Photochemical Damping of Inertio–Gravity Waves

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ABSTRACT

A simple model is developed for the coupling effect of radiative heating and ozone photochemistry on inertio-
gravity waves in the region 16–70 km. It is found that the photochemical damping rate is the weighted sum of
the rates caused by two individual processes. One, suggested by Craig and Ohring, involves the temperature
dependence of ozone reaction rate coefficients; the other, suggested by Leovy, involves the vertical gradient of
the ozone mixing ratio. The relative importance of these two processes depends on the ratio of dynamical and
chemical relaxation times. Calculations show that the most important region for coupling is near 48 km where
the total relaxation time is 8 days for equinox and 6 days for summer midlatitude.

1. Introduction

The influence of temperature dependent ozone photochemistry on the relaxation rate for temperature pertur-
bations in the stratosphere has been studied by many authors. Craig and Ohring (1958) first pointed out that
the marked temperature dependence of ozone reaction coefficients should have a stabilizing effect on both
the temperature and ozone concentration perturbations in the upper stratosphere. Since the ozone destruction rate
coefficient increases and the generation rate coefficient decreases as temperature increases, both tend to create
negative correlations between ozone caused heating and temperature perturbations. Lindzen and Goody (1965)
carried out a photochemical–radiative relaxation model for pure oxygen photochemistry, which was extended
by Blake and Lindzen (1973) to include catalytic hydroxogen and nitrogen chemistry. Their results showed
that for disturbances of large vertical scale damping times were reduced from the purely radiative values of
about 10 days at 35 km and about 5 days at 50 km to 3–7 days at 35 km and to 1.5–2.5 days at 50 km. Hart-
mann (1978) pointed out that the thermal relaxation rates computed by Blake and Lindzen in the 30–40
km altitude range were too large by nearly a factor of 2 for large vertical scale disturbances, since Blake and
Lindzen neglected the shielding effect of the variation in column ozone above a given level. This correction
becomes small above 45 km. Shielding effects were also considered by Strobel (1977, 1978), who found that
for some values of the parameters photochemical acceleration of the thermal relaxation rate may reduce to
zero. Observations have confirmed that the thermal relaxation rate near 45 km is 0.35 day\(^{-1}\), which nearly
doubles the Newtonian cooling rate (0.18 day\(^{-1}\)) for large vertical scale disturbances (Ghazi et al., 1979).
However, all these works are concerned with the relaxation of large scale ozone and/or temperature pertur-
bations.

For upward traveling gravity waves with their short dynamical time scales, local changes of ozone concen-
tration caused by vertical advection in the regions of strong vertical gradients in ozone provide a coupling
between ozone and dynamics in addition to the temperature dependent photochemistry that has been con-
sidered by previous works. For these waves, correlations between heating and temperature perturbations are
determined by the vertical gradient of ozone and vertical parcel displacements separately. These correlations
could be positive or negative depending on whether the ozone mixing ratio increases or decreases with height.
Though Lindzen and Goody (1965) included this factor in their original linearized equations, it was
Leovy (1966) who first pointed out that photochemistry could damp or enhance upward propagating gravity
waves, when he studied gravity wave destabilization near the mesopause where atomic oxygen increases
with height.

In this paper we will combine the effects of temperature-dependent photochemistry and vertical advection
of ozone in a simple model of upward traveling inertio-
gravity waves, and examine the photochemical damping (and/or enhancing) of such waves by ozone in the
stratosphere and low mesosphere.

2. Model

The model equations are linear and are essentially the same as those of Lindzen and Goody (1965). The heat
source is proportional to the ozone mixing ratio, which is determined by dynamical and photochemical processes:
\[
\frac{\partial u'}{\partial t} + \frac{\partial u'}{\partial x} - \beta u' + \frac{\partial \phi}{\partial x} = 0,
\]

