Equatorial Wave-Mean Flow Interaction: A Numerical Study of the Role of Latitudinal Shear

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ABSTRACT

A time-dependent primitive equation model for an equatorial channel is used to study the interaction of equatorial Kelvin and mixed Rossby-gravity waves with the mean flow. The model employs a semia-implicit time-differencing scheme and a finite-difference grid in the meridional plane. The zonal dependence is represented by the mean plus a single Fourier wave component. The wave modes are forced by specified geopotential perturbations at the lower boundary. Initial mean wind profiles with westerly (easterly) shear with height are used for cases with Kelvin (mixed Rossby-gravity) wave forcing. Both mechanical and thermal dissipation are included to generate wave-driven mean flow modifications. Integrations are carried forward for 60 days after initiation of the boundary forcing. As expected, the wave-mean flow interaction process leads to the development of intense downward moving shear layers centered at the equator. However, contrary to some previous suggestions, latitudinal mean shears antisymmetric with respect to the equator tend to be reduced rather than amplified by the wave-driven mean flow accelerations. Thus, both Kelvin and mixed Rossby-gravity waves tend to produce mean flow profiles symmetric with respect to the equator which are qualitatively in accord with observations.

1. Introduction

During the past decade there have been a number of observational and theoretical studies on the role of vertically propagating wave modes in the general circulation of the tropical stratosphere. [See Wallace (1973) and Holton (1975) for reviews of the observational and theoretical aspects, respectively.] Observations indicate that two wave modes account for most of the large-scale wave variability in the lower tropical stratosphere. These are the Kelvin wave and the mixed Rossby-gravity wave. The Kelvin wave is an eastward propagating mode which has pressure and zonal velocity perturbations symmetric about the equator, while the mixed Rossby-gravity wave is a westward propagating mode with pressure and zonal velocity perturbations antisymmetric about the equator.

Both these modes apparently are generated by a variety of processes in the tropical troposphere and propagate vertically into the stratosphere where they are dissipated by both thermodynamic and mechanical processes. As the waves are damped the mean flow is accelerated through the action of so-called “radiation stresses” associated with the waves. This process of wave-mean flow interaction has been elucidated elegantly by Andrews and McIntyre (1976a), and independently by Boyd (1976). Andrews and McIntyre (1976a) also showed that transience associated with wave growth could lead to mean flow acceleration—even in the absence of damping. Earlier Holton and Lindzen (1972) showed how the mean flow acceleration driven by Kelvin and mixed Rossby-gravity waves could account for the observed quasi-biennial oscillation in the mean zonal winds in the equatorial lower stratosphere. In their numerical simulation Holton and Lindzen used a meridionally integrated formulation of the wave-mean flow interaction process based on asymptotic formulas developed by Lindzen (1971). Their model, therefore, gave no information concerning the latitudinal dependence of equatorial wave-mean flow interaction processes or the possible role of wave transience. In particular, they could not examine the influence of latitudinal mean wind shears such as those associated with the annual cycle.

Andrews and McIntyre (1976b) used an asymptotic theory to show that for steady waves in weak horizontal shear, with weak thermal dissipation and no mechanical dissipation, both the Kelvin and mixed Rossby-gravity modes would produce mean flow acceleration distributions which would tend to amplify an existing horizontal shear. Andrews and McIntyre speculated that this process could lead to barotropic instability of equatorial mean flows. They further pointed out that the actual departure of the mean flow acceleration field $\delta a/\delta t$ from equatorial symmetry could be regarded as due
to a combination of two factors. First, the local momentum flux convergence for a dissipating wave is proportional to the inverse of the Doppler-shifted phase speed of the wave, \( \dot{c}^{-1} \). This factor will always tend to amplify a horizontal mean wind shear since the acceleration will be most rapid where \( \dot{c} \) is already small. Second, the latitudinal dependence of the mean wind causes an alteration in the structure of the wave fields which in the weak shear case treated by Andrews and McIntyre tends also to amplify a preexisting horizontal shear, at least close to the equator.

The relevance of this instability generating mechanism to actual conditions in the tropical stratosphere was not established by the limited calculations of Andrews and McIntyre. Simmons (1978) extended the results to strong latitudinal shear cases in a model which did not include vertical shear or mechanical dissipation. He also found a tendency for the mean flow accelerations to amplify latitudinal mean wind shears near the equator. This tendency was especially pronounced for the mean flow acceleration due to a mixed Rossby-gravity wave in a mean field with strong linear horizontal shear at the equator.

