Atmospheric Science 582—Winter 2014

Homework 2

1. Show that the phase speeds and stability properties of numerical solutions to the forward-backward approximation (4.17)–(4.18) to the linearized shallow-water equations with no mean flow are unchanged if the order in which u and h are updated is reversed by first stepping h forward and then updating u with a backward time difference.

2. The linearized one-dimensional Rossby adjustment problem for an atmosphere with no mean wind is governed by the equations

$$\frac{\partial u}{\partial t} - fv + g\frac{\partial h}{\partial x} = 0, \qquad \frac{\partial v}{\partial t} + fu = 0, \qquad \frac{\partial h}{\partial t} + H\frac{\partial u}{\partial x} = 0.$$

Compare the approximate solution to these equations obtained using leapfrog differencing on an unstaggered mesh

$$\begin{aligned} \delta_{2t}u - fv + g\delta_{2x}h &= 0, \\ \delta_{2t}v + fu &= 0, \\ \delta_{2t}h + H\delta_{2x}u &= 0 \end{aligned}$$

with those obtained using forward–backward time-differencing on the staggered mesh shown in Fig. 4.1:

$$\begin{aligned} \delta_t u_j^{n+\frac{1}{2}} - fv_j^n + g\delta_x h_j^n &= 0, \\ \delta_t v_j^{n+\frac{1}{2}} + fu_j^{n+1} &= 0, \\ \delta_t h_{j+\frac{1}{2}}^{n+\frac{1}{2}} + H\delta_x u_{j+\frac{1}{2}}^{n+1} &= 0. \end{aligned}$$

Define v, at the same points as u. Let the spatial domain be periodic on the interval $0 \leq x \leq 5000$ km, but show your solutions only in the subdomain $1200 \leq x \leq 3800$ km. Let $f = 10^{-4}$ s^{-1} and $c = \sqrt{gH} = 10$ ms^{-1}. For initial conditions choose $u(x,0) = v(x,0) = 0$, and let the height field be given by a slightly smoothed unit-amplitude square wave. Beginning with

$$h(x,0) = \begin{cases} 1 & \text{if } 0 \leq 2500\text{km} \\ -1 & \text{otherwise} \end{cases},$$

obtain a slightly smoothed square wave by four iterative applications of the filter

$$\phi_j^f = \frac{1}{4}(\phi_{j+1} + 2\phi_j + \phi_{j-1})$$

to the pure square wave. Let $\Delta x = 5$ km. Initialize the leapfrog scheme with a single forward step.

(a) Show solutions for all three fields at the time step closest, but not larger than, $t = 11\times10^4$ s (about 30 hr). Use a Courant number ($c\Delta t/\Delta x$) of 0.9. Discuss the quality of the two solutions. Explain the source of the difference between the two different numerical solutions.

(b) Recompute both solutions with $c\Delta t/\Delta x = 0.1$. What happens using the smaller Courant number? Why?

(c) Eliminate the smoothing step from the initialization and discuss the impact on the solution for the simulations with $c\Delta t/\Delta x = 0.9$.

(Note that analytic solutions to this problem are given in Gill, 1982: *Atmosphere-Ocean Dynamics* Sections 7.2–7.3.)

Due Friday, February 6