

Parameterizations of Single Scattering Properties of Frozen Hydrometeors at Millimeter-wave Frequencies

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1. Introduction

Physical approaches in microwave remote sensing to measure non-spherical frozen hydrometeors are frequently parameterized as spherical particles, assuming either dielectric mixing approximation or equivalent spheres, to make Mie-theory applicable. However, the applicability of those simplified approximations in millimeter-wave radar and radiometric remote sensing of frozen hydrometeors need to be evaluated.

To seek a parameterization to represent the electromagnetic (EM) properties of frozen hydrometeors at millimeter-wave frequencies, this study analyzes the Discrete Dipole Approximation (DDA) method calculated single scattering from nonspherical snow crystals at millimeter-wave frequencies (95, 140, 183, 220, and 340 GHz). Sizes, shapes, density, and refractive indices for five snow crystal models (hexagonal column, hexagonal plate, sector plate, planar rosette, and spatial rosette) employed in this study are described in [1]. The results shown in this study assume that all particles are randomly oriented.

By comparing with the DDA calculated single scattering properties, this study evaluates fast parameterization methods to represent the EM properties of snow at millimeter-wave frequencies. Dielectric mixing theory, which is widely used by the microwave remote sensing community, and the equivalent sphere approach, which is commonly used by infrared (IR) and ultraviolet (UV) remote sensing community are considered. In addition, equivalent cylinders and equivalent ellipsoids are also tested to examine how much the single scattering properties depend on the detailed shapes of ice crystals.

2. Discrete Dipole Approximation (DDA) method

The DDA is a flexible technique for calculating the EM scattering and absorption by particles with arbitrary shapes and composition. The DDA approximates the actual particle by an array of the dipoles. Each of the dipoles is subject to an electric field which is the sum of the incident wave and the electric fields due to all of the other dipoles. Through the solution of the electric field at each dipole position, the scattering and absorption properties of the target can be obtained. Because the DDA replaces the solid particle with an array of point dipoles occupying positions on a cubic lattice, and the lattice spacing must be small compared to the wavelength of the incident light in the particle, the DDA requires large computer storage and

CPU time. This study employs the DDA code developed by [2]. The DDA code is known to work well for materials with $|m|kd < 1$ where m is the complex refractive index of the target material and $k \equiv 2\pi/\lambda$, where λ is the wavelength in vacuum and d is the spatial interval of dipoles.

3. Approximation with equivalent spheres

Models to calculate absorption and scattering of light by nonspherical ice crystals in cloud and snow commonly represent the crystals by equivalent spheres because of their ease of use and the saving in computation time. Three choices of equivalent spheres have been used ([3], [4]). A nonspherical particle may be represented by a sphere of the same volume, V , or by a sphere of the same surface area, A , or by a collection of spheres with the same volume-to-surface area ratio, V/A . Fu et al. [5] showed that the equal- V/A prescription was superior to the other two presentations at IR and UV frequencies. For weakly absorbing particles the absorption is proportional to the volume, but the scattering is proportional to the surface area. The equal- V/A spheres have the same volume and surface area as nonspherical particles by abandoning the requirement that the number of nonspherical particles (n) is same as the number of equivalent spheres (n_s). For a particle of volume V and surface area A , the equivalent V/A spheres have radius of

$$r_{V/A} = 3 \frac{V}{A} \text{ and number of } \frac{n_s}{n} = \frac{3V}{4\pi r_{V/A}^3}.$$

These equivalent sphere approximations have been widely used in IR/UV remote sensing community but those accuracies and applicability have not yet been evaluated at millimeter-wave frequencies. Therefore, we compare applicability of equivalent sphere approximations at infrared and at millimeter-wave frequencies. Figure 1 and Figure 2 show the single scattering parameters for hexagonal columns with aspect ratio of 1/5 at IR and millimeter-wave frequencies, respectively. The thicknesses of columns are 20 μm and 500 μm at IR and millimeter-wave frequencies, respectively, to make size parameter ranges similar in comparisons. The real (imaginary) parts of dielectric constants for ice crystals at infrared wave frequencies range between 1.2 and 1.9 (between 0.05 and 0.55) while real (imaginary) parts of dielectric constants for ice crystals at millimeter-wave frequencies range between 1.78 and 1.79 (0.006-0.016).