The most complete analytic study of equatorial waves in mean shear flow is the work of Boyd (1978). Boyd applied a very accurate expansion procedure to examine the effects of both linear and parabolic latitudinal shear on various equatorial wave modes. Boyd’s analysis, unlike that of Andrews and McIntyre, was not limited to weak shears, and unlike Simmons he included both mechanical and thermal dissipation by specifying Rayleigh friction and Newtonian cooling with equal rate coefficients. The inclusion of mechanical dissipation is an important aspect of this work since Andrews and McIntyre (1976a) previously had shown that only a moderate amount of mechanical dissipation completely changes the latitudinal dependence of the mean flow acceleration for the mixed Rossby-gravity wave. Physically, the role of mechanical dissipation is to destroy the quadrature relationship between the zonal and meridional velocity perturbations, \( u' \) and \( \upsilon' \), and thus to produce a meridional momentum flux \( u'\upsilon' \) which has strong divergence at the equator.

Mechanical dissipation, however, is not the only process which can produce a meridional momentum flux in equatorial waves. Wave transience (i.e., temporal changes in wave amplitude) may play a similar role. In fact, as pointed out by Andrews and McIntyre (1976a) the mean flow acceleration profile given by their theory for an exponentially growing wave is identical to that for a steady wave subject to damping by Rayleigh friction and Newtonian cooling with equal rate coefficients. Mechanical dissipation and/or wave transience must be important in the atmosphere since the observed quasi-biennial oscillation has its maximum at the equator in both the westerly and the easterly phases, and only when mechanical dissipation or transience are included can a mixed Rossby-gravity wave produce a mean flow acceleration which is a maximum at the equator.

However, there is at present little evidence that the horizontal shear amplification mechanism discussed above is of significance in the atmosphere. In fact, although he does not mention it, Fig. 12 in Boyd (1978) shows that for his model (which has parameters appropriate to the tropical stratosphere) the mean wind acceleration for a mixed Rossby-gravity wave in the presence of linear latitudinal shear has a profile which tends to reduce the latitudinal shear. Apparently, for the situation which he considered, the alteration of the structure of the wave fields by the mean shear produced changes in the momentum flux distribution which more than compensated for the \( \dot{c}^{-1} \) dependence of the momentum flux convergence discussed by Andrews and McIntyre. Andrews (personal communication) has shown that the horizontal shear enhancement tendency turns out to be very sensitive to the ratio of mechanical to thermal dissipation. Thus, when mechanical dissipation vanishes the wave-driven acceleration enhances horizontal shear for sufficiently long waves. But, when mechanical dissipation is of the same magnitude as thermal dissipation the shear is diminished. From the argument of Andrews and McIntyre (1976a) stated above, wave transience should also lead to shear diminution.

The analytic models of Andrews and McIntyre (1976b) and Boyd (1978) were both based on a “two scales” approach in which the horizontal and vertical dependencies were separated by assuming that the mean wind field \( \bar{u}(y,z) \) varied slowly in height compared to the vertical scale of the waves. Previous work (e.g., Lindzen, 1971) has shown that this is an excellent approximation. However, analytic studies to date have also all assumed that the mean flow profile remained fixed in time. Only second-order initial mean flow accelerations have been calculated. This approximation is probably less justified in modeling equatorial waves since in the descending shear zones of the quasi-biennial oscillation, where wave-mean flow interaction is intense, the time scale for changes in the mean flow is not much different from the time scale for the waves.

The adjustment of wave amplitude to the changing mean wind in the shear zone creates a wave transience which is unrelated to changes in the wave forcing. The role of this type of transience can probably only be investigated numerically. Thus, the primary purposes of the present work are to examine the influence of wave transience and latitudinal shear on the wave-mean flow interaction process by employing an initial value approach using a numerical model so that the finite-amplitude time evolution of the wave-mean flow interaction process can be studied.

This numerical approach is not without its own limitations. The principal weakness of the method is the presence of truncation errors due to finite differen-
Table 1. Fourier coefficients for the dynamical field variables.

<table>
<thead>
<tr>
<th>$f(\lambda)$</th>
<th>$u$</th>
<th>$v$</th>
<th>$w$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_s$</td>
<td>$U_s$</td>
<td>$V_s$</td>
<td>$W_s$</td>
<td>$\psi_s$</td>
</tr>
</tbody>
</table>

Cing. Such errors are a particular problem in the simulation of vertically propagating modes since inadequate vertical grid resolution can cause spurious wave reflections to occur. Fortunately, the numerical solutions can be checked at least qualitatively by comparing the model generated wave fields with those given in Boyd’s analytic work, at least in cases where wave transience is small.