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where \( r' = ([O_3]/[M])' \) is the perturbation of the volume mixing ratio of ozone. For further detail concerning the model and notation see also Holton (1975). Here terms \( Ar' \) and \(-C\phi_z'\) represent the coupling between radiative and photochemical processes. We have omitted a term \( A' r \) (3) which comes from the shielding effect (Hartmann, 1978). We will see that the shielding effect is not important to traveling gravity waves. Assuming the existence of wave solutions of the form

\[
y = \left( \frac{1}{\omega} \right) \exp \left( i(kx + ly - \omega t) + \frac{z - z_0}{2H} \right)
\]  

and slowly varying mean states, we can derive dispersion relation for vertical wavenumber \((m)\)

\[
m^2 = \frac{K^2(N^2 + \tilde{A}r_f)}{(\omega^2 - f^2)[1 + i(\alpha + AC)/\tilde{\omega}]} - \frac{1}{4H^2},
\]

where

\[
\tilde{A} = (B + i\tilde{\omega})A^*, \quad A^* = \tilde{A}/(B^2 + \tilde{\omega}^2)
\]

and \( K^2 = k^2 + f^2. \) Note that \((2Hm) < 0.1\) when the vertical wavelength is less than 25 km. We thus conclude that the stratification effect is mainly included in the factor \( \exp\left( (z - z_0)/2H \right) \) of the wave solutions (6) and the last term in (7) is negligible.

In the real atmosphere we will see that \( BCA^* \ll |\tilde{\omega}| \) and \( \alpha \ll |\tilde{\omega}|; \) with these assumptions in (7), we get

\[
m^2 \approx \frac{K^2}{(\omega^2 - f^2)} \left\{ N^2 \{1 - (\alpha + CA^*)/(B + \alpha)\} \right\}
\]

\[
m_1 \approx \frac{K^2}{2m_1(\omega^2 - f^2)} \times \left[ \tilde{\omega}r_fA^* - \frac{(N^2 + Br_fA^*)(\alpha + BC^A^*)}{\tilde{\omega}} \right].
\]

These expressions determine the main properties of inertio–gravity waves in midlatitudes. The Doppler shifted frequency has a cutoff at \( |\tilde{\omega}| = f \sim 10^{-4} \text{ s}^{-1} \), which restricts the Doppler shifted period \( 2\pi/|\tilde{\omega}| \) to be less than one day.

The coefficient \( \tilde{A} \) is proportional to the heating due to absorption of solar radiation by ozone. It can be expressed in present notation as (Lindzen and Goody, 1965)

\[
\tilde{A} = \frac{R^2c_2J_3}{C_pH\lambda}
\]

where \( C_p \) is the specific heat at constant pressure, \( J_3 \) the photolysis rate of ozone, \( \lambda \) the averaged wavelength of the solar radiation absorption and \( c_2 \) the second radiation constant

\[
c_2 = \frac{hc}{k} = 0.0144 \text{ mK} \quad (k \text{ is Planck's constant here}).
\]

Figure 1 gives \( \lambda \) as a function of path length \( u_3 \) calculated by integrating important ozone bands. The curve can be fit very well (relative errors < 1%) by the following empirical formulae

\[
\lambda = 0.5(\lambda_1 + \lambda_2) + 1.1(\lambda_1 - \lambda_2)\frac{S}{1 + S^2}
\]

where

\[
\lambda_1 = 0.27 + \frac{u_3^{0.995}}{1.2 \times 10^{19} + 3.1u_3^{0.995}} \quad (14a)
\]

\[
\lambda_2 = 0.27 + \frac{u_3^{1.006}}{1.2 \times 10^{19} + 3.1u_3^{1.006}} \quad (14b)
\]

\[
S = \log_{10} u_3 - 18.7 \quad (14c)
\]

in which \( u_3 \) is in \( \text{cm}^{-2} \) and \( \lambda \) in microns.

In order to estimate the coefficients \( B \) and \( C \), we consider for simplicity the following photochemical model for ozone:

\[
O_2 + h\nu \rightarrow O + O, \quad J_2
\]

\[
O + O_2 + M \rightarrow O_3 + M, \quad k_{12}
\]
O + O₃ → O₂ + O₂, \( k_f^3 \) (17)
O₃ + hv → O₂ + O₂, \( J_3 \), (18)

As in Hartmann (1978), we have included reactions involving nitrogen, hydrogen and chlorine compounds in (17) by adjusting \( k_f^3 \). Here we adjust \( k_f^3 \) by multiplying the pure oxygen destruction rate \( k_{11} \) by an amplification factor \( e \), which is the ratio of the total ozone destruction rate including the catalytic cycles to the destruction rate by pure oxygen.

