On the Relation Between Kelvin Waves and the Quasi-Biennial Oscillation

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Abstract

The long term behavior of Kelvin waves is studied, using monthly data on the variance of zonal wind and temperature at Canton Island, for the period September 1954–March 1965. Increased variances are found to accompany the descent of the westerly wind regimes associated with the quasi-biennial oscillation. This is indicative of increased Kelvin wave activity within the shear zone which marks the leading edge of the westerlies. There is evidence of a convergence of westerly momentum within the shear zone, which supports the hypothesis that Kelvin waves may play an essential part in the momentum budget of the quasi-biennial oscillation.

1. Introduction

Recent articles in this journal by Yanai and Maruyama (1966), Maruyama and Yanai (1967), and Maruyama (1967) have described westward propagating wave motions in the tropical stratosphere with periods of about four days and horizontal wavelengths of 10,000 km. Lindzen and Matsuno (1968) have identified these waves as the gravest asymmetric mode* of the family of waves which represent solutions of the equations of motion on an equatorial beta plane.

More recently, Wallace and Kousky (1968) have presented observational evidence of eastward propagating waves with periods of about 15 days and wavelengths of 20,000 km or longer in the same region. The distinguishing characteristic of this latter wave type is the marked absence of fluctuations of the meridional wind component; hence, the name “Kelvin waves.” These correspond to the gravest symmetric mode of the same wave family (Matsuno (1966), Holton and Lindzen (1968)).

Thus far, these are the only two members of the wave family to be identified. Lindzen and Matsuno (1968) have suggested that other modes may be present, but only these gravest modes have sufficiently large vertical wavelengths to render them detectable with the present observing system. It has been pointed out by a number of authors that wave disturbances in the tropical stratosphere must be responsible for the large zonal accelerations associated with the quasi-biennial oscillation. Wave disturbances can produce zonal accelerations in two ways:

1. by transporting westerly momentum toward or away from the equator, or,
2. by producing a convergence (or divergence) of the vertical flux of westerly momentum.

Kelvin waves cannot produce zonal accelerations by the first mechanism, since they have no meridional wind component. The easterly waves should not produce a meridional transport of momentum either if, as theory predicts, the zonal and meridional wind components in the waves are 1/4 wavelength out of phase. Maruyama’s (1968b) calculation of poleward momentum transport by these waves confirms this expectation.

Thus, if either type of wave disturbance influences the zonal flow it is probably by means of the second mechanism, which involves vertical flux of zonal momentum. This transport may be expressed quantitatively as

$$ w'w' = \frac{r w^* w^*}{2} $$

where $u^*$ and $w^*$ are the average amplitudes of the zonal and vertical motion components in the waves, and $r$ is the correlation coefficient between

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* The latitudinal distribution of these waves is given by the lowest order Hermite polynomial having odd symmetry about the equator.
the same two components. $u^*$ has been estimated directly from zonal wind data presented by Wallace and Kousky (1968) for the Kelvin wave and by Maruyama (1967) for the easterly wave. The value of $u^*$ for the Kelvin wave can be obtained from the first law of thermodynamics in the same manner as in Maruyama (1968b).

From an inspection of Fig. 7 in Wallace and Kousky (1968) it can easily be shown that the only advection term which can have any net effect on the temperature changes is the one involving the average zonal velocity. This term can be eliminated if we measure the frequency in a frame of reference moving with the mean zonal wind. For an eastward moving wave, this Doppler shifted frequency will be highest at the level of peak easterlies and lowest in regions of westerlies. We will return to this point in section 6. The value of $w^*$ given in Table 1 is based on an intermediate value of frequency (1 cycle per 15 days) corresponding to a zero zonal wind. The value of $w^*$ for the easterly waves is based on Maruyama (1967). The correlation coefficient is assumed to be nearly +1.0 for both wave types in accordance with both theoretical expectations and synoptic evidence.

It is seen that both wave types transport westerly momentum upward, with the Kelvin waves having about four times as large an effect.

The latitudinal dependence of the vertical momentum flux is somewhat different in the two wave types. In the Kelvin waves both zonal and vertical velocities have a latitudinal distribution given by $e^{-\alpha y^2}$ where $y$ is distance from the equator. Thus, the momentum flux distribution will be bell-shaped with a maximum at the equator. The easterly waves, on the other hand, give no upward flux of westerly momentum at the equator since the zonal velocity fluctuations vanishes there. The upward flux is proportional to $y^2 e^{-\lambda y}$, reaching a maximum several degrees away from the equator and then dying away in a manner similar to the Kelvin waves. It is not possible at this time to draw any firm conclusion regarding the relative latitudinal extent of the two wave types but preliminary indications are that the Kelvin waves may extend to somewhat greater distances from the equator. When the vertical flux of westerly momentum is integrated over latitude, it is apparent that the Kelvin waves, with their larger zonal motions and broader meridional profiles, give by far the larger contribution, perhaps by as much as an order of magnitude.