2. The basic equations of the model

The numerical model, which is similar to that described by Holton (1976), is based on the primitive equations in spherical coordinates with a log-pressure vertical coordinate $z = -H \ln(p/p_s)$, where $H$ is a constant scale height and $p_s$ a standard reference pressure. The primitive equations in this coordinate system are given in Holton (1975) and will not be repeated here. In the present model we expand the dependent variables in a series of zonal harmonics and truncate the series to exclude all modes except for the zonal mean and a single wave component with planetary wavenumber $s$. Thus, we let

$$ f(\lambda, \theta, z, t) = e^{i2\pi H}[\tilde{F_s}(\theta, z, t) + F_s(\theta, z, t)e^{i\lambda t} + F^*_s(\theta, z, t)e^{-i\lambda t}], $$

where $f$ stands for any field variable, $\tilde{F}$ is the zonal mean of $f$, and $F_s$ is the Fourier transform of $f$ defined by

$$ F_s = \frac{e^{-i2\pi H}}{2\pi} \int_{-\pi}^{\pi} f(\lambda, \theta, z, t)e^{-i\lambda \theta} \, d\lambda, $$

so that $F_{-s} = F^*_s$, where the asterisk denotes a complex conjugate. Here the transformed variables $\tilde{F}$, $F_s$, and $F^*_s$ are all weighted by a factor $[e^{i2\pi H}]$ proportional to the inverse square root of the basic-state density.

Using the notation given in Table 1 the equations of the model can be written as follows:

$$ \frac{\partial}{\partial t} f\tilde{U} = -e^{i2\pi H}\left[\frac{1}{\cos \theta} \frac{\partial}{\partial \theta} (\tilde{U}\tilde{V} \cos \theta) + \frac{\partial}{\partial z} (\tilde{U}\tilde{W})\right] + F_w + D_1(u), \quad (2.1) $$

$$ \frac{\partial}{\partial t} f\tilde{V} = -e^{i2\pi H}\tilde{U}\tan \theta + D_2(\tilde{V}), \quad (2.2) $$

$$ \frac{1}{\cos \theta} \frac{\partial}{\partial \theta} (\tilde{V} \cos \theta) + \left[\frac{\partial}{\partial z} + \frac{1}{2H}\right] \tilde{W} = 0, \quad (2.3) $$

where $Q_s$ represents diabatic heating processes. Note that in (2.2), (2.5), (2.6) and (2.7) all terms involving advection by the mean meridional circulation $(\tilde{V}, \tilde{W})$ have been neglected. Little error is involved in this approximation.
In the above equations the terms specified by the operators \( D_{\alpha}(\cdot) \) and \( D_{\beta}(\cdot) \) represent various damping effects whose precise forms will be given in Section 4.

The system (2.1)–(2.8) together with the boundary conditions given in Section 3 can be approximated numerically in an efficient manner by using a semi-implicit time-integration scheme, and finite-differencing the system on a staggered grid in the meridional plane. This scheme has been described by Holton (1976); the reader is referred to that work for the details. In the present calculations the grid consisted of 25 points in latitude and 17 points in the vertical with horizontal grid increments of \( \Delta \theta = 3^\circ \) for the Kelvin wave cases and \( \Delta \theta = 4^\circ \) for the mixed Rossby-gravity wave cases. In all cases reported here the vertical grid spacing was \( \Delta z = 1 \text{ km} \) and the time increment was \( \Delta t = 1 \text{ h} \).

3. Boundary and initial conditions

The natural lateral boundaries for the system (2.1)–(2.8) are at the poles \( \theta = \pm \pi/2 \). However, the present study is concerned solely with modes trapped near the equator. Thus, for numerical efficiency we replace the poles by rigid walls at latitudes \( \theta = \pm \theta_w \), where \( \theta_w = 36^\circ \) in the Kelvin wave cases and \( \theta_w = 48^\circ \) in the mixed Rossby-gravity wave cases. In order to prevent wave reflections, or generation of spurious modes due to these artificial walls, a Rayleigh friction damping term with latitudinal dependence

\[
\kappa_1 = (2 \times 10^{-6} \text{ s}^{-1}) (\theta/\theta_w)^4
\]

is added to the wave momentum equations (2.5) and (2.6). This assures that all wave modes will be damped strongly near the lateral boundaries but that there will be negligible effects near the equator.