We note that below 70 km the photochemical lifetime of odd oxygen (O₃ + O) is much longer than those of ozone and atomic oxygen (Brasseur and Solomon, 1984) and thus the partition between odd oxygen species is determined by two fast reactions (16) and (18)

\[ J_3[O_3] = k_{12}[O][O_2][M]. \] (19)

Thus, for periods longer than a few minutes the partition between \([O_3]\) and \([O]\) is given by (19) and the rate of change of \([O_3]\) is governed by (Lindzen and Goody, 1965; Brasseur and Solomon, 1984)

\[ \frac{d}{dt} \frac{[O_3]}{[M]} = \frac{dr}{dt} = \left(2J_3 \frac{[O_3]}{[M]} - \frac{2k_f^3 J_3 r^2}{k_{12}[O_2]}\right)(1 + p)^{-1}, \] (20)

where

\[ p = \frac{[O]}{[O_3]} = \frac{J_3}{k_{12}[O_2][M]} \] (21)

is the partition ratio in odd oxygen.

In the present problem it is assumed that the inverse damping time is much smaller than the local Doppler-shifted frequency, so that the motion can be considered as adiabatic over one cycle as the waves propagate vertically. We thus have

\[ \frac{[O_3]}{[O_2]} = \frac{[M]}{[M]} = \rho/\bar{\rho} = \left(1 + \frac{C_v}{R} \frac{T'}{T}\right), \] (22)

where \( \rho \) is air density, \( T \) is temperature and \( C_v \) the specific heat at constant volume. This assumption is different from that of Lindzen and Goody (1965) where isobaric (rather than adiabatic) variation of density was assumed since they were interested in the long time scales of planetary scale perturbations. Linearizing Eq. (20) gives coupling coefficients \( B \) and \( C \):

\[ B = 4 \frac{J_3}{[O_2]} G(T) \bar{r}(1 + p)^{-1}; \]
\[ C = \frac{B H F}{2RT^2} \left(- \frac{d}{d(1/T)} \frac{d}{d(1/T)} \frac{C_v}{R} \bar{r}\right) \] (23)

where

\[ G(T) = \frac{k_f^3}{k_{12}} e e^E \left(\frac{T}{300}\right)^n \exp(-D/T). \] (24)

In deriving (23), we have omitted very small contributions by \( p' \) and thus assumed that \( p = \bar{p} \). On the basis of recent data (NASA/JPL, 1983), we let \( E = 1.333 \times 10^{28} \) molecules m⁻³, \( n = 2.3 \) and \( D = 2218 \) K in (24); the amplification factor \( e \) is calculated from Brasseur and Solomon (1984). The temperature dependence of \( G \) is reduced when catalytic cycles are included. Haigh and Pyle (1982) suggest that the following expression can be used

\[ \frac{d}{dT} \left(\frac{d}{d(1/T)} \frac{d}{d(1/T)} \frac{C_v}{R} \bar{r}\right) = \frac{a + b f_1 + c f_2 + d f_3}{2 + f_1 + f_2 + f_3} \] (25)

where \( f_1, f_2 \) and \( f_3 \) are the ratios of the rate destruction of odd oxygen due to HO₃, NOx, and ClOx, respectively, to the rate of destruction by the pure oxygen reactions. The other constants are \( a = 2810 \) K, \( b = 510 \) K, \( c = 1200 \) K. The correction by ClOx is very small (\( d < 275 \) K) and can be neglected.

