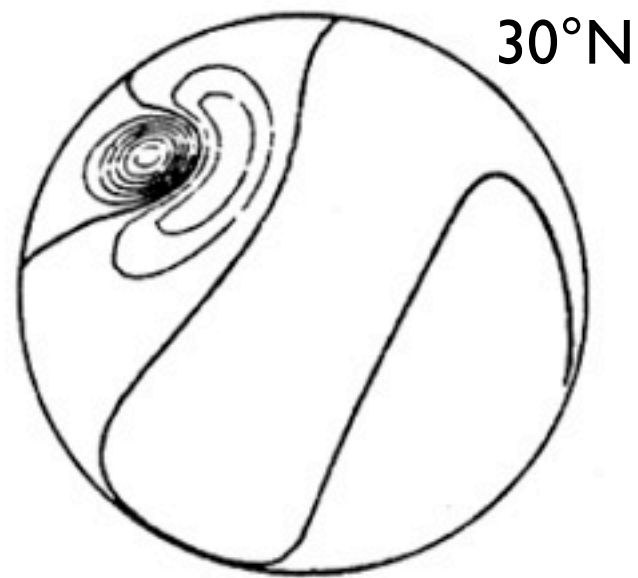
Consecutive 5-day mean  
wintertime 500 hPa height

As in left panel but 5 days later



*After Wallace and Hsu, JAS, 1983*





Day 2

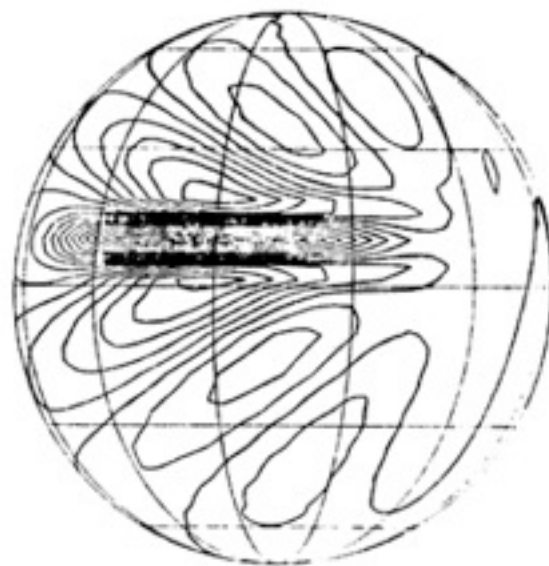
Barotropic vorticity equation

Basic state flow consisting of superrotation