The complete boundary conditions for the mean flow equations can be expressed most conveniently if we define a meridional streamfunction \( \bar{X} \) by letting

\[
\bar{W} \cos \theta = \partial \bar{X}/\partial y, \quad \bar{V} \cos \theta = \left( \frac{\partial}{\partial z} - \frac{1}{2H} \right) \bar{X}.
\]

The lateral boundary conditions are then

\[
\bar{X} = \bar{U} = \bar{V} = \partial \bar{\Psi} / \partial y = 0 \quad \text{for} \quad \theta = \pm \theta_w.
\]

The vertical extent of the domain of integration (16 km) is intended to encompass the lower stratosphere—the region where Kelvin and mixed Rossby-gravity waves interact with the mean flow to form the quasi-biennial oscillation. The lower boundary is set at 18 km \( (z=0) \) and the upper boundary at 34 km \( (z=\pi \gamma) \). Boundary conditions at these horizontal bounding surfaces are

\[
\bar{U} = \bar{U}_b(\theta), \quad \bar{V} = 0 \quad \text{at} \quad z=0.
\]

Thus the zonal mean velocity \( \bar{U}_b(\theta) \) as well as the geopotential height deviation \( h_\epsilon(\theta,t) \) must be specified at the lower boundary. Once \( \bar{U}_b(\theta) \) has been specified the mean streamfunction at the lower boundary \( \bar{\Psi}_b(\theta) \) can be determined from the gradient wind balance

\[
\frac{\partial \bar{\Psi}_b}{\partial \theta} = f \bar{U}_b + \bar{U}_b^2 \tan \theta - e^{2\pi \gamma H}.
\]

At the upper boundary the vertical shear of the mean zonal wind is assumed to vanish and the motion is specified to be in gradient wind balance so that the mean flow boundary conditions are

\[
\frac{\partial}{\partial z} (\bar{U} e^{i\pi \gamma H}) = -\frac{\partial}{\partial z} (\bar{\Psi} e^{i\pi \gamma H}) = 0 \quad \text{at} \quad z=\pi \gamma.
\]

For the waves it is sufficient to set \( \bar{\Psi}_\epsilon = 0 \) at \( z=\pi \gamma \).

In order to prevent this “rigid lid” boundary condition from generating spurious wave reflections the upward propagating waves are damped near the upper boundary by a strongly height-dependent linear damping introduced in Eqs. (2.2), (2.4), (2.5), (2.6) and (2.7). The functional form for the damping coefficient is

\[
\kappa_2(z) = (5 \times 10^{-6} \text{ s}^{-1}) \exp[\gamma(z-\pi \gamma)],
\]

where \( \gamma = 5 \times 10^{-4} \text{ m}^{-1} \), and \( z \) is in meters. That this damping layer prevents any significant reflection from the upper boundary can be seen by examining the structure of the wave fields (e.g., Fig. 6).

In addition, boundary conditions are also required for the vertical momentum and heat flux terms in (2.1) and (2.4), respectively. For simplicity we assume that the flux divergence vanishes at \( z=0 \):

\[
\frac{\partial}{\partial z} (\bar{U} \bar{W}) = -\frac{\partial}{\partial z} (U \bar{W}_z + U \bar{W}_z) = 0
\]

(with a similar expression for the heat flux) and that the fluxes themselves vanish at \( z=\pi \gamma \). We then have from (2.4) \( \bar{W} = \partial \bar{X} / \partial y = 0 \) at \( z=\pi \gamma \). This completes the specification of the boundary conditions.

Initial mean wind profiles for the various cases reported here were specified in terms of analytic functions with smooth variations in both latitude and height. The initial geopotential field was assumed to be in gradient wind balance with the specified mean wind field. Wave perturbations were excited by slowly impressing a geopotential perturbation along the lower boundary by letting \( h_\epsilon(\theta,t) \), the geopotential height at \( z=0 \) for the \( \epsilon \) wave mode, be given by

\[
h_\epsilon(\theta,t) = A_\epsilon(\theta)e^{-\omega t}(1 - e^{-it}),
\]

where \( \omega \) is the frequency of the forced wave, \( A_\epsilon(\theta) \) the latitudinal distribution of the forced geopotential height disturbance, and \( \epsilon \) the inverse time scale for
development of the boundary forcing. It was found that with \( \varepsilon^{-1} = 10^6 \) s there was relatively little high-speed gravity wave noise introduced by the switch on of the boundary forcing.

4. Parameterization of dissipation processes

Kelvin and mixed Rossby-gravity waves are dissipated in the stratosphere by both thermal and mechanical processes. The thermal damping, due primarily to emission of infrared radiation, can be represented with reasonable fidelity by a Newtonian cooling approximation. In the equatorial stratosphere the thermal relaxation time varies substantially with height from a value of about 21 days at 18 km to about 10 days at 34 km. However, in the present model we have chosen a constant Newtonian cooling coefficient \( \alpha = 1/7 \) (7 days) in order to intensify the wave-mean flow interaction in the region covered by our grid domain.