Lindzen and Holton (1968) have proposed a mechanism involving interactions between the wave motions and the mean zonal flow whereby alternating, downward propagating easterly and westerly wind regimes similar to those observed in the quasi-biennial oscillation may be produced. In this mechanism westerly regimes descend by absorbing the westerly momentum transported upward by waves with westerly phase speeds, while easterly wind regimes propagate downward by absorbing easterly momentum transported upward by waves with easterly phase speeds. The Kelvin waves have been shown to transport enough momentum to account for the descent of westerly regimes (Wallace and Kousky, 1968). The waves discovered by Yanai and Maruyama are exceptional in that they are easterly waves which transport westerly momentum upwards. Just what role they would play in Lindzen and Holton's scheme is not clear. All that can be said is that their effect upon westerly regimes is certainly much less than that of the Kelvin waves. Lindzen and Holton's mechanism depends upon

<table>
<thead>
<tr>
<th></th>
<th>$u^*$</th>
<th>$w^*$</th>
<th>$\bar{u}'w'$</th>
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<tbody>
<tr>
<td>Kelvin Waves</td>
<td>8 m sec$^{-1}$</td>
<td>0.1 cm sec$^{-1}$</td>
<td>.004 m$^2$ sec$^{-2}$</td>
</tr>
<tr>
<td>Easterly Waves</td>
<td>.2 m sec$^{-1}$</td>
<td>0.1 cm sec$^{-1}$</td>
<td>.001 m$^2$ sec$^{-2}$**</td>
</tr>
</tbody>
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* Maruyama's (1967) estimate of .04 cm sec$^{-1}$ was based on an erroneous static stability. The value given in the table has been corrected.

** Maruyama (1968b) estimated $\bar{u}'w'$ due to easterly waves from actual data and obtained a value of roughly .002 m$^2$ sec$^{-2}$. In his analysis, all stations but one exhibited values somewhat lower than this, with considerable scatter. Hence, we do not feel that there is any essential contradiction between this result and the value given in the table. For purposes of consistency we prefer to use the latter value for this discussion.
yet undiscovered higher modes of wave motions to provide the vehicle for the upward transport of easterly momentum.

2. Relation of wave amplitudes to the phase of the quasi-biennial oscillation

Maruyama (1968a) has studied the long term behavior of the easterly waves in relation to the phase of the quasi-biennial oscillation. For the three year period which he investigated, there was a tendency for amplitudes to be largest in regions where the absolute value of the mean zonal wind speed was decreasing with height.

The purpose of this paper is to investigate the long term behavior of the Kelvin waves. If these waves are instrumental in providing the momentum for the descent of westerly regimes, we should expect to observe the following:

(1) During the descent of westerlies the wave amplitudes should be large enough to account for the accelerations which are taking place.

(2) There should be a convergence of westerly momentum in the shear zones which precede the descending westerly regimes.

Since Kelvin waves are the largest amplitude wave disturbances observed in the tropical stratmosphere and since they have periods shorter than a month, it should be possible to gain some insight into their long term behavior by examining data on monthly variances of zonal wind and temperature. Months with intense Kelvin wave activity should be marked by relatively high variances of both these parameters. The advantage of using monthly statistics rather than daily data is that a much longer period of record can easily be obtained. A major difficulty is the fact that the trend during the month can at times contribute significantly to the variance. This effect is particularly troublesome because the largest trends in zonal wind and temperature are taking place precisely when Kelvin waves are expected to have the largest amplitudes.

3. The data

Monthly stratospheric wind and temperature statistics were provided by the U. S. Naval Weather Research Facility for Canton Island (3°S) up to and including the 30 mb level for the period September 1954–March 1965. During this period at least 10 and usually 20 or more observations per month were available up to the 30 mb level. During the later years the data are also available for some of the higher levels.

An attempt was made to compensate for trends in zonal winds and temperatures by fitting a curve to the mean monthly values and deducting the variance of this curve from the total variance for each month. (The details of this procedure are given in the appendix.) The trend corrections proved to be large enough to significantly lower the variances during periods of rapid transition between easterly and westerly regimes.

