1 Introduction

In this chapter we revisit one of the "classical" topics of atmospheric dynamics: the maintenance of the zonal flow relative to the rotating Earth. The literature on this topic, discussed at length in the monograph of Lorenz (1967), dates back over 300 years and includes work of Edmund Halley (of Halley's comet), George Hadley (of the Hadley cell), and William Ferrell (of the Ferrell cell). Most of the early work on this topic was restricted to consideration of the role of mean meridional circulations in the transport of angular momentum within the atmosphere, a line of inquiry that proved inconclusive because mean meridional circulations are not the only component of the atmospheric circulation that plays a role in the transport. It was not until the mid-20th century that the role of synoptic and planetary- scale eddies in the transport of atmospheric angular momentum was fully appreciated, and upper air data became available for making quantitative estimates of the eddy transports.

The first estimates of the poleward eddy fluxes of westerly momentum were made using the formalism

$$[\overline{uv}] = [\overline{u}]\,[\overline{v}] + [\overline{u}^*\overline{v}^*] + \left[\overline{u'v'}\right]$$

which is derived in Appendix 1. Estimates of time-averaged terms $\overline{u}$, $\overline{v}$, and $\overline{u'v'}$ derived from time series of station data[1] and subsequently zonally averaged [( )]. In these estimates, $\left[\overline{u'v'}\right]$ was interpreted as the transient eddy term, under the assumption that the term $\overline{[u]'\,[v]'}$ is negligibly small. This formalism was popular before the days of gridded datasets because it was much less labor intensive than the other decompositions, in which the laborious zonal averaging must performed multiple times before the time averaging.

Over an interval of about 25 years, beginning in the 1960s, diagnostics based on the atmospheric angular momentum balance provided more reliable estimates of the structure and intensity of the mean meridional circulation than direct estimates of $[\overline{v}]$. With the advent of increasingly sophisticated data assimilation schemes, that situation has changed: the dynamical consistency and overall reliability of gridded datasets such as the NCEP, ERA-40 and JMA reanalyses is good enough to provide useful direct estimates the time-varying mean meridional circulations. Calculation of the eddy fluxes from these datasets is straightforward and does not require neglecting small terms.

Another relatively recent innovation is the use of precise measurements of length of day which reveal short term changes in the angular momentum of the solid Earth due to exchanges with the atmosphere. Because the zonally symmetric flow in the oceans is largely restricted to the mass flux through the Drake Passage, the storange of angular momentum in the oceans plays only a minor role in the Earth's angular momentum budget.To first order, the exchange of angular momentum between the atmosphere and oceans is balanced by the exchange between the oceans and the solid Earth due to the pressure differences

[1] interpolated onto a regular grid using either hand-drawn analyses or, later on, univariate objective analysis.

between the east and west coasts of the continents. For example, the easterly tradewinds along the equator in the Atlantic and Pacific basins exert a westward torque on their respective oceans, causing sea-level to slope upward toward the west. Under steady state conditions the eastward pressure gradient force within the ocean balances the westward torque at the surface. Hence, sea-level is higher along the east coast of tropical South and Central America than along the west coast. The resulting pressure difference across the continent exerts an easterly torque on the solid Earth, slowing the Earth's rotation. These changes can occur without necessarily affecting the storage of angular momentum in the oceans. To a first approximation, the ocean serves as a conduit for the passage of angular momentum between the atmosphere and the solid Earth. [Check for papers relating changes in the strength of the Antarctic Circumpolar Current to changes in global angular momentum.]

2 Angular momentum conservation for the "Earth system"

The angular momentum per unit mass of an air parcel is given by

$$m = R_E \cos\phi[\Omega R_E \cos\phi + u] \tag{2.1}$$

where R_E is the radius of the Earth (6.37 x 10^6 m), ϕ is the latitude, Ω is the angular rotation rate of the Earth (7.29 $\times$ 10^{-5} s^{-1}), and u is the zonal wind. By definition, m is positive where the relative angular momentum is in the same sense as the Earth's rotation: in regions of westerlies the atmosphere is rotating slightly faster than the solid Earth and in regions of easterlies it isn't rotating quite as fast. The first term in (2.1) may be recognized as the angular momentum associated with the Earth's rotation and the second as the angular momentum of the air parcel relative to the rotating Earth. Since $\Omega R_E = 465$ m s^{-1}, it is evident that the first term is dominant, except at very high latitudes. The total atmospheric angular momentum M is obtained by integrating (2.1) over the mass of the atmosphere, which can be performed in three steps: a vertical integration

$$\oint m R_E \cos\phi d\lambda = 2\pi R_E \cos\phi[m]$$

a vertical integration

$$\frac{2\pi R_E \cos\phi}{g} \int_0^{p_0} [m] dp$$

and, finally, a pole-to-pole integration, which yields

$$M = \frac{2\pi R_E^2}{g} \int_{-\pi/2}^{+\pi/2} \int_0^{[ps]} [m] \cos\phi d\phi dp \tag{2.2}$$

where $[p_s]$ is zonally averaged pressure at the Earth's surface. Combining (2.1) and (2.2), we obtain

$$M = \frac{2\pi R_E^4 \Omega}{g} \int_{-\pi/2}^{+\pi/2} [p_s] \cos^3 \phi d\phi dp + \frac{2\pi R_E^3}{g} \int_{-\pi/2}^{+\pi/2} \int_0^{[ps]} [u] \cos^2 \phi d\phi dp \quad (2.3)$$

Atmospheric angular momentum is expressed in units of kg m^2 s^{-1}.

For the atmosphere, oceans and solid earth as a single system, total angular momentum is conserved, but for tidal interactions with the moon, which are conspicuous because they are quasi-periodic and predictable on the basis of theory. The angular momentum of the solid earth can be monitored by taking precise measurements of the length of the day (l.o.d.). Rosen and Salstein (1983) have shown that if the angular momentum of the molten core of the earth is assumed to be constant on time-scales of a year or less, and if the exchange of angular momentum with the oceans is neglected, changes in length of day and angular momentum of the solid earth obey the linear relation

$$\Delta \text{lod} = 1.68 \times 10^{-29} \Delta M \quad (2.4)$$

where Δlod is expressed in units of seconds. The mean length of the day has varied by several milliseconds on the decadal time scale over the course of the past century which, according to the above relation, imply gains and losses in angular momentum an order of magnitude larger than the atmosphere could have possibly experienced on these time-scales. These gradual changes are believed to be due to core-mantle coupling (Lambeck, 1980).