“Mountain” turned on on Day 0



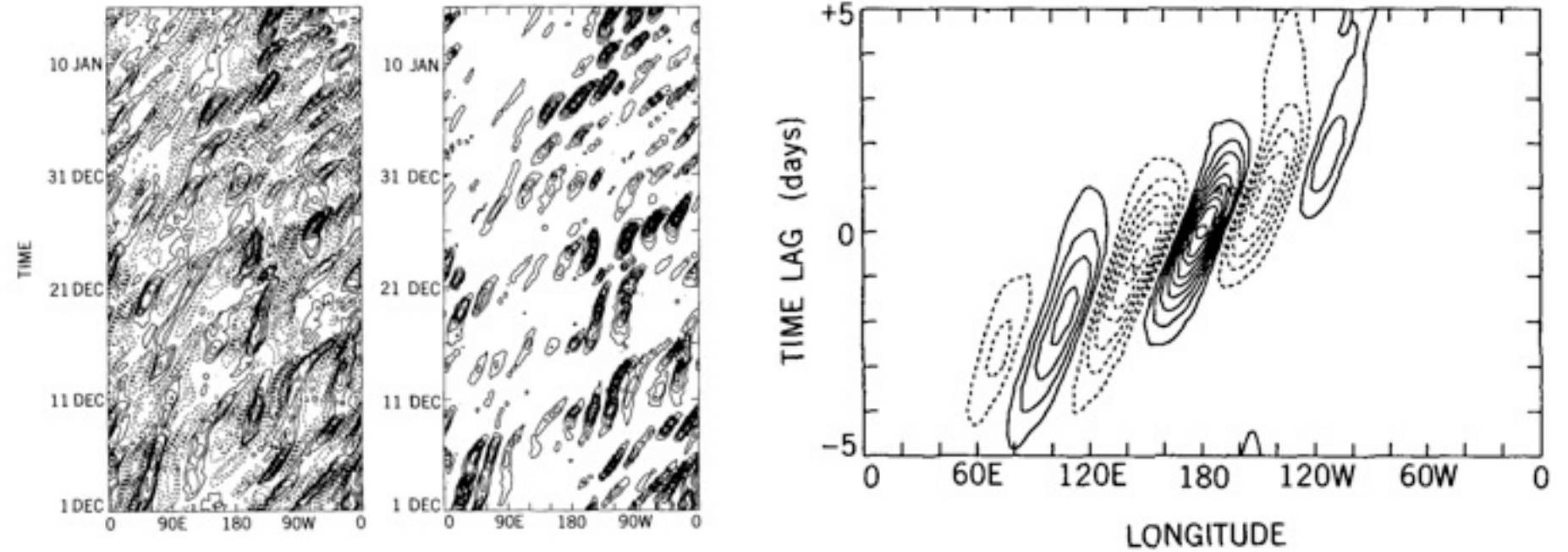
Day 5



Day 8

*Courtesy of B. J. Hoskins*

# Highpass: periods shorter than a week



Summary:

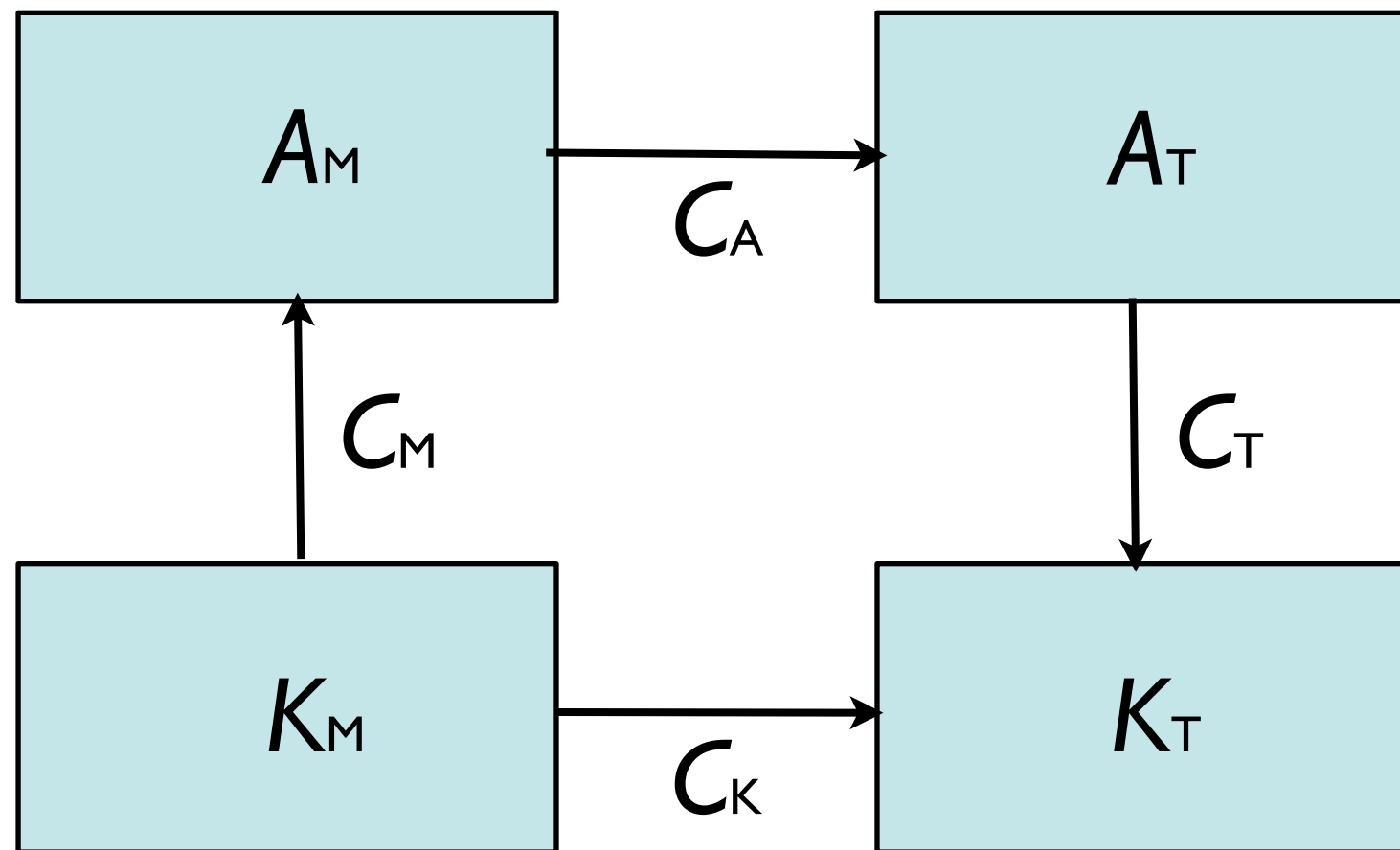
Phase propagation  $\bar{u} \frac{\partial}{\partial x}$  dominates;  $\overline{v'^2} > \overline{u'^2}$

