On the Interaction Between Kelvin Waves and the Mean Zonal Flow

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ABSTRACT

The descent of westerly wind regimes of the quasi-biennial oscillation in the equatorial stratosphere is marked by two regions of strong zonal wind fluctuations in the 8–20 day period range: one in the easterly regime below the westerly shear zone and another in the upper level westerlies. There is a distinct minimum wave activity within the shear zone. The lower waves are identified as Kelvin waves, while the upper waves are shown to require the existence of a meridional wind component.

The vertical transport of zonal momentum by the waves is computed and compared to the balance requirements. It is shown that the distribution of momentum flux divergence due to Kelvin waves is in good agreement with the observed zonal accelerations during the descent of westerly regimes of the quasi-biennial oscillation. The Kelvin waves give up much of their momentum to the zonal flow in the shear zone which marks the leading edge of the descending westerly regime, even though they apparently do not encounter a “critical layer” within this zone. The occurrence of strong Kelvin waves appears to be confined to periods of descending westerly regimes.

1. Introduction

In a previous paper (Wallace and Kousky, 1968a), the authors discussed a type of wave disturbance in the equatorial lower stratosphere which resembled the theoretical model of the atmospheric Kelvin wave. The observed waves exhibited a strong correlation between zonal wind and vertical motion, in the sense that westerly momentum was being transported upward. It was inferred from simple scaling arguments that this upward flux of momentum might be large enough to explain the strong westerly accelerations which precede the westerly phase of the quasi-biennial oscillation.

Almost concurrent with these findings, Lindzen and Holton (1968) produced a model of the quasi-biennial oscillation based upon interactions between vertically propagating, equatorial wave disturbances and the mean zonal flow. In essence, this model postulates a steady source of wave energy entering the equatorial stratosphere from below. Each wave mode penetrates upward until it encounters a “critical level” [a concept introduced by Booker and Bretherton (1967)] where its phase speed is equal to the speed of the mean zonal flow. At this level, the Doppler-shifted frequency of the wave goes to zero and the wave momentum and energy are absorbed by the zonal flow. Since the Kelvin wave prop-

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2 Present affiliation: Department of Meteorology, University of Utah.
3 The atmospheric Kelvin wave is the gravest mode of the family of zonally propagating waves which are solutions of the equations of motion on an equatorial beta-plane (Matsuno, 1966; Holton and Lindzen, 1968).

agates eastward with respect to the mean zonal flow, its critical layer must be imbedded in a westerly shear zone. Absorption of the westerly momentum carried upward by the wave should cause the shear zone to propagate downward.

This theory has gained further observational support from studies of Wallace and Kousky (1968b) and Maruyama (1969) which show that the occurrence of both of the prominent types of equatorial stratospheric disturbances is related to the phase of the quasi-biennial oscillation in a manner consistent with momentum balance requirements. However, there is some question as to whether the Kelvin type waves actually encounter a critical level in the westerly shear zones.

The purposes of the present paper are: 1) to provide more documentation concerning the latitudinal dependence, vertical and zonal wavelengths, and phase speed of the waves; 2) to present a more positive proof that the disturbances under investigation are, in fact, Kelvin waves; 3) to provide further evidence concerning the relationship between wave amplitude and the zonal flow pattern; and 4) to clarify the nature of the interaction between the waves and the zonal flow in westerly shear zones.

2. Kelvin wave characteristics

The tentative identification of Kelvin waves in the previous studies cited above was based on the following

4 The Kelvin wave and the mixed Rossby-gravity wave.
observational evidence:

(a) The existence of well-defined peaks in the time spectra of zonal wind and temperature in the 10–20 day period range, and the absence of a corresponding peak in the spectrum for the meridional wind component (Wallace and Kousky, 1968a; Maruyama, 1969).

(b) Significant coherence between the wind and temperature oscillations, with warmest temperature occurring 1/4 cycle in advance of the maximum westerlies (Wallace and Kousky, 1968a, b; Maruyama, 1969).

These disturbances can be positively identified as Kelvin waves if, in addition, it can be established that:

(c) the wave amplitude decreases with distance from the equator, (d) the phase speed of the waves is eastward with respect to the mean zonal wind, and (e) the zonal and vertical wind fluctuations satisfy a two-dimensional continuity equation.

Holton and Lindzen (1968) and Wallace and Kousky (1968a) have discussed the basis for the above criteria.

We shall now consider the waves observed during the period A (4 July–15 October 1963), which is shown schematically in Fig. 1. Detailed time-height sections of zonal wind and temperature for this period have been presented by Wallace and Kousky (1968b). Figs. 2–4 show the power spectra of the zonal wind component

Fig. 1. Time-height section of mean zonal wind at Canton Island showing periods of intensive analysis: A (4 July–15 October 1963) and B (10 January–23 April 1966). The vertical lines at the top of the diagram delineate the intervals used in power spectrum analyses at Ascension Island (see Fig. 11).