The strong correspondence between atmospheric angular momrntum and length of day is documented in a separate series of figures based on work of David Salstein, Richard Rosen and collaborators.

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Questions

(2.1) Why are we justified in ignoring time variations in the earth's rotation rate Ω in evaluating M in (2.2)?

(2.2) Show that for a zonally symmetric ring of air, angular momentum per unit mass and circulation are linearly related.

3 The climatological mean distribution of zonal wind

Figure 2.1 shows pole-to-pole cross-sections of the climatological mean zonal wind $[u]$ based upon the ERA-40 Reanalysis from 1979 through 2001. Pressure is plotted on a logarithmic scale so that distance above the baseline is nearly proportional to geometric height. It should be kept in mind that the density drops off by about three orders of magnitude from the bottom to the top of these sections. Hence, the mass-weighted angular momentum is dominated by the features at the lower levels. Cross-sections are shown for December-February and June-August (DJF and JJA) which correspond to the summer and winter seasons in the respective hemispheres. The dominant features in the cross-sections are the westerly jetstreams at the tropopause level. The jet is stronger and located at lower latitudes in the winter hemisphere. The Northern Hemisphere tropospheric jet jet exhibits a much stronger annual cycle, which dominates the annual march of mass-weighted global atmospheric angular momentum. Easterlies are largely restricted to the tropics: only in the subtropical lower troposphere of the summer hemisphere do they exceed 5 m s^{-1}.

Fig. 2.2 shows climatological mean wind speed at the 250 hPa level In the regions of strong winds, scalar wind speed is virtually identical to the distribution of the zonal wind component , since jetstreams tend to be zonally oriented. Note the strong jet extending from the subtropical Atlantic across the Sahara, the middle East, and the Himalayas to a maximum over Japan, where wind speeds reach 70 m s^{-1}. A second, much weaker jet extends east-northeastward from Baja California to a maximum over Washington DC. These jets occur at the longitudes of the longwave troughs in the climatological mean 250 hPa height field.

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Questions

2.1 For what vertical coordinate is the vertical spacing between levels proportional to the fraction of the mass of the atmosphere contained within the layer. What would be the disadvantage of using this coordinate for plots such as Fig. 2.2?

2.2 For what horizontal coordinate would equal increments on the x axis correspond to equal areas on the earth's surface? What latitude would lie halfway between equator and pole in the diagram?

2.3 For what pair of x, y coordinates would the contribution of an element of area δA in the section be proportional to the relative angular momentum of the atmosphere is equal to $[u]\delta A$, regardless of its position on the sections? What latitude would lie halfway between equator and pole in the diagram?

4 Sources and sinks of atmospheric angular momentum

The atmosphere exerts a torque (i.e., a force acting at a distance $R_E \cos\phi$ from the Earth's axis) on the solid earth or ocean beneath it (or vice versa) through three processes: (1) the torques associated with zonal pressure differences across mountain ranges, (2) the frictional torques associated with zonal wind stresses at the Earth's surface, and (3) the torques associated with the breaking of orographically-induced gravity waves within the atmosphere.

4.1 Mountain torques

Pressure differences sometimes develop across mountain ranges as a hydrostatic response to temperature contrasts between the air masses on the two sides, as illustrated in Fig. 2.3, or as a dynamical response to strong flow over the mountains. In these situations, the atmosphere exerts a net horizontal force upon the mountain range and vice versa, even in the absence of any atmospheric motions whatsoever. The globally-integrated rate of transfer of angular momentum by the pressure torque is given by

$$-2\pi R_E^2 \int_{-\pi/2}^{+\pi/2} \left[p_s \frac{dH}{dx} \right] \cos^2 \phi d\phi$$

where H is the terrain height. As an example of how such a force can act to transfer angular momentum between the atmosphere and the solid earth, Fig. 2.4 shows composite sea-level pressure distributions for "high and low index" circulation types as defined the strength of $[u_g]$ averaged between 35° and 55°N, after Willett (1944). The "high index" pattern is characterized by higher pressure to the west of the Rockies and lower pressure to the east. Warm "chinook" winds are often observed along the lee slopes of the Rockies in these situations. The isobars are depicted as being very smooth on this map, but on more detailed analyses much of the east-west pressure gradient is concentrated in along ridges such as the Continental Divide in Montana and Colorado and the Cascades in Washington. It is evident that in these situations the atmosphere is exerting an eastward torque on the Rockies, so the solid Earth is gaining angular momentum from the atmosphere. In the contrasting "low index" pattern, pressures are higher over the Great Plains, to the east of the Rockies in association with a pattern typical of major cold outbreaks. In this case the atmosphere is exerting a westward torque on the Rockies, slowing down the rate of rotation of the Earth. A pattern very similar to the one in Fig. 2.4 was obtained by Iskenderian and Salstein (1997) based on composites of with large positive and negative tendencies in global atmospheric angular momentum (their Fig. 11).[2] Hence, the torque on the Rockies causes the atmosphere to lose angular momentum to the solid Earth in the high index composite and to gain angular momentum from the Earth in the low index composite. Since the atmosphere tends to lose angular momentum during "high index" (i.e., strong zonal wind) situations, the mountain torque tends (locally, at least) to damp fluctuations in atmospheric angular momentum.

In the climatological mean sea-level pressure pattern the zonal gradient across the Rockies is so weak that its sign is in questionable and the same is true of most of the other mountain ranges in the hemisphere. Hence, it does not appear that the mountain torque term plays a major role in the climatological mean angular momentum balance, though it may be of first order importance (perhaps as a negative feedback) on short time scales.

The frictional torque at the Earth's surface can be inferred from the distribution of climatological-mean surface wind shown in Fig. 2.5. Easterlies

[2] The same authors showed that the daily time series of a index cross-Rockies sea-level pressure difference was correlated with their time series on North American mountain torque at a level of 0.81 and with the time rate of change of global atmospheric angular momentum at a level of 0.67.

prevail over the tropical half of the globe (30°N-30°S) during both seasons. The strongest easterlies are observed in association with the Tradewinds over the Atlantic and Pacific and over the southern Indian Ocean. Westerlies prevail at higher latitudes and are particularly strong between 40°S and 60°S, the belt that was referred to as "the roaring 40s" in the days of sailing ships. Away from the equator the winds tend to be close to geostrophic balance whereas near the equator the tend to blow across the isobars from higher toward lower pressure.

