Atms 536: Homework 3

Exploring simple tornado vortex models for 2D horizontally non divergent flow.

Recall that the circulation around a curve C in a horizontal plane is equal to the vorticity normal to that plane integrated over the area bounded by curve C. Also the vorticity in "natural coordinates" is

$$\zeta = \frac{\partial v}{\partial n} + \frac{v}{R_s},$$

where v is the speed along the trajectory, n is distance along a coordinate 90°to the left of the flow, and R_s is the radius of curvature of the streamline (positive for cyclonic flow). [See Holton and Hakim, *An Introduction to Dynamic Meteorology,* Section 4.2.1.]

(Show your derivations.)

1. The tangential wind speed v in a Rankine vortex may be written as a function of radius r as

$$v = \begin{cases} \dfrac{Z}{2\pi r_0^2} r & r \le r_0 \\ \dfrac{Z}{2\pi r} & r > r_0 \end{cases}$$

where r_0 is the radius of maximum wind and Z is a constant determining the vortex strength.

(a) Let ζ be the vorticity perpendicular to the r–θ plane. What is $\zeta(r, \theta)$?

(b) What is the total vorticity integrated over the r–θ plane,

$$\int_0^{2\pi} \int_0^{\infty} \zeta r \, dr \, d\theta \ ?$$

(c) What is the strength a point vortex at the origin that will produce the same far-field flow (i.e., for $r > r_0$) as in the Rankine vortex?

2. The tangential wind speed in the Burgers-Rott vortex can be written in the functional form

$$v = \frac{Z}{2\pi r}\left(1 - e^{-r^2/r_o^2}\right).$$

(a) What is the radius of maximum wind for this vortex?

(b) What is $\zeta(r, \theta)$?

(c) What is the total vorticity integrated over the r–θ plane.

3. Submit a plot comparing $v(r)$ for the Rankine and Burgers-Rott vortices over the interval $[0, 5r_0]$. Discuss their differences and similarities.

Due Tuesday, February 28