3. Results

Given the vertical mean ozone distribution we can calculate \( \bar{r} \) from the solar heating rate by ozone. In this model we calculate \( \bar{r} \) by using the parameterization scheme suggested by Schoeberl and Strobel (1978) for the Hartley, Huggins and Chappius bands. With (11)–(14) we can get \( J_3 \) which, together with the mean temperature distribution, gives coefficients \( B \) and \( C \) from (23). Here we choose the midlatitude ozone model from the U.S. Standard Atmosphere, 1976 (Fig. 2) as our midlatitude equinox ozone distribution. Since there is still some uncertainty about seasonal ozone changes above 30 km and, in addition, observations show that changes of ozone in the midlatitude upper stratosphere between summer and equinox are much smaller than

![FIG. 2. Volume mixing ratio of ozone (solid line, in 10^-4) and its vertical derivative (dashed line, in 10^-10 m^-1). [From U.S. Standard Atmosphere, 1976.]](image-url)
between winter and equinox (Frederick et al., 1983), we also calculate these coefficients for the summer season using the same ozone distribution model for comparison. In calculations, diurnal averaging is accomplished by averaging the \( A \) and thus \( J_2 \) over eight zenith angles spaced between midnight and noon.

In Eqs. (9) and (10), which determine the wave’s main features, modifications by ozone photochemical processes may be classified as terms including \( \bar{F}_z \) and terms including \( C \). Terms with \( \bar{F}_z \) represent effects of ozone advection by displacement (hereafter called process 1). Terms with \( C \) represent acceleration of thermal damping owing to temperature dependence of reaction rates (hereafter called process 2). In both these terms vertical wavenumber \( (m_r) \) and wave amplitude may be corrected. In the case of \( \bar{F}_z < 0 \) an ascending parcel will contain more ozone than its environment and will be warmed diabatically and a descending one will be cooled (Fig. 3). This tends to increase the static stability of the mean state \( (N^2) \) and depress the wave amplitudes. On the other hand, if \( \bar{F}_z > 0 \), process 1 will decrease the static stability and increase the wave amplitude. Noting that \( G \), which increases with temperature, is the ratio of the rate coefficients for ozone destruction and generation, we can conclude that process 2 always decreases the static stability and depresses the wave amplitudes (Fig. 4). This damping effect is partly canceled by density variation since the ozone generation rate is proportional to the square of the air density which decreases (increases) as the parcel adiabatically cools (heats).

All these conclusions are implicitly included in replacing \( N^2 \) by \( N^2 + A \bar{F}_z \) and \( \alpha \) by \( \alpha + \bar{A} \) if we compare the modified dispersion relation (7) with the classic one (e.g., Gill, 1982). For small time-scale ozone perturbations, which are adjusted to equilibrium mainly by reactions (16) and (18), the damping process 2 is not important. On the other hand, process 2 becomes important as the time scale of perturbations becomes so large that thermal relaxation is mainly through reactions (15) and (17). So for planetary time-scale perturbations the damping by process 2 is usually important; for midlatitude gravity waves the photochemical damping process 1 is usually important.

The shielding effect discussed by Hartmann (1978) is produced by negative correlations between \( r' \) and \( A' \). It is not important for upward propagating gravity waves. The reasons are as follows: First, for the two processes, the variations in column ozone above a given level are reduced because of the cancellations between wave peaks and valleys. Second, the shielding effect may cause negative or positive correlations at that level depending on the wave phases. As the waves propagate upward, they may also cancel each other.

In the real atmosphere we have \( \alpha < 5 \times 10^{-5} \text{ s}^{-1} \) (Fels, 1982, 1984) and \( z^2 > f^2 \sim 10^{-1} \text{ s}^{-2} \) for midlatitude gravity waves. Under these conditions we may show that \( CA^* < 10^8 CA \text{ s}^{-2} \ll 1 \) and \( |\alpha \bar{F}_z A^*|, |B \bar{F}_z A^*| < 4 \times 10^{-3} \text{ s}^{-2} \ll N^2 \).