4.2 Frictional torques

The atmosphere exerts a frictional torque on the ocean or land below it in the same sense as the surface wind; i.e., it is acting to slow down the rotation of the earth equatorward of 30°, where easterlies prevail, and to increase it at higher latitudes, in the regions of surface westerlies. From the point of view of the atmosphere, the frictional torque is acting as a source of angular momentum at low latitudes and a sink at higher latitudes. This "frictional torque" is given by

$$2\pi R_E^3 \int_{-\pi/2}^{+\pi/2} [\tau_x] \cos^3 \phi d\phi$$

where the stress is determined from the surface wind using the bulk aerodynamic formula

$$\tau_x = \rho C_D (Vu)$$

where V is the scalar wind speed at the anemometer level and C_D is an empirically determined, dimensionless drag coefficient which is on the order of 1.2 x 10^{-3} over the sea[3] (Smith, 1980; Large and Pond, 1981). Note that for a time average such as a climatological mean involves both mean and transient terms. For example, if we ignore the contribution of the meridional wind component to the wind speed, the scalar value of the stress is given by $\tau_x = \rho C_D(\overline{u}^2 + \overline{u'^2})$. Note that because of the nonlinearity inherent in these relationships, the stress tends to be concentrated in regions of high surface wind speeds and, (other things being equal) it tends to be larger in regions of large temporal variability such as the northern oceans than over regions such as the trades, where the winds are much more steady from day to day.

5 Time variations in atmospheric angular momentum

Fig. 2.5 shows that the time tendency in global angular momentum is highly correlated with the sum of the mountain and frictional torques integrated over the globe. The gravity wave torque, which is missing in this comparison is

[3] Thr drag coefficient increases slightly with wind speed.

evidently of secondary importance in the angular momentum budget for the atmosphere as a whole, but it is important in the upper atmosphere.

Power spectra for the globally integrated mountain torques and frictional torques are shown in Fig. 2.6, together with the spectrum for the time rate of change of atmospheric angular momentum. At periods shorter than about a month the mountain torques dominate, whereas at periods longer than a year, the frictional and mountain torques tend to compensate such that the tendency is the smallest of the three terms. In the limit, as the time scale of the fluctuations becomes infinitely long, there must be a balance between the mountain, frictional, and wave breaking torques, integrated over the atmosphere as a whole.

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Questions

2.4 If the zonally averaged surface winds were either westerly or all easterly at all latitudes, the zonal winds in the atmosphere could not possibly be in a steady state. Explain.

2.5 Prove that on any pressure surface that doesn't intersect the ground the zonally averaged zonal pressure gradient force $[-\partial\Phi/\partial x\ ]$ is identically equal to zero.

2.6 Write an expression for the globally-integrated torque associated with the breaking of gravity waves. Let $[F_x]$ be the wave-induced zonally averaged force in the zonal direction. Note that this term depends upon the gravity wave parameterization.

2.7 In formulating the angular momentum balance for the upper atmosphere (e.g., the part of the atmosphere that lies above the 100 hPa level) what would processes would need to be considered in accounting for the sources and sinks of angular momentum?

6 Poleward transport of atmospheric angular momentum

Figure 2.7 shows the climatological-mean distribution of surface winds. Bearing in mind that frictional drag in the boundary layer is always acting to reduce the wind speed, it is evident that the portion of the atmosphere that lies within 30° of the equator, where the tradewinds prevail, is gaining angular momentum from the underlying oceans, while the part that lies poleward of 30° , where surface westerlies prevail, is losing angular momentum to the oceans.[4] Since the atmosphere is close to a steady state, it follows that there exists a balance requirement for the poleward transport of angular momentum across 30°, as

[4]The oceans, in turn transmit the torque to the solid Earth through horizontal pressure gradients that can be readily inferred from the gradients in sea level. In the tradewind belts, sea level tends to be higher on the east coasts of the continents than on the west side, so the ocean is pushing the continents westward and thereby gaining angular momentum that it is transmitting to the atmosphere through the surface wind stress. Poleward of 30 degrees the opposite situation prevails.

discussed in essays on the atmospheric general circulation by Jeffreys (1926) and Starr (1948) and in a related paper on the vorticity balance by Rossby (1947). The nature of the balance is illustrated schematically in Fig. 2.8. It can be expressed symbolically in the form

$$-2\pi R_E^3 \int_{eq}^{30^\circ} [\overline{\tau_x}] \cos^2 \phi d\phi = \frac{2\pi R_E \cos\phi}{g} \int_{30^\circ} [\overline{mv}]\, dp \qquad (2.5)$$

where τ_x is the stress exerted by the atmosphere upon the underlying surface.

The poleward transport of atmospheric angular momentum is given by the integral

$$\frac{2\pi R_E \cos\phi}{g} \int_{30^\circ} [\overline{mv}]\, dp = \frac{2\pi\Omega R_E^3 \cos^3\phi}{g} \int_{30^\circ} [\overline{v}]\, dxdp + \frac{2\pi R_E^2 \cos^2\phi}{g} \int_{30^\circ} [\overline{uv}]\, dxdp \qquad (2.6)$$

The integration is performed over an imaginary wall extending all the way around a latitude circle at 30° latitude, and from the ground to the "top" of the atmosphere. The overbar refers to time averages over some rather long interval like a season. The first calculations of these integrals were performed by Widger (1949).

The first term on the right-hand side represents the transport of the component of the angular momentum that can be attributed to the Earth's rotation. It is proportional to the net poleward flux of mass across the latitude circle: if the atmosphere as a whole, were to draw closer to its axis of rotation, it would rotate faster in the same sense as the Earth's rotation (from west to east) just as an ice skater spins more rapidly by drawing in his/her arms. The center of mass of the earth's atmosphere undergoes a small latitudinal shift with the seasons, but it isn't anywhere near large enough to account for the observed seasonal changes in zonal wind speed. When we examine the hydrologic cycle we will see that in the statistical average, water vapor is always being transported poleward through middle latitudes. However it constitutes such a small fraction ($\simeq 1\%$) of the atmospheric mass that its contribution to the angular momentum flux is minimal. Hence we conclude that this term cannot be a major contributor to the poleward flux of angular momentum in the earth's atmosphere.