4. Results of analysis of monthly data

Time height sections of corrected variance data are shown in Figures 1 (a) and 1 (b) with

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Fig. 1. Time–height sections showing variances of (a) zonal wind in m² sec⁻² and (b) temperature in °C². The superimposed shading indicates regions of westerly winds associated with the quasi-biennial oscillation.
shading superimposed to outline the westerly wind regimes.

It is seen that large zonal wind and temperature fluctuations accompany the descent of westerlies on every occasion. The most intense activity is usually confined to the shear zone which marks the leading edge of the westerlies and the region immediately above it. Maxima are notably absent in zones of strong easterly shear preceding the descent of easterly regimes. This is further evidence that the enhanced variances are real and not merely reflections of the trends in the zonal winds and temperatures.*

Analysis of the variances of the meridional wind component (not shown) do not exhibit similar periods of high values, or any apparent relationship to the mean zonal wind oscillation. This is a strong indication that Kelvin waves are primarily responsible for the periods of enhanced variances. Synoptic evidence of the presence of Kelvin waves during these periods is presented in the next section.

During the disturbed periods, variances of zonal wind often exceed 50 m² sec⁻², which would correspond to an oscillation with an amplitude of 10 m sec⁻¹, and variances of temperature often exceed 8°C², which would correspond to oscillation with an amplitude of 4°C. Thus during these times the Kelvin wave amplitudes are roughly the same size as those found by Wallace and Kousky (1968) during the descent of a westerly regime in early 1966. It was shown in that paper that amplitudes of this size are adequate to account for the observed westerly accelerations. At other times, variances of zonal wind and temperature

* Wallace (1967) has pointed out that westerly regimes descend more rapidly than easterlies. Thus, the largest trends are those occurring during the transitions to westerlies. If not compensated for by trend removal, this effect could partially account for the larger variances observed during these periods. However, it still could not account for the complete absence of variance maxima in zones of strong easterly shear.

Fig. 2(a). Time-height section of zonal wind. Contours are placed at increments of 5 m sec⁻¹. Westerlies are shaded.

Fig. 2(b). Time-height section of zonal wind. Contours are placed at increments of 5 m sec⁻¹. Westerlies are shaded.

Fig. 2(c). Time-height section of zonal wind. Contours are placed at increments of 5 m sec⁻¹. Westerlies are shaded.
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Fig. 2(d). Time–height section of zonal wind. Contours are placed at increments of 5 m sec\(^{-1}\). Westerlies are shaded.

Fig. 3. Time–height section of temperature. Contours are placed at intervals of 2\(^\circ\)C. Dashed lines indicate easterly maxima transcribed from Figure 2.

average about 20 m\(^2\) sec\(^{-2}\) and 4\(^\circ\)C\(^2\) respectively, about half the disturbed values.

5. Synoptic evidence

At times the zone of increased variance preceding a westerly wind regime is so narrow that one wonders whether it might merely be the result of incomplete removal of the trend. In order to verify that this is not the case, time-height sections for one of these situations were prepared from daily data. The period selected was January 1963–April 1964, which spans the briefest but most intense period of high variances preceding a westerly regime.

The section for zonal wind is shown in Figures 2 a–d. A band of strong fluctuations can be seen in the zone of strong wind shear above the easterly wind maximum throughout most of 1963. The intense activity migrates downward with the shear zone and culminates in a series of particularly strong fluctuations immediately prior to the disappearance of the easterly regime in December 1963. The descent of a new easterly regime at the higher levels during early 1964 is not accompanied by a similar enhancement of Kelvin wave activity. Enhanced wave activity is also noted during 1963 in the westerly regime above the shear zone. This is also in agreement with the variance data.

Figure 3 shows a temperature section for the four most active months the same period. Easterly wind maxima are indicated by dashed lines. There is a good correspondence between the wind and temperature fluctuations, the phase relationship being the same as noted by Wallace and Kousky (1968). It should be noted that easterly waves are also present at times during this period, but the Kelvin waves are clearly the dominant feature.

6. Discussion

The monthly variance statistics and the zonal wind data both show that Kelvin wave activity is strongly influenced by the mean zonal wind profile. Wave amplitudes are strongly enhanced in the layers of strong westerly wind shear which precede the descent of westerly wind regimes. These results have several implications upon recent hypotheses concerning the quasi-biennial oscillation:

1. During the 5 periods of descending westerly wind regimes which were investigated, the Kelvin waves appear to be large enough to account for the observed increases of zonal momentum at the higher levels.