The results show that the equal- V/A spheres represent the scattering and extinction cross-sections significantly better

than the equal-V or equal-A spheres at IR frequencies while they generated smaller asymmetry factor. Equal-V spheres generated more consistent asymmetry factor to the DDA results for this case (Figure 1).

Equal-V/A spheres presented the extinction cross sections reasonably at millimeter-wave frequencies except resonance regime (Figure 2). It should be noted that all equivalent spheres significantly overestimated the resonance peaks of extinction cross sections. This is attributable to surface waves in spherical particles. Considering snow crystal sizes ranging 0.25 mm and 4 mm, the millimeter-wave frequencies used in this study include the resonance regimes while IR or UV frequencies are in geometric optics regime which suffers less constructive interference effects that are sensitive to the shape and size of a particle.

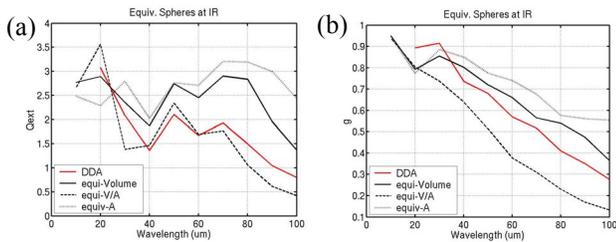


Figure 1. Comparisons of the DDA calculated (a) extinction efficiency and (b) asymmetry factors for hexagonal columns with equivalent sphere results at IR wavelengths.

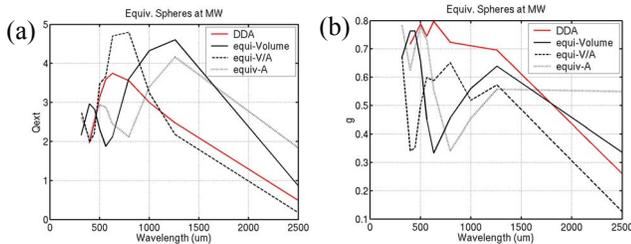


Figure 2. Comparisons of the DDA calculated (a) extinction efficiency and (b) asymmetry factors for hexagonal columns with equivalent sphere results at millimeter-wave wavelengths.

Figure 3 shows comparisons of scattering properties for hexagonal disks, hexagonal columns, planar rosette, and spatial rosettes calculated with the equivalent sphere approximations and the DDA. Equal-V/A and equal-V spheres can reasonably well present the extinction efficiency while equal-V/A spheres (equal-V and equal-A spheres) show significantly smaller (larger) asymmetry factor than the DDA results.

4. Approximation with equivalent cylinders

As an attempt to use a simpler geometry to approximate a more complex geometry, Lee et al. [6] examined whether a hexagonal geometry can be approximated by a circular cylinder at IR wavelengths. Their motivation was to use the T-matrix method, which only can be applied to particles with rotational symmetry and has much better computational

efficiency than the DDA method. However, the T-matrix is known not to give a converged solution for particles with extreme aspect ratio [7]. Based on quick tests with most popularly used T-matrix code ([7], [8]), we found that the aspect ratios of observed ice crystal in falling snow and cirrus clouds ([9],[10],[11],[12]) commonly have extremely large or small values beyond the aspect ratio limits that the T-matrix code allows. In case the cylindrical column, the limit of aspect ratio that the T-matrix code can give the converged solution was 0.4. It should be noted that the lower limit of aspect ratio used by Lee et al. [6] was also 0.4 which is much larger than the ice crystals in clouds (~0.2 in [10],[11])