The second term involves the component of the angular momentum associated with the zonal motion of air parcels relative to the rotating Earth. It can be identified with exchange processes (i.e., the exchange of equal amounts of mass containing differing amounts of relative angular momentum). The integrand $[\overline{uv}]$ can be evaluated by calculating the product uv at each gridpoint and time and averaging around the latitude circle and over time in either order. However, the nature of the dynamical processes responsible for the transport becomes more clearly evident if we expand it in terms of various types of exchange processes as explained in Appendix I.

$$[\overline{uv}] = [\overline{u}]\,[\overline{v}] + \overline{[u]^{'}\,[v]^{'}} + [\overline{u}^*\overline{v}^*] + \left[\overline{u^{*'}v^{*'}}\right] \qquad (2.7)$$

The first term in this expansion is identified with the climatological-mean mean meridional motions. Since the net vertically-integrated meridional mass flux is very small, we can assume that this term is associated with mean meridional circulation cells characterized by poleward motion at some levels and equatorial motions at others. The observed distribution of mean meridional cells in the Earth's atmosphere is shown in Fig. 2.9. In the winter hemisphere the mean meridional motions are rather strong, but note that there is no exchange of mass across 32°N, which marks the transition zone between the thermally direct "Hadley cell" in the tropics and the thermally indirect "Ferrel cell" which prevails in middle latitudes. Hence it is clear that the mean meridional cell term does not contribute to the required transport of angular momentum from the tropics into middle latitudes during the winter season. In the summer hemosphere, the mean meridional motions are so weak that even the sign of the transport of angular momentum is uncertain. The mean meridional circulations do produce fluxes of angular momentum across other latitude circles. Throughout most of the troposphere, zonally averaged temperature $[\overline{T}]$ decreases with latitude, and mean zonal wind $[\overline{u}]$ increases with height, as required by the thermal wind equation. Hence the transport of westerly (positive) angular momentum is in the same sense as the mean meridional motion in the upper branch of the cell; that is to say, the Hadley cell transports westerly angular momentum poleward and the Ferrel cell transports it equatorward. For example, in the upper branch of the Hadley cell (say, at 15° latitude of the winter hemisphere), $[\overline{u}]$ and $[\overline{v}]$ are both positive and hence $[\overline{u}]\,[\overline{v}]$ is positive there. In the lower branch, $[\overline{v}]$ is negative in the equatorward flow and $[\overline{u}]$ is negative in association with the tradewinds. Hence the product $[\overline{u}]\,[\overline{v}]$ is positive at low levels as well.

The second term $\overline{[u]^{'}\,[v]^{'}}$ in (2.8) involves temporal correlations between u and v in the transient component of the mean meridional circulations. This term is generally regarded to be small and is not included in conventional general circulation atlases. From the product moment formula

$$\overline{[u]^{'}\,[v]^{'}} = r\left([u]^{'}, [v]\right) \times \sigma\,[u]^{'} \times \sigma\,[v]^{'}$$

where $r\left([u]^{'}, [v]\right)$ is the temporal correlation coefficient between $[u]^{'}$ and $[u]^{'}$ and $\sigma\,[u]^{'}$ and $\sigma\,[v]^{'}$ are the respective standard deviations. The mean meridional circulations are on the order of 1 to 2 m s^{-1} for the Hadley cell and the fluctuating component $\sigma\,[v]^{'}$ is likely to be even smaller. The corresponding upper limit for

Having eliminated the possibility of the required transport of angular momentum across 30° latitude being accomplished by the climatological-mean mean meridional circulations (the first and second terms in Eq. (2.8)) we must conclude that it is accomplished by eddy transports represented in the second and third terms. Both terms are associated with the meridional tilt of the "eddies" or "waves" in the horizontal plane, as shown in Fig. 2.10. In disturbances that tilt eastward with increasing latitude, poleward moving air is characterized by larger (westerly) angular momentum than equatorward moving air. The exchange of equal masses of air containing differing amounts of angular momentum per unit mass results in a net poleward transport of angular momentum at a given level.

The second term in (2.10) represents the effect of standing eddies or stationary waves: longitudinally dependent features of the flow which appear on the time mean maps, while the third term represents the effect of the transients. It is evident from the sections presented in Fig. 2.10 and Fig, 2.11 that both terms contribute to the required poleward transport of angular momentum across 30°N. The fluxes are largest precisely where they are needed. Note also the weaker equatorward transports across 65°S, which marks the boundary between the belt of surface westerlies in middle latitudes and a ring of easterlies centered along the Antarctic coast. There is a suggestion of a similar feature in the Northern Hemisphere during wintertime. In all the sections the fluxes tend to be concentrated near the jet stream level: the lower troposphere contributes very little to the mass weighted transport. In both hemispheres and in both seasons the transients make the dominant contribution to the transport: only in the northern hemisphere during winter do the climatological mean stationary waves make an appreciable contribution. The Northern Hemisphere exhibits a much larger annual cycle in the magnitude of the fluxes.

The magnitude of the eddy transports has been compared against the amount of angular momentum required to balance the frictional torque associated with the middle latitude surface westerlies and, within the range of uncertainty of the estimates the agreement appears to be satisfactory. For example, the Southern Hemisphere exhibits much stronger summertime westerlies and it also exhibits a much stronger summertime poleward transport of westerly momentum.

Wherever the eddy flux of angular momentum is poleward, the zonal momentum of the poleward flowing air in the eddies must be larger than that of the equatorward flowing air. Regardless of the the direction or meridional shear of the zonal flow upon which the eddies are superimposed, this difference in zonal momentum implies that the axes of the waves or eddies shift eastward as one moves poleward, as pictured in Fig. 2.9. In this example, the individual contours are virtually sinusoidal and they are all identical. Therefore, it is not the shape of the contours but, rather, their eastward shift with latitude that makes the zonal momentum larger in the poleward flow than in the equatorward flow. From a careful inspection of Fig. 2.13 it is evident that at at latitudes around 30°N, the major features in the distribution of the 200 hPa streamfunction field in the Northern Hemisphere winter circulation exhibit the required northeast-southwest tilt, consistent with the large poleward standing eddy flux of angular momentum across this latitude in Fig. 2.12.