## 10-30d bandpass



Eastward dispersion dominates; great circle routes

# Energetics of the transients

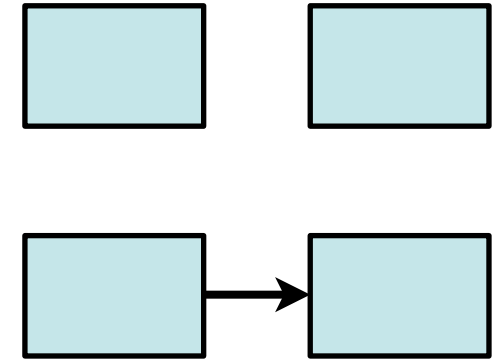


Lorenz kinetic energy cycle: time mean vs. transients

Time mean includes both zonally symmetric flow and stationary waves.

Transients may be decomposed into frequency ranges.

$$C_K = -\overline{u'u'} \frac{\partial \bar{u}}{\partial x} - \overline{u'v'} \left( \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right) - \overline{v'v'} \frac{\partial \bar{v}}{\partial y}$$



If we assume that the time mean flow is nondivergent,

$$C_K = \left( \overline{v'v'} - \overline{u'u'} \right) \frac{\partial \bar{u}}{\partial x} - \overline{u'v'} \left( \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right)$$

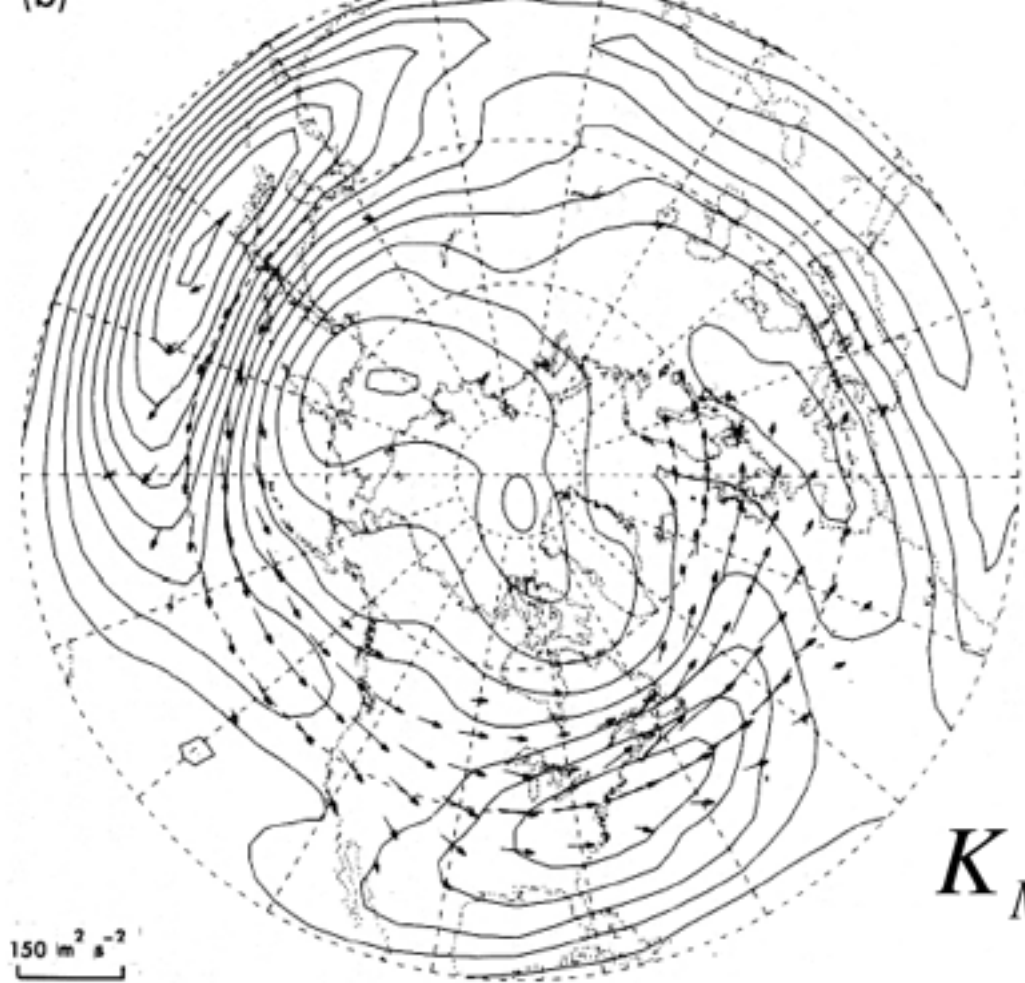
and that  $\partial \bar{u} / \partial y \gg \partial \bar{v} / \partial x$

