Atmospheric Science 441, Homework 8

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1. Synoptic-scale vertical velocities are very small. Understanding why this is so is important.

> (a) Show that the scale for vertical motions in midlatitude synoptic-scale systems is $R_o HU/L$ where U is the scale for the horizontal velocities, H is the depth of the troposphere, L is the horizontal length scale of the disturbances, and $R_0 = U/(f_0 L)$ is the Rossby number. Here f_0 is a constant representative value of the Coriolis parameter. For simplicity when dealing with the continuity equation, *assume the density is the constant* ρ_0. Also let the geostrophic winds be defined using the constant value of the Coriolis parameter f_0. Finally you may also assume that we have already established that the scale of the horizontal ageostrophic wind is $R_0 U$.
>
> (b) Using numerical values for these scales representative of mid-latitude synoptic-scale motions, what is the numerical value for the scale for w?

2. Do problem 4.10 on p. 123. To answer this you need the answer to 4.9, which is that the depth at radius r satisfies

$$h(r) = H + \frac{\Omega^2 r^2}{2g}.$$

Note that the relative vorticity requested in this problem is the vorticity relative to the rotating tank.

3. Professor Rhines' large tank is rotating at 0.5 radians per second. Approximate the top of the surface of this tank as flat. A mesa-shaped mountain with smooth sides is in the tank with a height above the bottom of 5 cm. Away from the mouton, the fluid depth is 20 cm, which we will take as shallow compared to the horizontal scales of any disturbances. If the fluid is initially in solid body rotation with the tank, but the tank is very gradually decelerated so that the column initially above the mountain slides completely off the mountain, what is the relative vorticity in the column after it has left the mountain? (Neglect any changes in numerical value of the rotation rate of the tank.)

4. The Exner-function pressure is defined as $\pi = (p/p_s)^{r/c_p}$ (note that $T = \pi\theta$). Using the identity

$$\frac{1}{\rho}\nabla p = c_p \theta \nabla \pi,$$

which facilitates the mathematics of linearization, the inviscid momentum governing equations for flow in a fluid in which there are no variations along the y-axis (i.e., in an x–z plane) may be written exactly as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} + c_p\theta\frac{\partial \pi}{\partial x} = 0. \tag{1}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + w\frac{\partial v}{\partial z} = 0. \tag{2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} + c_p\theta\frac{\partial \pi}{\partial z} = -g. \tag{3}$$

Define perturbations (denoted by the primes) with respect to a basic state (denoted by the subscript "0") as follows.

$$u = u_0(z) + u'(x,z,t), \quad v = 0 + v'(x,z,t), \quad w = 0 + w'(x,z,t)$$

$$\pi = \pi_0(z) + \pi'(x,z,t), \quad \theta = \theta_0(z) + \theta'(x,z,t).$$

(a) Linearize these equations using the perturbation method described in Section 5.1.

(b) What relation must the basic state satisfy to ensure that it is a solution of the governing equations?

(c) Rewrite the linearized vertical momentum equation, using the relation determined in (b) to eliminate $\partial\pi_0/\partial z$. What does this give as an expression for the buoyancy force?