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Questions

2.8 The poleward flux of angular momentum across 30° latitude, as estimated by in a number of different studies, is on the order of 30×10^{18} kg $m^2 s^{-2}$ (e.g., see Obasi, 1963 and the references therein) Make your own estimate based on the wind stress equatorward of 30° latitude. [Hint: start by assuming representative zonal wind and scalar wind speeds in the tradewind belt.]

2.9 On the basis of the data presented in this section, estimate the poleward transport of angular momentum across 30° latitude. Compare your results with those from Problem 2.7.

2.10 On 30°N at the jetstream level during DJF, $[u'^2] = [v'^2] = 30$ m^{2}s^{-2}, and $\overline{u'v'}$ =200 m^{2}s^{-2}. Calculate the correlation coefficient between u and v in the transients.

2.11 Suppose that the eddies consisted of waves in which the perturbations in u and v both had amplitudes of 10 m^{2}s^{-2} and were 60° out of phase. Calculate $[u^*v^*]$ and the correlation coefficient between u^* and v^* in the eddies.

6.1 Contribution of the high frequency transients to the transport

Nearly half the transient eddy transport of angular momentum across 30°N is due to baroclinic waves, which are distinguished from the other temporal variability by their relatively short periods (or high frequencies). Baroclinic waves dominate the variability with periods shorter than a week and can therefore be isolated from other kinds of variability by the simple application of a highpass digital filter such as the one applied by Blackmon (1976). The design of the filter isn't critical: e.g., taking departures from a five day running mean is suffient to isolate tham perfectly well. Fig. 2.14 shows the distributions of transient eddy kinetic energy at the jetstream (250 hPa) level. In both hemispheres, highpass filtered so as to emphasize the spatial signature of baroclinic waves. The kinetic energy tends to be concentrated within zonally elongated bands centered along 45° latitude. In the Southern Hemisphere the band can be traced all the way around the laritude circle, but the variability is largest in the Indian Ocean sector. In the Northern Hemisphere the zonal asymmetries in the distribution are more prominent. The bands of largest variability of the high frequency transients are concentrated over the Atlantic and Pacific sectors, whereas the Eurasian sector is relatively quiescent. The bands of high variability are suggestive of the notion of "storm tracks". Similar features are observed in all seasons and at all levels. The same banded structure is apparent in the temporal variances (or standard deviations) of the highpass filtered geopotential height, vorticity, and vertical velocity fields. Note that the axes of the "storm tracks" do not lie along 30°, the latitude of the strongest poleward fluxes of angular momentum: they lie along 45° latitude.

It is possible to get further insight into the structure of the high frequency transients by taking a "reference (highpass filtered) time series" of some variable at a specified gridpoint located along the axis of one of the stormtracks and correlating it with (or regressing it upon) time series of any desired field over an entire array of gridpoints and plotting the corresponding correlation (or regression) coefficients in the form of a map. Fig. 2.15 shows such a pair of maps, constructed by taking highpass filtered 500 mb height at the gridpoint (41°N, 178°E, along the Pacific stormtrack) as the reference time series and correlating it with unfiltered 250 and 850 hPa height over the entire Pacific sector, using DJF data only. The correlations in the figure are modest but the pattern that emerges is remarkably clean nonetheless. It is suggestive of a zonally oriented wavetrain in which the disturbances exhibit a zonal wavelength of around 50° of longitude (∼4000 km). The patterns at the two levels are simillar, but upon close inspection it is evident that the various features at the 250 hPa level are displaced slightly to the west of their counterparts at the 850 hPa level, indicating that the waves tilt westward with height. The phase speed of the waves, as deduced from the displacements of the primary "centers of action" on lag correlation maps is 10-15 m s^{-1}, which is comparable to the wind speed at the 700 hPa level (Wallace et al., 1988). Hence, the structure and evolution of the high frequency transients is consistent with an interpretation in terms of baroclinic waves.

The wavetrains at the two levels are oriented slightly differently: at the upper level it extends in an west-northwest—east-southeast direction, whereas at the lower level it follows the 40°N latitude circle almost exactly. Because of this minor difference in orientation, the individual positive and negative centers at the 250 hPa level tilt poleward with increasing latitude much more than those at the 850 hPa level. This difference is consistent with the much larger poleward fluxes of angular momentum at the higher level in Fig. 2.11. The tilt is largest around 30°N (halfway between the two southernmost latitude circles), precisely where the poleward flux of angular momentum is largest. The characteristic "kidney bean" shape of the individual centers is a reflection of the fact the poleward transport of westerly momentum is much stronger on the southern flank of the stormtrack than on the northern flank, which is also consistent with the previous figure.

Hence, the distribution of the flux of angular momentum in zonally averaged cross-sections is a reflection of the horizontal shape of the disturbances that produce it. From an inspection of correlation patterns such as the one shown in Fig.2.15 it is possible to deduce the sense of the fluxes. These results suggest that waves that disperse equatorwards tend to be associated with poleward momentum fluxes and vice versa.

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Questions

2.12 Consider the balance requirements for the conservation of angular momentum within a tropical cyclone, which can be assumed to be in a steady state. Assume that the storm exhibits circular symmetry about the "eye" and imagine a porous circular "wall" at a radius large enough to enclose most of the strong winds. The large loss of angular momentum associated with frictional drag at the ocean surface must be balanced by the radial transport through the wall. Assume that the inflow is mostly occurring within the planetary boundary layer and that the outflow is just below the tropopause, and neglect the fluxes associated with the large-scale flow pattern in which the hurricane is embedded. Does the angular momentum associated with the earth's rotation play an important role in the transport?

2.13 Consider the balance requirements for the conservation of angular momentum within an intense and rapidly deepening extratropical cyclone. In particular, consider the transport across a circular "wall" concentric with the center of the cyclone at the earth's surface and enclosing the strong and intensifying low-level circulation. Assume that the associated trough at the jetstream level lies about one quarter of a wavelength to the west of the deepening cyclone. Take into account the transports associated with the nondivergent component of the wind at upper levels.

2.14 By means of sketches analogous to 2.11, show that the result deduced from that figure is valid regardless of whether the background flow is from the west or from the east, or whether is is present or not.

