Atmospheric Science 536—Winter 2017

Homework 4

1. Recall that for steady shallow-water flow over a obstacle of height $b(x)$ the flow speed $u(x)$ and fluid depth $h(x)$ are related such that

$$\left(1 - \mathrm{F_r}^2\right) \frac{\partial h}{\partial x} = -\frac{\partial b}{\partial x}, \qquad (1)$$

where $\mathrm{F_r} = u/\sqrt{gh}$ is the Froude number.

(a) Show that if a transition to supercritical occurs over a mountain with a single crest, the point at which the flow becomes critical ($\mathrm{F_r} = 1$) must occur at the crest.

(Note that if an evolving flow temporarily becomes critical at some point on the upstream slope, a wave of elevation will form and propagate upstream, deepening the oncoming flow so that at steady state, the transition from sub- to supercritical occurs over the crest.)

(b) The mere fact that an initially subcritical flow becomes critical at the crest does not obviously imply that a transition to supercritical will occur downstream of the crest. Rule out the possibility of solutions that are symmetric about the crest (such as everywhere subcritical or everywhere supercritical flows) by demonstrating that $\partial h/\partial x$ cannot be zero at the crest if the flow at the crest is critical. *Hint:* take the x-derivative of (1).

2. Suppose that the standard analysis for trapped waves in a two layer atmosphere is extended to include a third upper layer. The mean windspeed is constant a U_0 at all levels, and the Scorer parameter in lowest layer of depth H_1 is ℓ_1, in the middle layer of depth H_2 it is ℓ_2, and in the upper layer it is ℓ_3.
Suppose

$$\ell_1^2 - \ell_2^2 \geq \frac{\pi^2}{4H_1^2}.$$

(a) In the case $\ell_3 = \ell_2$, would you expect to see trapped waves *downstream* of an appropriately sized ridge (i.e., one whose height and width are potentially appropriate for generating trapped waves). Why or why not? (You can use results from the lectures or other resources without mathematically deriving a result from first principles.)

(b) Suppose that $\ell_3 = \ell_1$, how would this change the nature of any waves that might appear *downstream* of the mountain? How does what one would observe depend on H_2? Provide a rigorous, but qualitative, argument without going through a lot of math.

3. Show that the maximum perturbation horizontal wind speed near the surface in a linear mountain wave scales like Nh_0, where N is the Brunt-Väisälä frequency and h_0 is the maximum mountain height. Assume the flow is steady, Boussinesq, two-dimensional (in the x-z plane), and that N and the background cross-mountain wind U are constant. (see next page)

Hint: it is not necessary to obtain an actual linear solution. Use some of the linearized of the governing equations, the boundary conditions, and the dispersion relation for steady gravity waves in which the perturbation fields are proportional to $\exp(i(kx + mz))$.

Due by 4 PM Tuesday March 14th