When only thermal damping is included the wave-mean flow interaction process tends to generate extremely strong vertical shears in the mean wind profile which in a numerical model can create spurious wave reflections for small vertical-scale wave modes. In the atmosphere the magnitude of the vertical shear is apparently limited by the occurrence of shear instabilities which produce rapid turbulent mixing across the shear layer. Observational evidence for such instabilities in association with Kelvin wave-mean flow interactions in the stratosphere has been presented by Kousky and Wallace (1971). In a numerical model this type of shear-dependent vertical diffusion could be represented using a nonlinear (shear-dependent) formulation for the vertical eddy viscosity coefficient. However, to avoid the numerical complexities involved in such an approach we have in this study chosen the simple expedient of representing mechanical dissipation by a fourth-order vertical diffusion. Thus, in the zonal mean momentum equation we include a term

\[-A_s \frac{\partial^4 U_z}{\partial z^4}, \]

where \( A_s/\Delta z^4 = 2 \times 10^{-7} \) s\(^{-1}\). This yields a highly scale-selective damping which has a time scale of about 3 days for motions with vertical scale \( 2 \Delta z \), but increasing to 48 days for a vertical scale of \( 4 \Delta z \) (i.e., the time scale increases by a factor of 16 for each doubling of the space scale). In addition to the vertical diffusion it was found necessary to include a small horizontal smoothing to suppress nonlinear computational instability. For simplicity we use the same type of fourth-order diffusion operator as used for the vertical diffusion. However, the inverse time scale \( A_s/\Delta y^4 = 2 \times 10^{-4} \) s\(^{-1}\) is set an order of magnitude smaller than in the vertical diffusion term. Thus \( 2 \Delta y \) wavelength disturbances are damped on a 30-day time scale, while larger meridional scales are virtually unaffected by the damping. The complete diffusion terms for the mean flow and wave components thus take the forms

\[ D_1(\vec{U}) = -\frac{A_v}{\cos^2 \theta} \frac{\partial^4 \vec{U}}{\partial y^4} \left( \frac{\partial \vec{U}}{\cos \theta} \right), \]

\[ D_2(U_z) = -\frac{A_v}{\cos^2 \theta} \frac{\partial^4 U_z}{\partial y^4} \left( \frac{\partial U_z}{\cos \theta} \right), \]

(4.1)

(4.2)

The form for horizontal diffusion in (4.1), i.e., diffusion of angular velocity, is required to insure that horizontal diffusion does not change the mean angular momentum on a pressure surface (see Mahlman and Manabe, 1976). The diffusion terms in (2.2), (2.3), (2.6) and (2.7) are all analogous to (4.2). Specification of these terms plus the Newtonian cooling

\[ Q_x = -\alpha \left( \frac{\partial}{\partial z} + \frac{1}{2H} \right) \Psi, \]

(4.3)

in (2.7) completes the model equations.

5. Kelvin wave-mean flow interaction

Observed stratospheric Kelvin waves have an average phase speed relative to the ground of \( \varepsilon = 25 \) m s\(^{-1}\). The zonal scale of the observed waves corresponds to planetary wavenumbers \( s = 1 \) and \( s = 2 \). In the present study we have specified a value of \( \omega \) in (3.9) corresponding to \( s = 2 \) and a phase speed \( \varepsilon = 25 \) m s\(^{-1}\). The mean zonal wind at the lower boundary at the equator is fixed at \(-20 \) m s\(^{-1}\) so that the Doppler-shifted phase speed \( \varepsilon = +45 \) m s\(^{-1}\) at the lower boundary. The meridional distribution of the geopotential perturbation at the lower boundary is then set equal to the theoretical distribution for a pure Kelvin mode with \( \varepsilon = 45 \) m s\(^{-1}\). Thus (see Holton, 1975) we set

\[ A_s(\theta) = A_0 \exp(-\xi \theta/2), \]

(5.1)

with \( \xi = \theta(2\Omega / \varepsilon)^4 \). In the cases reported here we set \( A_0 = 20 \) m as the amplitude of the geopotential height perturbation at the lower boundary at the equator.

