Homework Set 1

1. Consider a clear convective boundary layer driven by strong surface heating. Such boundary layers are observed to be *well-mixed*, i. e. the ensemble-averaged $\bar{\theta}$ and $\bar{q}$ do not vary with height. Assume that both the mean wind and the geostrophic wind are zero. Reference BL density is $\rho_R = 1.2$ kg m^{-3}; reference BL virtual potential temperature is $\theta_{vR} = 300$ K, BL depth $h = 1$ km.

   (a) Consider the ensemble-averaged equations for $\bar{\theta}$ and $\bar{q}$. Assume S_θ and S_q are negligible. The well-mixed assumption implies that the tendencies of $\bar{\theta}$ and $\bar{q}$ must be hieght-independent within the BL. Deduce that the vertical fluxes of θ and q must vary linearly with height in the BL, and hence that the buoyancy flux $B = (g/\theta_{vR})\,\overline{w'\theta_v'} = (g/\theta_{vR}) \left(\overline{w'\theta'} + 0.61\theta_{vR}\overline{w'q'}\right)$ must vary linearly with height.

   (b) If the surface latent and sensible heat fluxes are both 300 W m^{-2}, what is the surface buoyancy flux B_0? (Note $L_v = 2.5\cdot10^6$ J kg^{-1} and $C_p = 10^3$ J kg^{-1} K^{-1})

   (c) Take the TKE equation and vertically integrate it over the BL, assuming:
      i) there is no vertical flux of TKE through the ground and the BL top.
      ii) the upward energy flux $\overline{w'p'}$ by gravity waves generated above the BL by BL eddy motions is negligible, and that advection and tendency of $\bar{e}$ are negligible.
      iii) TKE dissipation rate $D = -V^3/h$, where the BL depth h is a characteristic large-eddy size and V is a characteristic convective large-eddy velocity.
      iv) the BL-averaged TKE tendency is negligible.

      Show that the vertically-integrated shear and transport sources of TKE are zero, and that

$$V = \left(\int_0^h B(z)dz\right)^{1/3}.$$

   (d) In clear convective BLs the 'entrainment' buoyancy flux at the BL top is observed to be approximately $-0.2B_0$. Estimate V and D this case for the parameters in (b), using the result from (a) that $B(z)$ is a linear function of z.

   (e) At mid-height in the BL, most of the fluxes are carried by the large eddies. Suppose we idealize the vertical motion, buoyancy, temperature and moisture patterns associated with these eddies as being sinusoidal in the horizontal, with amplitudes V, δb, $\delta\theta$, δq:

$$w' = V\sin(\pi x/h),\ b' = \delta b\sin(\pi x/h),\ \theta' = \delta\theta\sin(\pi x/h),\ q' = \delta q\sin(\pi x/h),$$

   Show that the corresponding buoyancy flux would be $B(z = h/2) = V\delta b/2$. Argue from (a) and (e) that $B(z = h/2) = 0.4B_0$ and use this approach to estimate δb. Assume also that the moisture flux does not vary with height (i. e. it is equal to its near-surface value), and use the same approach to estimate δq. Lastly, back out the upward potential temperature flux at $h/2$ from the buoyancy and moisture fluxes, and deduce a typical large-eddy horizontal temperature perturbation $\delta\theta$.

   (f) A aircraft flies at 100 m s^{-1}, measuring vertical velocity 25 times per second. Using scaling arguments based on the turbulent energy cascade, estimate what the typical variability in w would be between successive samples. **Hint**: Assume this variability is dominated by eddies whose half-wavelength is the distance Δx between samples. Use the Kolmogorov energy cascade with a turbulence dissipation rate $-D$ to argue the corresponding typical eddy velocity for eddies of half-wavelength l is $(|D|l)^{1/3} = V(l/h)^{1/3}$.

2. Consider a BL of depth $h = 1$ km which is sufficiently well mixed that wind velocity $\mathbf{u}$ is uniform with height above the surface layer. Empirically, it is observed that the surface drag $\rho_R \overline{u'w'}|_0 = -\rho_R C_D|\mathbf{u}|\mathbf{u}$, where $C_D = 1.5 \cdot 10^{-3}$ and the reference density $\rho_R = 1.2$ kg m^{-3}. Assume that the tendency and horizontal advection of $\mathbf{u}$ are negligibly small, that the momentum flux vanishes at the BL top, and that the geostrophic wind $\mathbf{u}_g$ is uniform with height. This is the mixed layer analogue to an Ekman layer.