Then

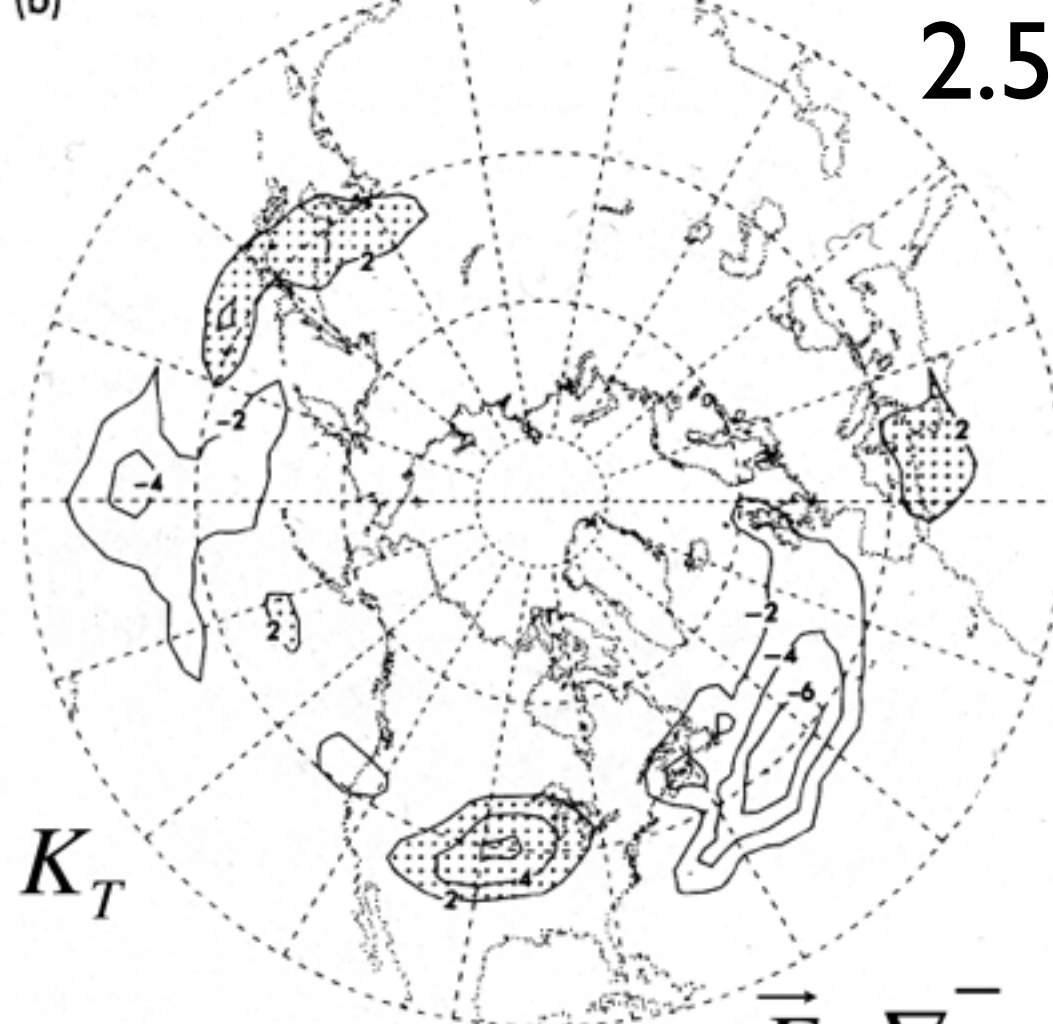
$$C_K = \left( \overline{v'^2} - \overline{u'^2} \right) \frac{\partial \bar{u}}{\partial x} - \overline{u'v'} \frac{\partial \bar{u}}{\partial y} = \vec{E} \cdot \nabla \bar{u}$$