2.15 Write an analytic expression for a flow similar to the one pictured in Fig. 2.11. Evaluate the eddy flux $[u^*v^*]$ and show that it depends upon the tilt of the wave axes with latitude.

2.16 The sketches below are based on a single contour, replicated many times. In the left panel the contours are shifted meridionally, whereas in the right panel they are shifted both meridionally and zonally. In which panel is the poleward flux of zonal momentum identically equal to zero? Why?

7 The vertical transport of angular momentum

We have seen that the atmosphere's major source of angular momentum is in the planetary boundary layer in the tradewind belt and its major sink is in the planetary boundary layer in the westerlies at much higher latitudes. Yet most of the poleward transport of angular momentum within the Earth's atmosphere takes place near the jet stream level. It remains to be shown how the angular momentum gets transferred upward from the boundary layer to the jet stream level equatorward of 30° and back down poleward of 30°. Is this vertical transport accomplished by some sort of turbulent mixing, as in the planetary boundary layer, or is some other mechanism involved?

7.1 A laboratory analog

A simple laboratory experiment yields insight into both questions. Consider a cylindrical tank, filled with a homogeneous liquid, in solid body rotation about its axis of symmetry. At some instant in time the motor for the turntable is turned off so that the walls of the tank abruptly become stationary. The fluid continues to rotate for a period of time under laminar flow conditions, until its angular momentum is removed by frictional drag at the walls. How long does this "spindown" process take? If the boundary effects were transmitted to the interior of the fluid by molecular diffusion, one should expect that the spindown time would be on the order of hours for a laboratory-size tank and minutes for a "teacup size" demonstration model. By comparison, the observed spindown time is remarkably short; minutes for a laboratory-size apparatus and much less than a minute for a teacup. Evidently some other much more efficient process is at work. The tendency for tea leaves to pile up in the middle of the cup during such an experiment provides a clue as to the nature of this process.

Let us consider the motions in the tank during the spindown process. Throughout most of the tank the tangential flow is close to a state balance between the inward directed horizontal pressure gradient force associated with the parabolic shape of the free surface (not shown) and the outward directed centrifugal force associated with the curvature of the trajectories. However, within the very thin molecular boundary layer adjacent to the bottom wall of the tank this balance is continually being upset by the frictional drag which is slowing down the tangential flow so that the centrifugal force is unable to balance the pressure gradient force. The situation is analogous to sub-geostrophic flow in the planetary boundary layer of the earth's atmosphere. As a result of the imbalance, fluid drifts inward toward the axis of rotation along the bottom of the tank; hence the pile of tea leaves in the middle of the teacup. This frictionally driven radial inflow requires a very slow return flow above the boundary layer, as shown in Fig. 2.16. The character of the return flow is highly dependent upon whether the fluid in the tank is homogeneous (constant density) or stably stratified. If it is homogeneous, then the flow must be barotropic: there is no way to generate horizontal density gradients so there can be no vertical shear of the tangential flow. In this case the radial outflow must be uniformly distributed throughout the interior of the tank. As an element of fluid drifts inward along the bottom of the tank it loses most of its angular momentum. Thus, by the time it completes a full circuit of the radial circulation cell, the spin-down of the tank should be almost complete. The stably stratified (baroclinic) case has been investigated by Holton (1965).

7.2 Role of mean meridional cells

The radial circulation cell in the tank is the analogue of mean meridional cells in the zonally symmetric atmospheric circulations. Let us examine how such cells transport angular momentum vertically in the atmosphere. The vertical flux of angular momentum through any horizontal plane in the atmosphere is given by

$$\frac{\Omega R_E^2}{g}\int\int -\overline{\omega}\cos^2\phi dxdy + \frac{R_E}{g}\int\int \overline{u\omega}\cos\phi dxdy$$

where x and y have their usual meanings defined in the context of a spherical coordinate system. Performing the indicated zonal integration, we obtain

$$\frac{2\pi\Omega R_E^3}{g}\int -\left[\overline{\omega}\right]\cos^3\phi dy + \frac{2\pi R_E^2}{g}\int -\left[\overline{u\omega}\right]\cos^2\phi dy$$

Expanding the $[\overline{u\omega}]$ term and combining the $[\overline{u}] \setminus [\overline{\omega}]$ part of it with the first term we obtain the total contribution of the mean meridional circulations to the vertical transport of angular momentum

$$\frac{2\pi R_E^2}{g}\int -\left[\overline{\omega}\right]\left(\Omega R_E\cos\phi + [u]\right)\cos^2\phi dy \qquad (2.8)$$

In the above expression the first term exhibits a much wider range of variation with latitude than the second. For example, for the latitude range of the Hadley cell (0° to 30° latitude), $\Omega R_E \cos\phi$ varies from ∼465 m s^{-1} to 402 m s^{-1}

and for the latitude range of the Ferrel cell (30° to 60° latitude) it varies from 402 to 232 m s^{-1}. Thus the air in the rising branch of the Hadley cell contains much more angular momentum per unit mass than the air in the sinking branch and hence the Hadley cell transports angular momentum upward from the planetary boundary layer into the upper troposphere, south of the jet stream. The eddies at the jet stream level transport this angular momentum poleward into middle latitudes.

The Ferrel cell then transports the angular momentum downward into the planetary boundary layer where it serves to maintain the surface westerlies in the presence of frictional dissipation, as shown in Fig. 2.17. During wintertime, there is an analogous, but much weaker transport of angular momentum from the polar cap regions into middle latitudes, as indicated in Fig. 2.17.

The mean meridional circulations extract angular momentum from the flow at one level and impart it to the flow at another level by means of the cross isobar flow which induces a zonal Coriolis force. For example, in the lower branch of the Hadley cell the equatorward flow induces a westward Coriolis force that maintains the easterly tradewinds against frictional dissipation. Meanwhile in the upper branch of the same cell, the poleward return flow induces an eastward Coriolis force that maintains the upper level westerlies in the presence of the westward acceleration due to the divergence of the eddy flux.. The Hadley cell, which is a thermally direct circulation, serves to increase the vertical shear of the geostrophic (zonal) flow and the Ferrel cell, which is a thermally indirect circulation, serves to decrease it.