In the absence of mean wind shear the meridional distribution of the Kelvin wave response at all levels is given by (5.1), while the vertical wavenumber for the wave is \( \lambda_z = N/\varepsilon \). Thus, both horizontal and vertical scales depend only on the Doppler-shifted phase speed, not on frequency and zonal wavenumber separately. Furthermore, provided that the Doppler-shifted frequency \( \phi \) is greater than the rate coefficient for Newtonian cooling, the vertical decay scale \( |\lambda_z|^{-1} \), for the Kelvin wave is given approximately by

\[ |\lambda_z|^{-1} = \lambda_z^{-1}(\phi/\alpha). \]

Thus, the structure for a Kelvin wave of wavenumber \( s = 2 \) and period 9 days, for example, will be the same as that for a Kelvin wave with \( s = 1 \) and period 18 days, provided that the Newtonian cooling rate in the former case is twice the value in the latter case. Al-
though this correspondence holds strictly only for constant \( \bar{a} \), it should be approximately true even in the presence of mean wind shear. Therefore, solutions for a single set of parameters \( s, \omega, \alpha \) also apply for certain other combinations of the parameters.

In all the numerical results reported here the basic-state parameters of the model have been assigned the following values based on observed temperatures in the equatorial stratosphere: \( T_e = 220 \) K, \( H = 6.5 \) km, \( N^2 = 5 \times 10^{-4} \) s\(^{-2}\), \( dT_e/\!dz = 2 \) K km\(^{-1}\). Since Kelvin waves interact strongly with the mean wind in westerly shear zones, initial mean zonal wind profiles with westerly shear are chosen for the Kelvin wave simulations. Two cases are discussed in this section. In the first case (for convenience denoted as K1) the initial mean wind \( \bar{u}_{K1} \) is symmetric about the equator:

\[
\bar{u}_{K1} = U_0 (-1 + \tanh [(z-5)/2]) \exp [-(y/L)^2/2].
\] (5.2)

Here \( U_0 = 10 \) m s\(^{-1}\), \( L = 2000 \) km, and \( y \) and \( z \) are expressed in kilometers. The profile (5.2) has a very weak latitudinal dependence in the region of significant Kelvin wave amplitude near the equator, and a moderate vertical shear centered at \( z = 5 \) km (see Fig. 1 for the initial flow profile at the equator). The Gaussian dependence of \( \bar{u} \) on \( y \) in (5.2) is included to suppress possible noise which might be generated by large vertical mean wind shear near the lateral boundaries.

For the second case (K2) we superpose an initial mean flow which is antisymmetric about the equator onto the initial mean flow of case K1:

\[
\bar{u}_{K2} = \bar{u}_{K1} + U_0 (1 + \tanh [(z-7)/2])
\times (y/L_1) \exp [-(y/L_1)^2/2],
\] (5.3)

where \( U_0 = 10 \) m s\(^{-1}\), \( L_1 = 1000 \) km, and \( y \) and \( z \) are again expressed in kilometers. The mean flow field given by (5.3) is shown in Fig. 2. Note that the initial mean flow profiles at the equator are identical in cases K1 and K2.

In each of the cases K1 and K2 the time-dependent lower boundary forcing (3.9) was applied with the meridional distribution given by (5.1). The two cases were integrated forward in time to simulate the wave-mean flow interaction for a 60-day period. Fig. 1 shows the mean flow profile evolution at the equator for case K1. As expected the Kelvin waves are strongly damped in the westerly shear zone, and as a consequence there is a strong mean flow acceleration which both intensifies the shear zone and moves it downward. The equatorial mean flow evolution in case K2 is very similar except that the zone of mean wind acceleration is somewhat narrower and descends a little more rapidly. This aspect is illustrated in Figs. 3 and 4 which plot \( \partial \bar{a}/\partial t \) at the equator as a function of time for the two cases. As these figures indicate, during the initial 15–20 days the acceleration is spread throughout a wide layer. But, following this transient phase caused by the wave development, the acceleration becomes concentrated in a narrow 2 km thick zone which descends at a rate of about 1 km per month—a typical descent rate for the westerlies in the observed quasibiennial oscillation.

The most significant aspect of these integrations is shown in Fig. 5 in which the meridional distributions of \( \bar{a} \) and \( \partial \bar{a}/\partial t \) at day 60 are plotted. It is evident from the figure that, at least for the asymmetric initial profile of Fig. 2, the wave-mean flow interaction process does not increase the initial cross-equatorial shear, but rather tends to reduce it so that the flow becomes more symmetric about the equator as the westerlies
descend. This tendency of the interaction process to establish a symmetry about the equator is clearly shown in the mean flow acceleration pattern which is slightly shifted into the hemisphere where $\delta$ is larger. Thus, the mean flow acceleration pattern has an asymmetry which is opposite to that which would occur if the interaction were simply proportional to $\delta^{-1}$ as intuition might suggest.