*If  $\vec{E}$  is up the gradient of  $\bar{u}$ , then the flux is downgradient and the transients are gaining KE at the expense of the time mean flow.*



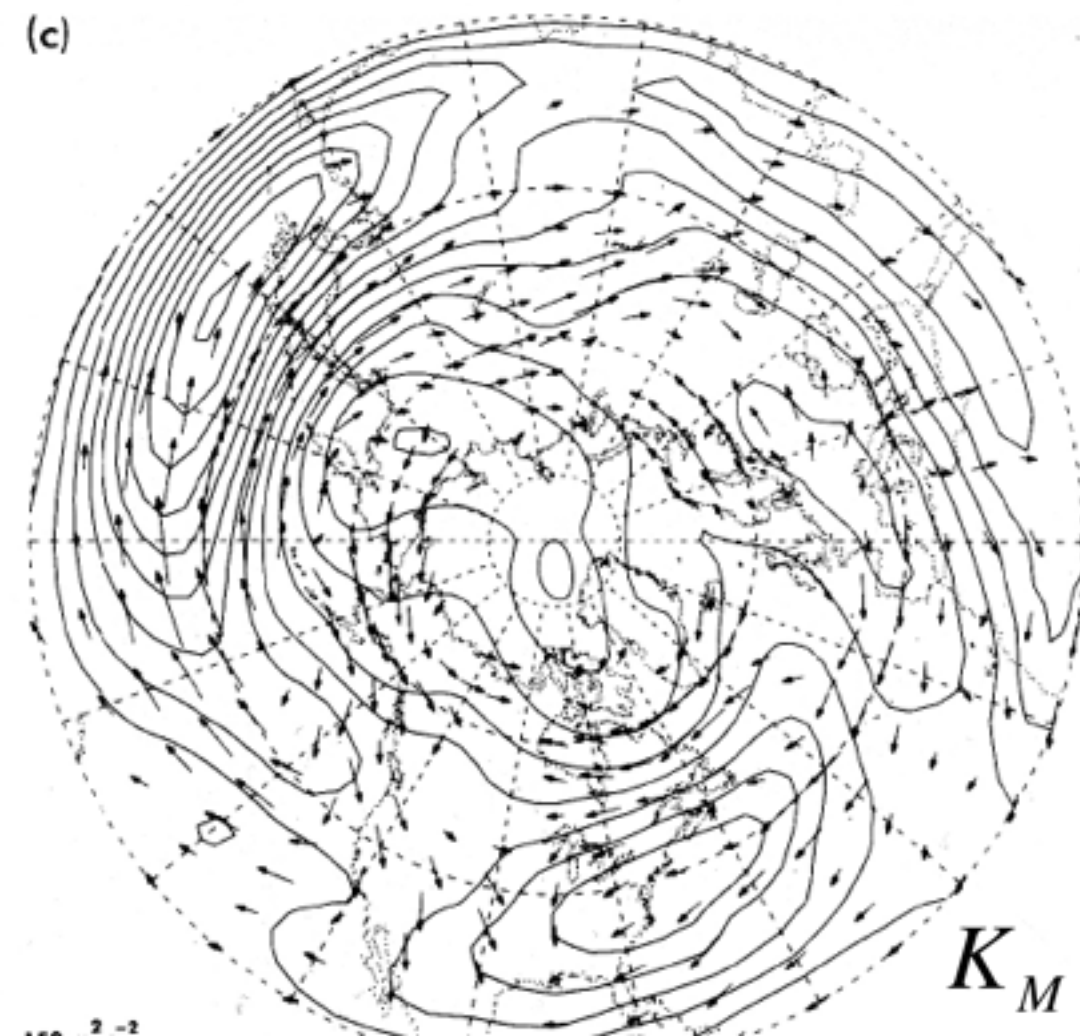


$$K_M \leftarrow K_T$$

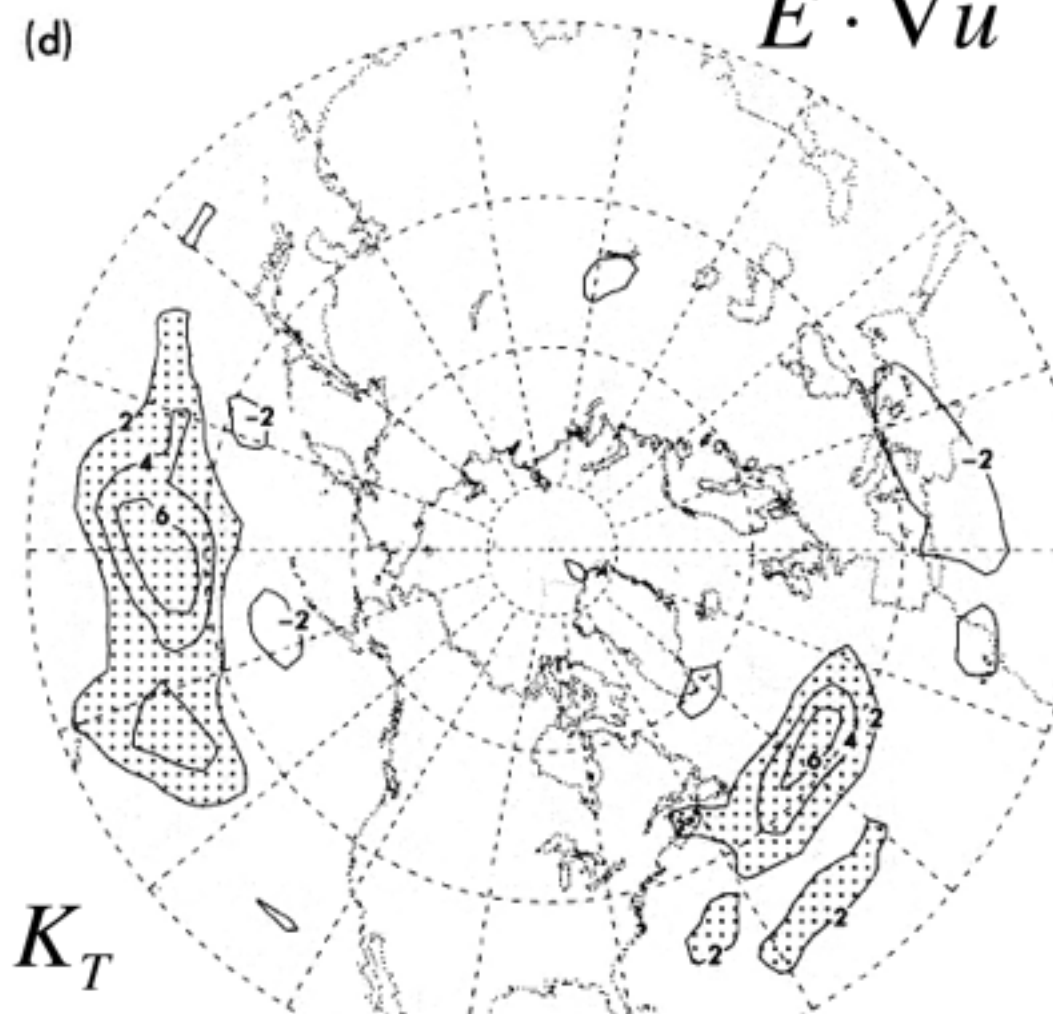


2.5 to 6 d

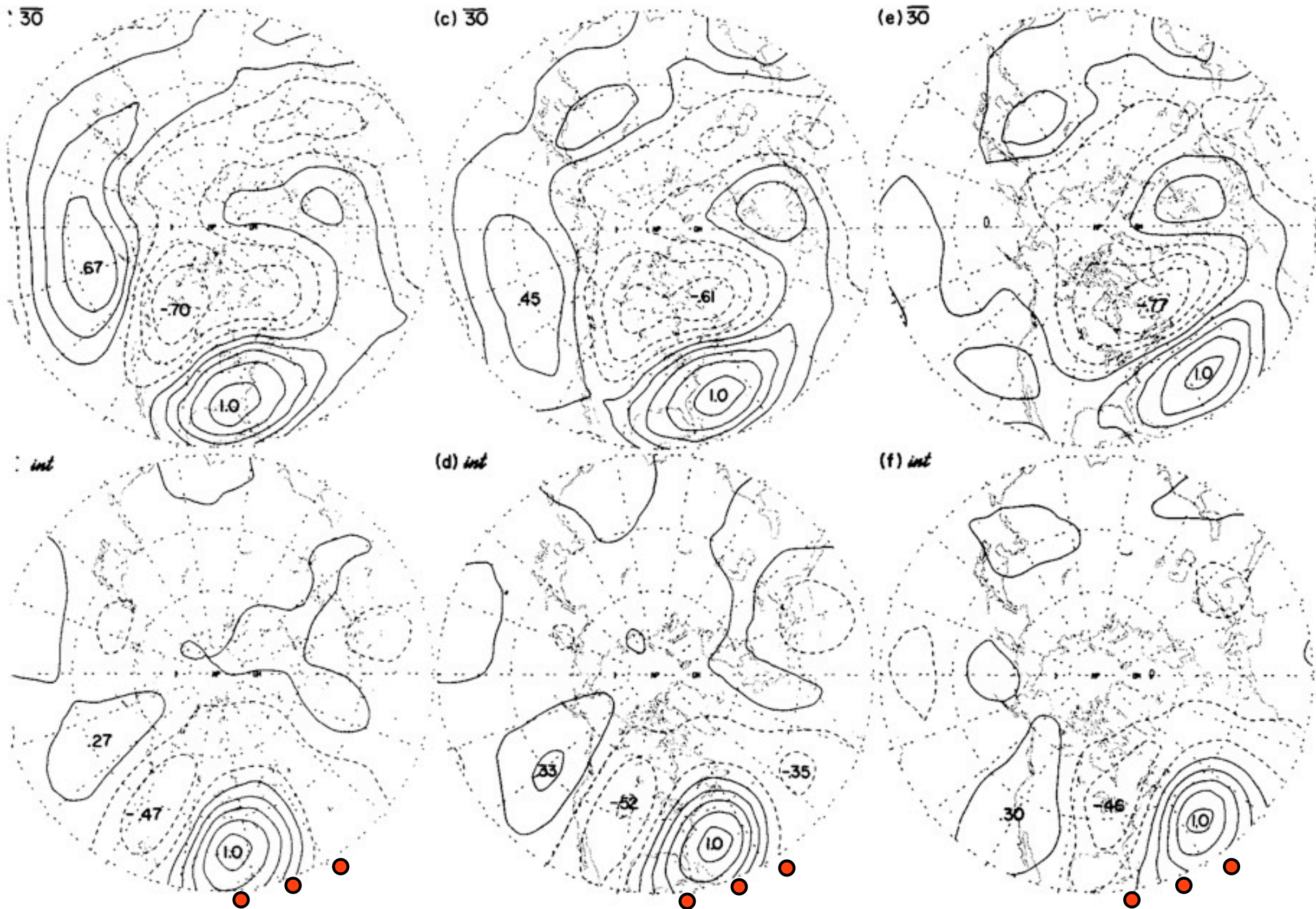