In order to obtain some estimate of the effectiveness of mean meridional circulations in the angular momentum balance of the earth's atmosphere, it is interesting to compute the "spin down time" of the westerlies in the upper branch of the Ferrel cell, say at 43°N, 250 mb during the winter season, where $[\overline{u}] \sim 25$ m s^{-1} and $[\overline{v}] \sim 0.4$ m s^{-1} (we will justify this estimate later). In the absence of any other influences

$$\frac{\partial [u]}{\partial t} = f [\overline{v}] \simeq 0.4 \times 10^{-4} \text{ m s}^{-1} \simeq 4 \text{ m s}^{-1}\text{d}^{-1}$$

Thus the "spindown time" is on the order of only a week. The "spin up time" for the Hadley cell is of the same order of magnitude.

7.3 Role of the eddies

Now let us consider the vertical transport of angular momentum due to the eddies. Figure 2.18 shows the transient eddy contribution to the transport. At low and middle latitudes, the transport is predominantly upward (i.e., $[u^*\omega^*] < 0$), which contrary to one might expect on the basis of an analysis if the structure of a typical baroclinic wave, as depicted in Fig. 2.19.[5]

[5] I just became aware of this discrepancy today. It suggests that the vertical transport of angular momentum is dominated by features other than baroclinic waves. Determing just what's causing the upward flux would be a good class term paper project for a student who can access gridded data for u and v, say, at the 500 hPa level.

Now let us consider the magnitude of the vertical eddy flux of zonal momentum. In order to be of comparable importance to the meridional flux, it needn't be nearly as large as the latter, since much shorter distances are involved in the vertical direction. On the basis of scaling considerations we can argue that the meridional and vertical fluxes should be of comparable importance in the tropospheric general circulation if $[u^*w^*] / [u^*v^*] \sim D/L$, where D is the depth of the troposphere (of order 10 km) and L is the distance over which the eddies transport zonal momentum in the meridional direction, say from 15° to 45° latitude, or 3000 km. Hence D/L ~1/300. Now if we assume that the correlation between u and w is comparable to that between u and v, it follows from the product moment formula that

$$\frac{[u^*w^*]}{[u^*v^*]} = \frac{\sigma(w^*)}{\sigma(v^*)}$$

where the latter ratio is a measure of trajectory slopes in the eddies, projected onto the meridional plane. Hence for meridional and vertical eddy fluxes to assume comparable importance in the transport, the trajectory slopes must be of the order D/L. However, the typical trajectory slopes in the mid-troposphere are much less than 1 part in 300. (We recall from the theory of baroclinic waves that they are inclined only about half as steeply as the isentropes which have a slope on the order of 1 part in 1000.) Furthermore, it can be demonstrated from quasi-geostrophic scaling arguments that w/v is of the order $Ro \times D/L$ where Ro, the Rossby Number is of order 0.15. The magnitude of the peak transports in Fig. 2.18 is more than three orders of magnitude smaller than the corresponding poleward transport in Fig. 2.11. Hence we conclude that as far as the angular momentum budget is concerned, the vertical eddy flux of zonal momentum is of second order importance in comparison to the meridional transport.

Vertical eddy fluxes of zonal momentum are not always of second order importance. In the equatorial stratosphere the fluxes produced by planetary-scale, vertically propagating Kelvin-waves and mixed Rossby-gravity waves interact with the background flow to produce the remarkable quasi-biennial oscillation (QBO) in zonal wind. Large upward fluxes of westerly momentum have been observed in association with the descent of the leading edges of successive westerly wind regimes. These fluxes play a critical role in Lindzen and Holton's (1968) theory of the QBO[6], which has been verified by Plumb's (1977) laboratory analogue.

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Questions

2.17 On the basis of the data presented in Figs. 2.11 and 2.18, estimate the ratio $[u^*w^*] / [u^*v^*]$, evaluating $[u^*v^*]$ at 30° latitude at the jet stream level and $[u^*w^*]$ at 45° latitude at the 500 hPa level.

[6] see also Holton and Lindzen (1972) and Holton et al. (1987)

8 The local, zonally averaged zonal momentum balance

In Appendix 2 it is shown that the time rate of change of zonally averaged zonal wind is given by

$$\frac{\partial [u]}{\partial t} = [v]\left(f - \frac{1}{\cos\phi}\frac{\partial}{\partial y}[u]\cos\phi\right) - [\omega]\frac{\partial [u]}{\partial p} - \frac{1}{\cos^2\phi}\frac{\partial}{\partial \phi}[u^* v^*]\cos^2\phi - \frac{\partial}{\partial p}[u^*\omega^*] + F_x \tag{2.9}$$

The terms that arise as a consequence of spherical geometry are of second order importance. Therefore, for purposes of discussion it will be useful to refer to the Cartesian approximation

$$\frac{\partial [u]}{\partial t} = [v]\left(f - \frac{\partial [u]}{\partial y}\right) - [\omega]\frac{\partial [u]}{\partial p} - \frac{\partial}{\partial y}[u^* v^*] - \frac{\partial}{\partial p}[u^*\omega^*] + F_x \tag{2.10}$$

The term in parentheses that multiplies $[v]$ in (2.9) and (2.10), which is proportional to the meridional gradient of angular momentum m, plays a role analogous to the static stability in the thermodynamic energy equation. For a zonally symmetric ring of air displaced meridionally under the conservation of angular momentum, it is readily verified that $D[u]/Dt = f[v]$, in analogy with the temperature of a sinking air parcel rising at a rate $D[T]/Dt = (\kappa T/p)\,[\omega]$. But the zonal momentum of the environmental air varies with latitude, and therefore, $\partial[u]/\partial t$ is given by the term in parentheses [just as $\partial [T]/\partial t$ is given by $\sigma \equiv (\kappa T/p - \partial T/\partial p)$]. Just as the static stability is a measure of the restoring (gravitational) force per unit vertical displacement, the term in parentheses is a measure of the meridional (inertial: i.e., Coriolis) restoring force per unit meridional displacement of a zonally symmetric ring of air. Just as the static stability is ordinarily positive (potential temperature increasing with height) the "inertial stability" is ordinarily positive (zonally averaged angular momentum decreasing with latitude). Note that the inertial stability for the zonally averaged flow is simply the zonally averaged absolute vorticity. If the anticyclonic shear of $[u]$ were strong enough to cancel f, the inertial stability would vanish. On the equatorward flank of the Pacific jetstream (Fig. 2.3) $\partial u/\partial y$ is so large that it does, in fact, approach f locally, but the absolute vorticity does not approach zero in the zonal average. Figure 2.20 shows the meridional profile of angular momentum (divided by the radius of the Earth), together with the corresponding profiles of zonal velocity for the solid earth and for the atmosphere (in absolute coordinates), at the 200 mb level which corresponds to the core of the tropospheric jetstream, in DJF. Equatorward of the jetstream, zonal velocity is almost uniform, but even there the angular momentum still exhibits a substantial meridional gradient.