This tendency of the waves to symmetrize the mean flow about the equator results from the modification in the structure of the wave fields due to the cross-equatorial mean wind shear. As Boyd (1978) has previously noted the Kelvin wave perturbation fields in the presence of horizontal shear are surprisingly little changed from the structures which would occur in the presence of vertical shear alone. This fact is illustrated by Fig. 6 which shows the perturbation zonal wind field for case K2 on day 60. The departure from symmetry about the equator is too small to be easily discernable in this type of diagram. In fact, the phase and amplitude distributions are very similar to those given by the asymptotic theory of Lindzen
(1971) for the mean wind profile at the equator. This tendency for the Kelvin wave distribution to be determined primarily by the mean wind field right at the equator was previously noted by Holton (1970).

The effect of the asymmetric horizontal mean wind shear is readily seen, however, in the structure of the meridional wind perturbation. In the symmetric mean wind case (K1) there is a small amplitude \( v' \) field (maximum velocity 0.4 m s\(^{-1}\)) which is antisymmetric about the equator. In the asymmetric mean wind case (K2) the meridional velocity perturbation field (Fig. 7) is doubled in strength and shifted almost entirely into the hemisphere where \( \delta \) is larger. Comparison of the phase lines in Figs. 6 and 7 reveals that the \( u' \) and \( v' \) fields are not in phase quadrature. In fact, there is a substantial southward directed horizontal momentum flux \( \langle \dot{u} \dot{v} \rangle < 0 \) which transfers momentum from the region of smaller \( \delta \) to the region of larger \( \delta \). It is this horizontal flux which accounts for the asymmetry of the \( \partial u/\partial t \) field shown in Fig. 5. Although the meridional velocity perturbation required to account for this flux is quite small, it is crucial for the overall wave-mean flow interaction process.

6. Mixed Rossby-gravity wave-mean flow interaction

The observed mixed Rossby-gravity waves in the lower equatorial stratosphere have periods in the range of 4–5 days and zonal wavelengths of order 10,000 km corresponding to zonal wavenumber \( s = 4 \). In the present study, therefore, we let \( s = 4 \) and choose a phase speed relative to the ground of \( c = -25 \) m s\(^{-1}\), yielding a period of about 4.0 days. The mean zonal wind at the lower boundary at the equator is fixed at \( +20 \) m s\(^{-1}\) so that the doppler shifted phase speed \( \bar{c} = -45 \) m s\(^{-1}\) at the lower boundary. The meridional distribution of the geopotential perturbation at the lower boundary is then set equal to the theoretical distribution for a pure mixed Rossby-gravity mode with \( \bar{c} = -45 \) m s\(^{-1}\). Thus (see Holton, 1975)

\[
A_s(\theta) = A_M \xi \exp(-\xi^2/2)
\]

with \( \xi = \theta[2\Omega(1+s\partial/2\Omega)]^{1/2} \), where \( \partial = \partial/\partial \theta \). In the cases discussed here we set \( A_M = 27 \) m which gives a latitudinally integrated generalized momentum flux (Holton, 1975) equal to 1.23 of that in cases K1 and K2. All basic state numerical parameters are the same as indicated in Section 5. The coefficient for Newtonian cooling is again set at a constant value of \( \alpha = 1/(7 \text{ days}) \). However, because the latitudinal scale of the wave perturbations is wider for the mixed Rossby-gravity waves studied here than for the Kelvin waves, the grid distance has been set at \( \Delta \theta = 4^\circ \) with boundaries at \( \theta_w = 48^\circ \).

Since mixed Rossby-gravity waves interact strongly with the mean wind in easterly shear zones the vertical shears in the initial mean wind profiles for the mixed Rossby-gravity wave cases have the opposite sign to those for the Kelvin wave cases. Again two cases are considered. In case M1 the initial mean flow profile, which is shown in Fig. 8, is symmetric about the equator and is nearly constant in latitude for \( |\theta| < 12^\circ \). This profile is given by the analytic expression

\[
\bar{u}_{m1} = \{U_0 - 3U_0 \tanh[(z-5)/2]\}[1 - (\theta/\theta_w)^s],
\]

where \( U_0 = 5 \) m s\(^{-1}\), \( s \) is in kilometers and \( \theta_w = 48^\circ \). In case M2 a mean flow field which is antisymmetric about the equator is superposed on the profile \( \bar{u}_{m1} \) to yield the mean flow shown in Fig. 9. The analytic
expression for this profile is

$$\tilde{a}_{M2} = \begin{cases} \tilde{a}_{M1} & \text{for } z < 6 \text{ km} \\ \tilde{a}_{M1}(1 - \theta/\theta_z) & \text{for } z \geq 6 \text{ km} \end{cases}$$

(6.3)

where $\theta_z = 12^\circ$ and $\theta$ is expressed in degrees. Note that the profiles (6.2) and (6.3) are equal at the equator and have the same latitudinal means at every level.