Thus, (9) and (10) may be simplified further in the range 16–70 km as

\[
m_r \approx \frac{-NK}{\omega(1-f^2/\omega^2)^{1/2}};
\]

(26)

\[
m_i \approx \frac{-K^2 N^2}{2m \omega(\omega^2 - f^2)} \left\{ \frac{\omega^2 \bar{F}_z A^*}{N^2} - C B A^* + \alpha \right\} \]

(27)

where we choose the negative sign for upward propagating gravity waves. Equations (26) and (27) show that there is no significant photochemical influence on vertical wavelength; however, the amplitude of upward propagating waves may be enhanced or damped. The influences of photochemical enhancement or damping

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**Fig. 3.** Photochemical damping by process 1. Vertical advection of ozone gives a negative correlation between ozone caused heating and temperature perturbations as ozone mixing ratio decreases with height.
on these waves may be comparable to the Newtonian cooling effect. The photochemical damping coefficient is given by

$$\alpha_p = \alpha_{p1} + \alpha_{p2}$$

(28)

where

$$\alpha_{p1} = \frac{\omega^2 F_{p1} A^*}{N^2} = -\frac{F_{p1} A}{N^2 (1 + (B/\omega)^2)}$$

(29)

is the damping caused by process 1 and

$$\alpha_{p2} = CBA^* = \frac{C\tilde{A}}{B\{1 + (\omega/B)^2\}$$

(30)

is the damping caused by process 2. Now the total damping coefficient $\alpha_t$ is given by

$$\alpha_t = \alpha_p + \alpha_N,$$

(31)

where $\alpha_N$ is the Newtonian cooling coefficient.

From (29) and (30) we see that photochemical damping coefficients are dependent on the Doppler-shifted frequency, or more precisely on the ratio of the dynamical and chemical relaxation times $B/[\omega]$. In the limiting case when $B^2 \ll \omega^2$, process 1 dominates and

$$\alpha_p \approx \alpha_{p1} = -\frac{F_{p1} A}{N^2}.$$

(32)

On the other hand, when process 2 dominates ($\omega^2 \ll B^2$)

$$\alpha_p \approx \alpha_{p2} = \alpha_B/B.$$

(33)

In general, from (28)–(30) we see that photochemical damping coefficient is the weighted average of these two limiting process coefficients, i.e.,

$$\alpha_p = w_1\alpha_{p1} + w_2\alpha_{p2}$$

(34)

where $w_1 + w_2 = 1$.

Figures 5 and 6 give these two limiting photochemical damping coefficients for equinox and summer seasons, respectively. We see that they both reach the same order of maximum value in the region of 46–52 km, where the photochemical relaxation time is about 8 days for the equinox and about 6 days for the summer. Note that (34) implies that $\alpha_p$ always lies within the region bounded by $\alpha_{p1}$ and $\alpha_{p2}$. Thus, the photochemical damping coefficient in the region 46–52 km is approximately independent of the Doppler-shifted frequency.
in midlatitude we would expect, as a rule, that damping by process 1 is more important than damping by process 2.

Below 50 km, where there is little gravity wave breaking (Lindzen, 1981, 1985), Newtonian cooling is usually considered to be an important damping factor. It is well known that the Newtonian cooling coefficient is scale-dependent for radiative processes that are non-local (Wehrbein and Leovy, 1982; Fels, 1982, 1984). For upward traveling gravity waves, however, vertical wavelength and frequency are related by (26) and thus Newtonian cooling is frequency dependent for given gravity waves. In the upper stratosphere, it may vary from 0.1 to 1.0 day\(^{-1}\) and the cooling coefficient for typical gravity waves of 10 km vertical wavelength is about 0.3–0.5 day\(^{-1}\) (Fels, 1982). We thus conclude that photochemical damping may give a significant correction (with 25% ∼ 50% increases for the typical gravity waves) to the radiative damping rate represented by the Newtonian cooling coefficient.

4. Conclusions

Photochemical damping of gravity waves in the middle atmosphere is examined by using a simple model. The calculated maximum photochemical damping rate coefficients are about \(\frac{1}{9}\) to \(\frac{1}{6}\) day\(^{-1}\) in the region of 40–50 km. Thus, photochemical damping near the stratopause is usually of secondary importance compared to Newtonian cooling damping; but it is not negligible. The damping is caused by the vertical gradient of the ozone mixing ratio (process 1) and the temperature dependence of ozone reaction coefficients (process 2). The relative importance of these two processes depends on the ratio of dynamical and chemical relaxation times. For midlatitude gravity waves, process 1 usually dominates since the wave propagation requires \(\omega^2 > f^2\).

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