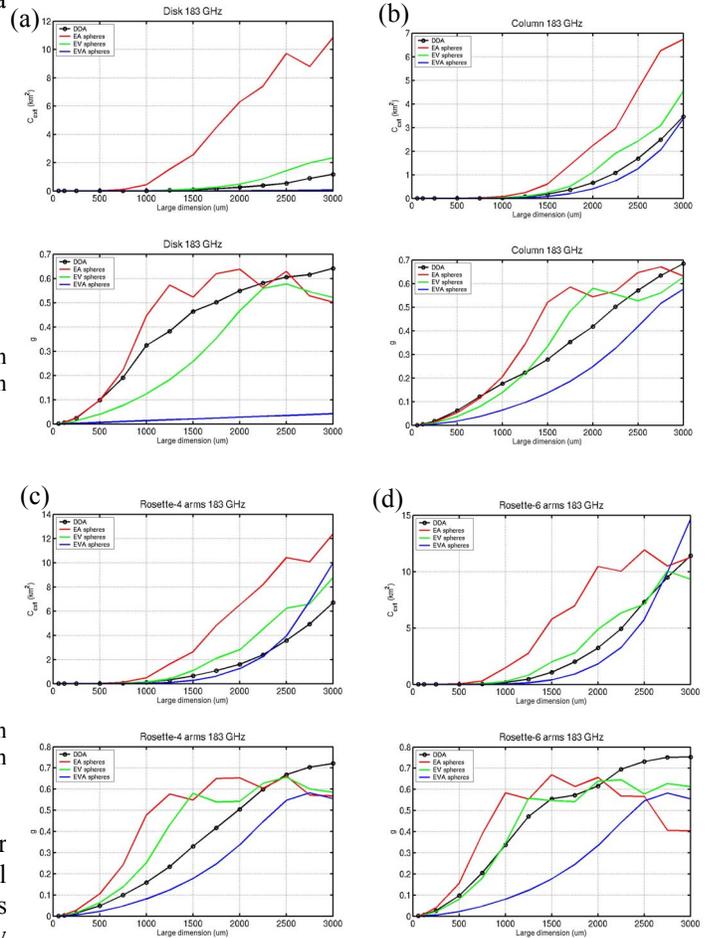


Figure 3. Comparisons of the DDA calculated extinction cross sections and asymmetry factors for (a) hexagonal disks, (b) hexagonal columns, (c) planar rosette, and (d) spatial rosettes with equivalent sphere results at 183 GHz.

Even we cannot benefit the computational efficiency of T-matrix method, we still can save computational time significantly even with the DDA method if the detailed edge is not important and the hexagonal prism can be approximated as spherical cylinders.

To define the equivalence of a circular cylinder and a hexagonal column in scattering calculations, the two particles are assumed to have the same aspect ratio, that is, $a/L = R/H$,

where R and H are the radius of the cross section and the length of a circular cylinder, respectively and a and L is the semi-width of the cross section and length of a hexagonal column. For this condition the cross-sectional radius of a circular cylinder with an equivalent volume, an equivalent projected area, or an equivalent ratio of volume to projected area is given by $R_v = \left(\frac{3\sqrt{3}}{2\pi}\right)^{1/3} a$, $R_A = \left(\frac{3\sqrt{3}a + 6L}{2\pi(a+L)}\right)^{1/2} a$, $R_{vA} = \frac{\sqrt{3}(a+L)}{\sqrt{3a+2L}} a$, respectively.