Mean flow profiles at the equator for days 0, 30 and 60 are shown for cases M1 and M2 in Figs. 10 and 11, respectively. The mean flow change at the equator is significantly larger in the M2 case, despite the fact that the initial cases differ only in the antisymmetric component of the mean wind field. Although the same tendency was present in the Kelvin wave cases (Figs. 3 and 4) it is much more pronounced for the mixed Rossby-gravity waves. The meridional profiles of the mean zonal flow acceleration fields for cases M1 and M2 at day 60 are shown in Figs. 12 and 13, respectively. In the symmetric case the acceleration is a maximum at the equator which is in agreement with the analytic solutions of Andrews and McIntyre (1976a) who showed that if the waves are damped by both thermal and mechanical dissipation or are growing in amplitude, the mean flow acceleration pattern changes from a distribution with separate maxima in each hemisphere.

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**Fig. 9.** As in Fig. 8 except for case M2.

**Fig. 10.** Mean zonal wind profile at the equator on days 0, 30 and 60 for case M1.

**Fig. 11.** As in Fig. 10 except for case M2.

**Fig. 12.** Mean wind and mean wind acceleration profiles for case M1 on day 60. Units: wind, m s$^{-1}$; acceleration, 10$^{-9}$ m s$^{-2}$. 
(the pattern which exists when only thermal damping is present) to the single equatorial maximum found in case M1.

In order to determine whether it is mechanical dissipation or transience which primarily determines the $\partial \bar{u}/\partial t$ profile, case M1 was rerun for 40 days with the vertical diffusion set to zero in the wave equations. (Vertical diffusion was retained, however, in the zonal mean momentum equation.) The latitudinal profile of the mean flow acceleration in this case was almost identical to that of the mechanically damped case. Thus, in this model wave transience effects apparently control the latitudinal profile of the mean flow acceleration.

In the asymmetric mean wind case M2 the meridional profile at day 60 (Fig. 13) shows that just as in the K2 case the wave-mean flow interaction process causes the most rapid acceleration to occur on the side of the equator where $|\varepsilon|$ is larger. Thus, the mean flow tends to be symmetrized as the easterly shear zone descends. An interesting aspect of case M2 shown in Fig. 13 is that a critical level ($\varepsilon=0$) has formed in a small region centered north of the equator at about 24 km. The critical level inhibits the upward propagation of the waves in this region and thus produces a minimum in the mean wind acceleration field above the critical level. The meridional profiles of the amplitudes of the perturbation wind components $u'$ and $v'$ as well as the $\bar{u}$ and $\partial \bar{u}/\partial t$ distribution for $z=24$ km in cases M1 and M2 on day 60 are shown in Figs. 14 and 15, respectively. Clearly, there are substantial differences in the perturbation fields. For the M1 case the fields are nearly identical to those given by the asymptotic theory of Boyd (1978). For case M2 the $u'$ and $v'$ fields are qualitatively in agreement with the asymptotic theory; however, the meridional profiles in this case tended to change with time in an irregular fashion during the course of the 60-day simulation, indicating that the wave motion in this case was inherently unsteady. Analysis of a few other simulations for mixed Rossby-gravity wave-mean flow interaction with a forced wave of zonal wavenumber $s=2$ indicated, however, that the waves always act to reduce the asymmetric component of the mean wind shear, regardless of the details of the particular mean profile used.

7. Conclusions

We have shown using a numerical model that forced equatorial waves interact with the mean flow to produce equatorial jets characterized by downward moving westerly (Kelvin wave forcing) and easterly (mixed Rossby-gravity wave forcing) shear zones, respectively.
Furthermore, these results indicate that, for parameters characteristic of the observed waves in the equatorial stratosphere, the wave-mean flow interaction process always reduces the amplitude of any initial cross-equatorial mean wind shear. Thus, the mean flow profile tends to become symmetric about the equator as the interaction process continues. This "symmetrizing" tendency of the Kelvin and mixed Rossby-gravity waves may help to explain the remarkable repeatability of the observed quasi-biennial oscillation in the presence of time-varying latitudinal mean wind shear due to the annual cycle.

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