< 6 d



$$K_M \rightarrow K_T$$

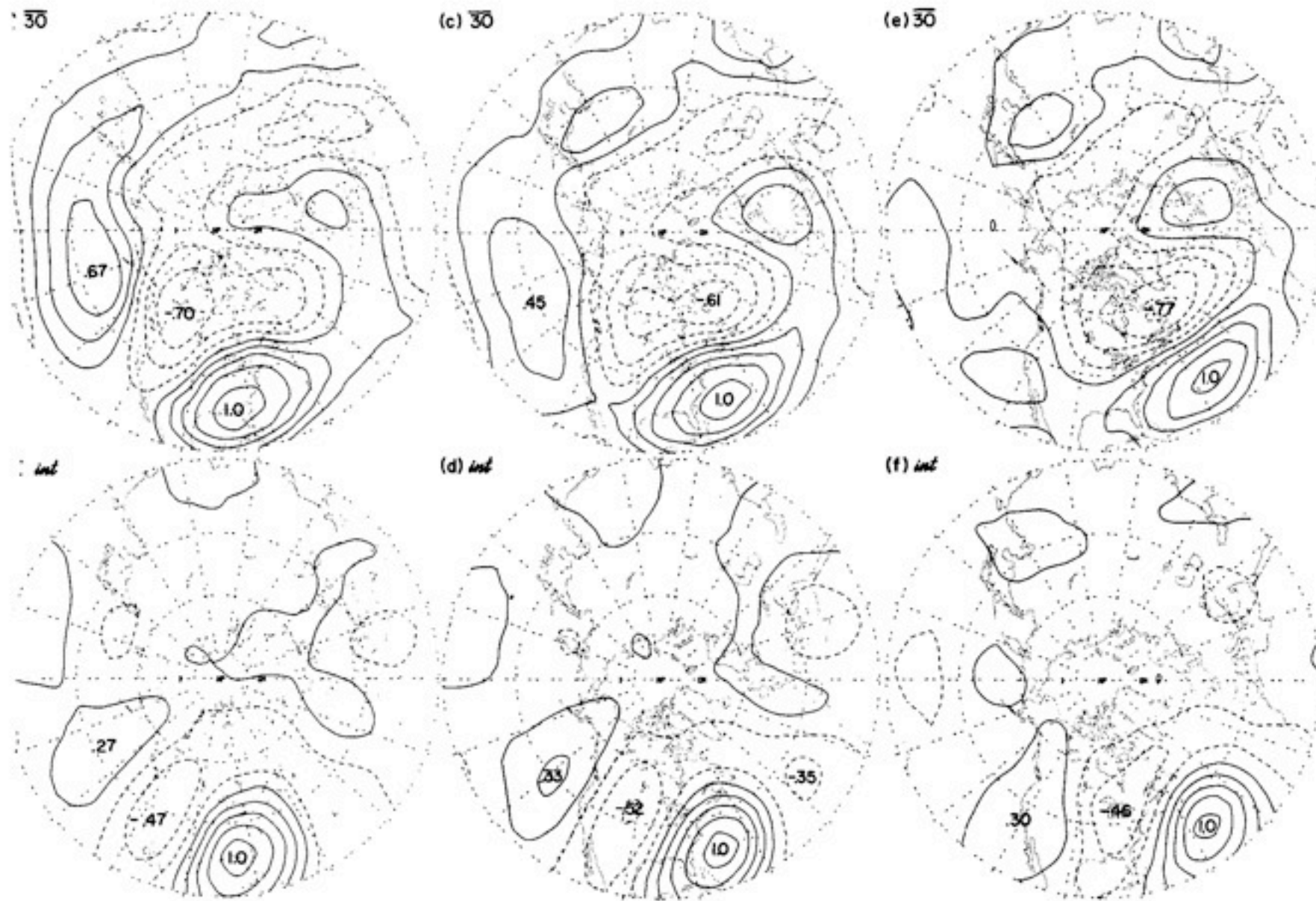


> 30 d



After Blackmon et al., JAS, 1984a





*Indication of  
geographically-fixed  
patterns*

*Pattern moves with  
reference grid point*

Suggests the existence of “teleconnection patterns”

*After Blackmon et al., JAS, 1984a*