The lengths associated with these radii are given by

$$H_v = \left(\frac{3\sqrt{3}}{2\pi}\right)^{1/3} L, H_A = \left(\frac{3\sqrt{3}a + 6L}{2\pi(a+L)}\right)^{1/2} L, H_{vA} = \frac{\sqrt{3}(a+L)}{\sqrt{3a+2L}} L, \text{ respectively.}$$

Figure 4 shows a comparison of single scattering parameters at 340 GHz frequency for hexagonal ice columns and the circular cylinders of same dielectric constants. Evidently, the sharp edges of side faces of hexagonal geometry are not important in specifying the electromagnetic scattering properties for snow crystals at millimeter-wave frequencies. Equal-V (equal-V/A) cylinders generated more (less) consistent scattering properties with the DDA results. Results show that, if real ice particles happen to be hexagonal, they may be approximated by circular cylinders at millimeter-wave frequencies.

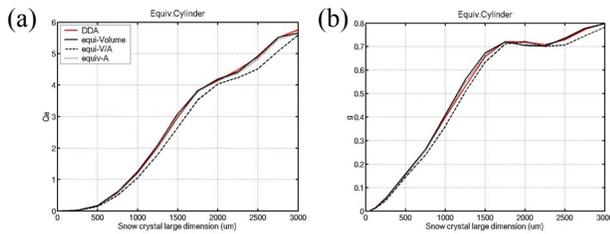


Figure 4. Comparisons of the DDA calculated (a) extinction cross sections and (b) asymmetry factors for hexagonal columns with equivalent cylinder results at 340 GHz.

5. Approximation with equivalent ellipsoids

Another possible attempt to use a simpler geometry to approximate a more complex geometry is approximating a hexagonal geometry by a ellipsoids. Macke and Mishchenko [13] reported the similarities and differences in light scattering by axis-equivalent, regular, and distorted hexagonal cylindrical, ellipsoidal, and circular cylindrical ice particles with sizes much larger than wavelength using the geometric optics approximation. They found that at a nonabsorbing wavelength of 0.5 um, ellipsoids (circular cylinders) have a much (slightly) larger asymmetry parameter than regular hexagonal cylinders. They suggested that the scattering by regular particle is not necessarily representative of real atmospheric ice crystals at nonabsorbing wavelengths.

Three equivalent ellipsoids are employed: equal volume-aspect ratio spheroid, equal volume and length spheroid, and equal axes ellipsoid. The minor (x) and major axes (h) of equal

volume and aspect-ratio spheroid are given by $x = \left(\frac{9\sqrt{3}}{8\pi}\right)^{1/3} a$

and $h = \left(\frac{9\sqrt{3}}{8\pi}\right)^{1/3} L$, respectively. The minor(x) and major axes

(h) of equal volume and length spheroid are given by $x = \left(\frac{9\sqrt{3}}{8\pi}\right)^{1/2} a$ and $h=L$, respectively. Following Macke and

Mishchenko (1996), the three axes (x1, x2, h) of equivalent-axes ellipsoid are given by $(x1, x2, h) = \left(\frac{\sqrt{3}}{2} a, a, \frac{L}{2}\right)$.

Figure 5 shows a comparison of single scattering properties for equivalent ellipsoid approximations and for the hexagonal column calculated by the DDA method. The results show that the equivalent ellipsoid approximations underestimate the single scattering properties including asymmetry parameter at millimeter-wave frequencies. This is opposite what Macke and Mishchenko [13] at optical frequencies that the equivalent ellipsoid approximations generated larger asymmetry parameters than the hexagonal columns.

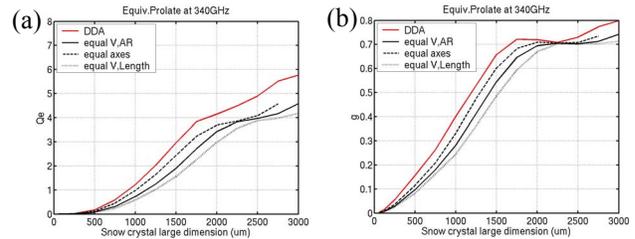


Figure 5. Comparisons of the DDA calculated (a) extinction cross sections and (b) asymmetry factors for hexagonal columns with equivalent ellipsoids results at 340 GHz.