2. There is reason to expect a convergence of westerly momentum in the shear zone which precedes the descending westerly regimes. Usually, within this region, there is a rapid decrease in wave amplitude with height. Even at times when this decrease does not occur (e.g., throughout much of 1963), the sharp decrease in Doppler shifted frequency with height may be large enough to cause a sizable decrease in upward momentum flux through the westerly shear zone. For example, the Doppler shifted period of a wave with westerly phase speed of 30 m sec\(^{-1}\) and a wavelength of 40,000 km (wave no. 1) increases from 7 1/2
days at the core of the easterly regime (assuming a mean zonal wind, \(\bar{u} \sim -30\, \text{m sec}^{-1}\)), to 15 days in the middle of the shear zone (where \(\bar{u} \sim 0\)), to 30 days in the upper layer of westerlies (where \(\bar{u} \sim +15\, \text{m sec}^{-1}\)). For constant amplitude this four-fold decrease in frequency results in equally large decreases in vertical motion and in upward momentum flux through the shear layer. For shorter wavelengths, the fractional reduction is even larger. Similar reasoning would suggest a divergence of westerly momentum in easterly shear zones, when Kelvin waves are present.

An unexpected feature which can be seen in Figure 2 is the apparent phase reversal of the Kelvin waves as they pass through the shear zone above the easterly maximum. Westerly maxima occur in the westerlies over easterly maxima in the easterlies. This feature becomes even more apparent in data which have had the higher frequency oscillations filtered out. (These will be presented in a later paper.) We know of no explanation of this phenomenon but we believe it to be of sufficient importance to warrant further study.

Appendix

Correction of variances for trends

The monthly variance due to waves whose period is less than some specified value is given by

\[
\text{var}^*(u) = [u - \bar{u}]^2 \tag{1}
\]

where \(u\) is the instantaneous value of the zonal wind, \(\bar{u}\) the smooth trend curve, from which all waves of interest have been eliminated, and \([\cdot]\) denotes the monthly average. Unfortunately, this expression could not be evaluated since daily wind data were not used in this study.

Instead we chose to compute a different corrected variance, \(\text{var}^{**}(u)\), in the form

\[
\text{var}^{**}(u) = \text{var}(u) - [u - \bar{u}][\bar{u} - \bar{u}] \tag{2}
\]

With some manipulation, the difference between the two expressions can be shown to be equal to

\[
\text{var}^{**}(u) - \text{var}^*(u) = 2[u - \bar{u}] [\bar{u} - \bar{u}] \tag{3}
\]

This difference is negligible provided that within the month there is little or no correlation between the deviation of the instantaneous wind from the trend curve and the trend curve itself. Thus, equation (2) should be appropriate if the trend is a good fit to the data.

Trend removal procedure

Each month a quartic equation was fit to the five data points adjacent in time at each level. This trend curve was determined in such a way that it would pass through the five adjacent monthly means at the midpoints of the five respective months. Denoting this trend curve for zonal wind by \(\bar{u}\), we have

\[
\bar{u}(t) = \bar{u} + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 \tag{4}
\]

where time \((t)\) is measured in months before or after the midpoint of the month in question, and \(\bar{u}\) denotes the mean for that same month. The coefficients were determined by substituting \(t = \pm 1\), and \(\pm 2\) into (4). The contribution of the trend curve to the variance for month \(n\) is then given by

\[
\left[\bar{u} - \bar{u}\right]^2 = \int_{n-1/2}^{n+1/2} \left[\bar{u} - \bar{u}\right]^2 dt = \frac{a_1^2}{12} + \frac{a_2^2}{80} + \frac{a_3^2}{448} + \frac{a_4^2}{2304} + \frac{a_1a_2}{40} + \frac{a_2a_3}{224} \tag{5}
\]

This was evaluated for each month (except the first and last two) and subtracted from the total variance to give the corrected variance.

The identical procedure was applied to the temperature data.

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ケルヴィン波と準2年周期振動との関係について

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1954年9月から1965年3月の期間にわたって、カントン島での帯状風と気温のvarianceについての月々のデータを用いて、ケルヴィン波の長期間における動静を調べた。準2年周期振動に関連した偏西風域の下降に伴って、varianceの増加が見出された。これは、偏西風の先端的な高気温であるshear zone内で、ケルヴィン波の活動が活発化したことを示している。更にshear zone内で偏西風の運動量の収束があることの証拠があり、そのことは、準2年周期振動の運動量の収支において、ケルヴィン波が本質的な役割を演じているという仮説を支持するものである。