Hence, the inertial stability for the zonally symmetric flow is positive throughout the troposphere with the possible exception of a narrow region within a few degrees of the equator. Note that the jetstream at 30° marks a break in the profiles.

For the reasons discussed in the previous section, the terms involving the vertical motions are smaller than their counterparts that involve meridional motions by a factor of the Rossby Number. Neglecting these terms, (2.10) becomes

$$\frac{\partial [u]}{\partial t} = [v]\left(f - \frac{\partial [u]}{\partial y}\right) + G + F_x \tag{2.11}$$

where G represents the meridional convergence $(-\partial [u^* v^*] / \partial y)$ of the northward flux of westerly momentum by the eddies.

Now let us apply this equation at points A–D in Fig. 2.21. We will assume that poleward eddy fluxes are important only in the upper troposphere (points B and C), and that frictional effects are important only within the planetary boundary layer (points A and D).

At point A in the figure there must be a balance between the $f[v]$ term, which is producing an easterly (negative) acceleration and the frictional drag, which is trying to decelerate the surface easterlies. Thus, the easterly tradewinds are maintained against frictional dissipation by the equatorward flow in the lower branch of the Hadley cell. In a similar manner, the surface westerlies in the middle latitudes are maintained against frictional dissipation by the poleward flow in the lower branch of the Ferrel cell. Thus there is a good reason why the transition between easterly and westerly surface winds, near 30° latitude, coincides exactly with the transition between Hadley and Ferrel cells.

At point B the poleward eddy flux of zonal momentum is increasing rapidly with latitude. Hence, $\partial [u^* v^*] / \partial y >> 0$; that is to say, there is a strong divergence of eddy flux out of this region. Thus at B, the $f[v]$ term associated with the poleward flux in the upper branch of the Hadley cell supplies westerly momentum at exactly the rate that the eddies are removing it.

At point C the picture is just the reverse. The zonal momentum fluxed poleward by the eddies is converging into this region so that $\partial [u^* v^*] / \partial y << 0$. The Coriolis term associated with the equatorward flow in the upper branch of the Ferrel cell removes the westerly momentum as fast as the eddies bring it in, thus maintaining a steady state.

We have purposely applied the above equation to latitudes near 15 and 45 degrees, where the signs of the terms are unambiguous. Note that this equation doesn't tell us anything about the balance in the vicinity of 30°, where all the terms are near zero.

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Questions

2.18 Zonally symmetric inertial instability is possible in a flow with a(n) (westerly, easterly) jet centered on the equator.

2.19 Show how the zonal momentum balance is satisfied at point D in Figure 2.19.

2.20 Taking into account the meridional variation of f and [u] in (2.9), show that the Hadley cell must be stronger than the Ferrel cell by at least a factor of 3.

2.21 Making use of Figures 2.9 and 2.10, discuss the seasonal differences in the distribution of mean meridional circulations and relate them to relative the strength of the "eddy forcing" term G in (2.10).

2.22 Prove that if u^* and v^* are nondivergent, $G \equiv -\partial\left[u^* v^*\right]/\partial y = [\zeta^* v^*]$, where ζ^* is the eddy component of the relative vorticity.

2.23 During the peak of the QBO, $[u]$ on the equator reaches values of 20 m s^{-1} . Prove that only eddy processes could be responsible for the presence of such strong westerlies in the equator.

9 Concluding remarks

That the time-mean, global, tropical, and extratropical angular momentum balances yield such small residual terms testifies to the validity of the budget equations, the reliability of the data and parameterizations that are used in evaluating the atmospheric angular momentum, the accuracy of estimates of the sources and sinks of atmospheric angular momentum due to wind stress and mountain torques at the Earth's surface and estimates of the transport of angular momentum within the atmosphere. It has been instructive to see how the conservation of angular momentum plays out in the atmospheric general circulation under steady state conditions.

Although we have listed the breaking of gravity waves among the sources and sinks of angular momentum, we have not been able to detect its effect on the angular momentum balance for the atmosphere as a whole, given the uncertainties in the other terms. Wave breaking plays a more significant role in the angular momentum balance of the stratosphere and mesosphere where quasi-stationary gravity waves "launched" by flow over mountainous terrain tend to break under the influence of exponentially increasing amplitude with height[7] and decreasing Doppler-shifted phase speed [8]. A favored region for wave breaking is above the tropospheric jet stream in midlatitudes, in the latitude belt of the Himalayas and Rockies.

It should be acknowledged that consideration of the global angular momentum balances leaves unanswered many fundamental questions about the observed distribution of zonal wind. For example, it does not explain the existence of the tropospheric jet stream. The zonal momentum balance, as expressed in Eq. (2.8) is satisfied in the core of the (zonally averaged) jet stream, but that balance doesn't yield any useful information because all three terms are very close to zero. Nor does the angular momentum balance, in and of itself, place any constraints on the total atmospheric angular momentum; e.g., atmospheres with substantial super- or sub-rotation could equally well exist under steady state conditions, provided that the globally averaged torque at the Earth's surface was zero. The conservation of angular momentum tells us that there must

[7] In the absence of wave breaking energy density tends to be conserved, so as density decreases with height, the wind velocities increase.

[8] As the doppler-shifted phase speed decreases, so does the vertical wavelength of the waves. Hence, as they approach their critical level the vertical wind shear in the waves increases. Wave breaking occurs when the shear reaches the point where the Richardson number drops below 1/4.

be a poleward transport of angular momentum across 30 degrees latitude in the time mean, given the observed distribution of surface winds, but it does not, in and of itself, yield any insight as to why that transport is poleward. We will address that question in Chapter 5. For now we we will have to be content with merely having balanced a budget.

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