

Homework Set 2 (Monin-Obukhov theory)

1. (Adapted from Arya, p. 180) The following measurements of mean wind and potential temperature were taken around noon during the 1968 Kansas field program:

z (m)	2	4	8	16	32
u (m s ⁻¹)	5.81	6.70	7.49	8.14	8.66
θ (K)	307.20	306.65	306.28	305.88	305.62

We wish to calculate the surface fluxes of heat and moisture using the Monin-Obukhov relations $\phi_h = \phi_m^2 = \{1 - 16z/L\}^{-1/2}$ for an unstable surface layer. To proceed:

- (a) Calculate the gradients of u and θ by differencing between successive heights. In the surface layer, the profiles tend to vary roughly logarithmically with height, so it is better to difference using $\ln(z)$ as the height coordinate. Note that $du/dz = z^{-1} du/d(\ln z)$ and similarly for θ . Numbering the levels 1 (2 m) to 5 (32 m), the estimated gradient of u between levels 1 and 2 would be

$$\left(\frac{\Delta u}{\Delta z}\right)_{21} = \frac{1}{z_m} \left(\frac{\Delta u}{\Delta \ln z}\right)_{21} = \frac{1}{z_m} \left(\frac{u_2 - u_1}{\ln z_2 - \ln z_1}\right)$$

at a height z_m such that $\ln z_m = (\ln z_1 + \ln z_2)/2 = (\ln 2 + \ln 4)/2$, i. e. at $z_m = 2.82$ m. This works out to a gradient $(\Delta u/\Delta z)_{12} = 0.45$ s⁻¹. Neglecting virtual effects on buoyancy, use your calculated gradients to find Ri vs. z .

- (b) From this data, estimate the Obukhov length L (note that $Ri = z/L$ in an unstable surface layer).
- (c) Using the data from the lowest two heights and your L from (b), calculate the friction velocity, the sensible heat flux, and the surface roughness length z_0 . Does the implied roughness length seem appropriate for a field of wheat stubble? Take the air density = 1.2 kg m⁻³ and $C_p = 10^3$ J kg⁻¹ K⁻¹. Can we also estimate the thermal roughness length z_T from the given data?
2. An oceanographic research ship is stationed 100 km off the California coast. It is measuring a wind speed of 10 m s⁻¹ and an air temperature of 286.9 K at a height of 10 m above sea level (i.e. 287 K if adiabatically displaced to the sea surface). The ocean surface temperature is also 287 K. Neglect virtual effects, so we have a neutral surface layer.
- (a) Using the bulk aerodynamic approach, with $C_{DN} = (0.75 + 0.067u_{10}) \times 10^{-3}$, find the surface stress and the friction velocity (use the same reference air density as before)?
- (b) What is the roughness length z_0 (use Charnock's formula)?
- (c) If you based C_{DN} on this z_0 , rather than the bulk formula of (a), how different would the result be (this checks the consistency of the two approaches).
- (d) The saturation mixing ratio for salt water at the sea-surface is 9.7 g kg⁻¹, while the mixing ratio at 10 m elevation is 7.4 g kg⁻¹. Using a bulk formula with $C_{qN} = 1.3 \times 10^{-3}$, calculate the surface latent heat flux.
- (e) An airplane flies overhead at 30 m elevation. Assuming this is still within the surface layer, what mean wind speed, temperature and mixing ratio would the aircraft measure?
3. Now the ship moves over a coastal upwelling zone, where the 10 m wind and air temperature remain as above but the SST is cooled 3 K to 284 K.

- (a) Starting with the neutral bulk formula of (a) and $C_{HN} = 1.3 \times 10^{-3}$, make a first guess at the sensible heat flux and Obukhov length L . This won't be exact, but will be good enough for our purposes.
- (b) Using this to estimate z/L at the 10 m measurement height, use Garratt Fig. 3.7 (shown in the Lecture 5 notes; use the $z/z_0 = 10^5$ curve) to estimate by what percentage C_D is decreased from C_{DN} ? Based on this estimate, by what percentage is the surface wind stress changed by moving over the colder SST?