   (a) Write down equations for the BL wind components u and v in terms of the knowns, based on a boundary-layer averaged force balance.

   (b) Suppose $\mathbf{u}_g = 10$ m s^{-1} to the east (the $+x$ direction) and the Coriolis parameter $f = 10^{-4}$ s^{-1}, Calculate the speed and direction of $\mathbf{u}$, and draw the momentum flux profile.

3. The dataset `rf18L1.txt` downloadable from the class WWW page contains 1819 seconds (30 mins) of data from a horizontal aircraft leg flown in a large circle S of Australia 30 meters above the ocean surface, sampled 25 times per second. The aircraft is flying at roughly 100 m s^{-1}. The columns are labeled; u, v, w are the three velocity components, T is temperature and q is water vapor mixing ratio, while the last column, p (which we won't use) is the measured air pressure. In this problem, we will analyze this dataset using Matlab. Please write a script or scripts (to attach with your solutions) that implements the following:

   (a) Plot u, v, w vs. time, and note that all of them show mesoscale variability (on scales of 10 km, or 100 s of sampling time) as well as turbulent variability.

   (b) Now perform a spectral analysis of vertical velocity w and E-W velocity component u using the Matlab signal-processing toolbox function `psd` called according to the script `psduw.m` given in the class web page. This uses a Hanning window for data tapering and averages the tapered periodograms over overlapping intervals of 4096 samples (160 s, corresponding to 16 km, so variability in w on wavelengths longer than 16 km is not accounted for). Plot the analyzed w and u power spectral densities P_{ww} and P_{uu} vs. frequency f. Over what range in frequencies does P_{ww} exhibit a power law behavior, and does it have the expected exponent?

   (c) Based on the above analysis, we decide to isolate the turbulent component of the signal from its mesoscale component by high-pass filtering to remove frequencies of less than 0.05 s^{-1} (wavelengths longer than 2 km). To do this, we use a Butterworth filter. Download the script `highpassw.m` and use it to perform this filtering on w. What is the standard deviation (Matlab function `std`) of `whi`, the filtered w?

   (d) The Matlab function `w_lag = xcov(whi,'coeff')` calculates the autocorrelation sequence of the `whi` time series. If `whi` includes ns samples, then `w_lag(ns + lag)` will be the autocorrelation of `whi` with lag time `lag`/25 s. Plot the autocorrelation of `whi` vs. lag time for lag times between 0 and 2 s, and calculate the resulting integral timescale for w. To what characteristic updraft width does it correspond?

   (e) After high-pass filtering u, v, T, q similarly, calculate their variances, their correlations with w, and the corresponding momentum, sensible and latent heat fluxes. Comment on the assumption of perfect correlation assumed in problem 1e, based on this analysis. Use a nominal air density of $\rho_0 = 1.21$ kg m^{-3}, and the values of C_p and L_v from problem 1b to compute the fluxes. Is the buoyancy flux upward or downward? Calculate the friction velocity u_* and the Obukhov length L. At the measurement height of 30 m, is z/L in the stable ($z/L > 0.2$), neutral ($-0.2 < z/L < 0.2$) or unstable ($z/L < -0.2$) regime?

4. Fill in the steps of the following dimensional argument that interprets Obukhov length L as a height at which buoyancy effects become important to the structure of flux-carrying eddies near the ground. Assume that the near surface wind (and hence the near-surface momentum flux) are in the *x*-direction. We are given the surface momentum and buoyancy fluxes

$$\overline{u'w'} = -u_*^2, \quad \overline{w'b'} = B_0 > 0.$$

We assume that in the relevant eddies, the perturbations u', w' and b' near the ground are strongly correlated.

(a) Using these ideas, argue that near $z = 0$, a characteristic eddy updraft velocity is u_* and that a characteristic eddy updraft buoyancy is B_0/u_*.

(b) Assuming that the updraft accelerates due to its buoyancy. At what height will it achieve an updraft velocity of $2u_*$? Show this height is proportional to $-L$.