6. Approximations with dielectric mixing theory

O'Brien and Goedecke [14] computed scattering cross sections of a dendrite snow crystal by approximating it as a homogeneous circular disk with a diameter equal to the maximum linear extent of the dendrite and an equal thickness, as an enveloping oblate spheroid with an axial ratio equal to the maximum aspect ratio of the dendrite, and as an oblate spheroid with the same aspect ratio and maximum linear dimensions as the dendrite. The effective refractive indices calculated by several mixing rules were applied and found that the Bruggemann mixing rule produced the best match. This study applies O'Brien and Goedecke [14] approaches to sector plates (Figure 6) and spatial rosette (Figure 7) for all size ranges between 0.25 mm and 3 mm.

Figure 6 shows comparisons of scattering properties for sector plates with fluffy cylinders and oblate spheroid results calculated with the DDA method. The oblate spheroids with the same aspect ratio and maximum linear dimensions as the sector plate represent scattering properties most consistently with the sector plates. This is consistent with the findings by O'Brien and Goedecke [14].

This approach is tested with spatial rosette by approximating it with fluffy spheres which have same large dimensions. Single scattering result comparisons (Figure 7) shows that it is not possible to extend the conclusions made for the sector plate to the spatial rosettes. The dielectric mixing approaches employed by the O'Brien and Goedecke for a dendrite seem not applicable to generate reasonable single scattering properties especially for complex snow flakes including large fraction of air inside. The inappropriateness of applying effective medium mixing theories to irregularly shaped hydrometeors with size parameter of large structural inhomogeneities was also cited by [15].

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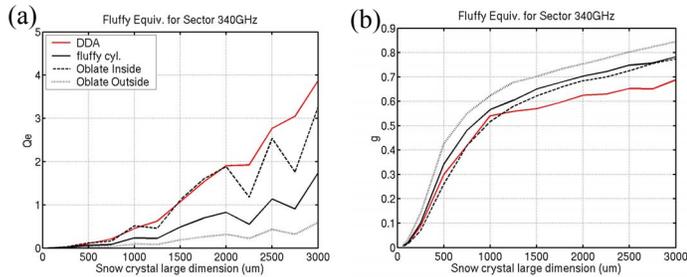


Figure 6. Comparisons of the DDA calculated (a) extinction efficiency and (b) asymmetry factors for sector plates with results from equal mass fluffy cylinders and oblates at 340 GHz.

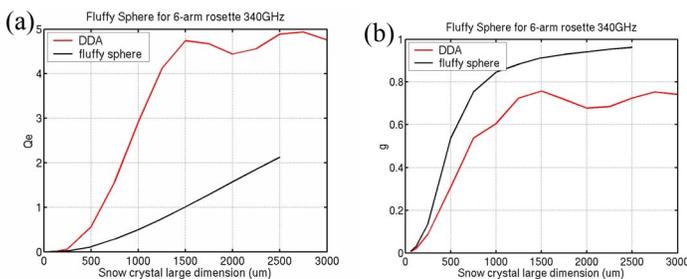


Figure 7. Comparisons of the DDA calculated (a) extinction efficiency and (b) asymmetry factors for spatial rosettes with results from equal mass fluffy spheres at 340 GHz.

7. Discussion and future work

Equal-V/A and equal-V spheres can reasonably well present the extinction efficiency while equal-V/A spheres (equal-V and equal-A spheres) show significantly smaller (larger) asymmetry factor than the DDA results. Equivalent cylinders and equivalent spheroids can be reasonable approximations for ice crystals of hexagonal prism shape. The dielectric mixing approaches employed by the O'Brien and Goedecke [14] for a dendrite seem not applicable to generate reasonable single scattering properties especially for complex snow flakes including large fraction of air inside. More comprehensive analyses are under way to determine the uncertainty range of single scattering properties of ice crystals caused by variations of snow density, shape, and size distribution.