Fig. 2. Power density contours (m$^2$ sec$^{-2}$ day$^{-1}$) of zonal wind (left) at Canton Island as a function of frequency and height during period A and the vertical profile (right) of mean zonal wind corresponding to this time period.
as a function of height for Canton Island (3S), Majuro (7N) and Ascension Island (7S), plotted with their respective vertical profiles of mean zonal wind. From these figures the following features are evident:

1) All equatorial stations show a concentration of power density in the 0.05–0.125 cycle per day (cpd) frequency band.

2) There are two distinct power maxima in the vertical. The lower is located near 20 km, the level of maximum easterlies; the upper is located above 25 km in the westerly regime. They are separated by a minimum near 23 km, which is located in the shear zone where the mean zonal wind is near zero. This is also evident in Figs. 5 and 6 which show meridional cross sections of power density in the 0.05–0.125 cpd frequency band and mean zonal speed, respectively.

3) Power density decreases with latitude in both hemispheres. Corresponding calculations have been made for temperature and the meridional wind component. The former results are similar to those for zonal wind; the latter show an absence of power in the 0.05–0.125 cpd frequency band. These results are omitted for the sake of brevity.

From time-height sections shown in Wallace and Kousky (1968b) it is evident that the zonal wind and temperature fluctuations are strongly related during this period with warmest temperatures preceding maximum westerlies by $\frac{1}{4}$ cycle. Cross-spectrum analysis
Table 1, List of stations.

<table>
<thead>
<tr>
<th>Station</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Station Identifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Koror</td>
<td>7N</td>
<td>134E</td>
<td>K</td>
</tr>
<tr>
<td>Guam</td>
<td>14N</td>
<td>144E</td>
<td>G</td>
</tr>
<tr>
<td>Truk</td>
<td>7N</td>
<td>151E</td>
<td>T</td>
</tr>
<tr>
<td>Ponape</td>
<td>7N</td>
<td>158E</td>
<td>P</td>
</tr>
<tr>
<td>Majuro</td>
<td>7N</td>
<td>171E</td>
<td>M</td>
</tr>
<tr>
<td>Canton</td>
<td>3S</td>
<td>172W</td>
<td>C</td>
</tr>
<tr>
<td>Balboa</td>
<td>9N</td>
<td>80W</td>
<td>B</td>
</tr>
<tr>
<td>Ascension</td>
<td>8S</td>
<td>14W</td>
<td>A</td>
</tr>
</tbody>
</table>

(not shown) substantiates this result. Thus it is clear that these disturbances fulfill criteria (a), (b) and (c) above for Kelvin waves. We now turn our attention to the remaining two criteria.

Wallace and Kousky (1968a) were unable to determine the wavelength and direction of propagation for the waves observed during early 1966 because the three stations available to them were not suitably spaced. This calculation is now repeated for period A, using data for the stations given in Table 1. The results are shown in Fig. 7. Each point represents the phase difference between the zonal wind oscillation at the pair of stations indicated by the station identifiers as a function of longitudinal separation between stations. The phase plotted is the fraction of a cycle by which the fluctuations at the western station lead those at the eastern station. Thus positive phase differences are indicative of eastward propagation. The phase differences are computed for the 0.05-0.125 cpd frequency band and averaged for the 80-, 70-, 60- and 50-mb levels and the 30-, 25-, 20- and 15-mb levels. Only phases correspond-

![Fig. 6. Meridional cross section of mean zonal wind for period A. The analysis is based on the stations whose identifiers (see Table 1) are given at the top of the diagram.](image)

![Fig. 5. Meridional cross section of power density of zonal wind in the frequency band 0.050-0.125 cpd during period A. The analysis is based on the stations whose identifiers (see Table 1) are given at the top of the diagram. The values on the isopleths may be multiplied by \((2\pi \times 0.125)\) to obtain the variance \((m^2 \text{ sec}^{-2})\).](image)

The coherence square is a measure of the fraction of the variance of one series which can be explained on the basis of the other series.

![Fig. 7. Phase difference in the 0.050-0.125 cpd frequency range plotted as a function of longitudinal difference between pairs of stations for period A. Positive slopes indicate eastward propagation. The sloping lines correspond to the phase-longitude relationships for wavenumbers 1 and 2.](image)
We now apply the above estimates of the zonal and vertical wavelengths of the observed waves \((L\text{ and }Z)\) to the continuity equation

\[
\frac{\partial}{\partial x}(\rho u) + \frac{1}{\cos \phi} \frac{\partial}{\partial y}(\rho \cos \phi) + \frac{\partial}{\partial z}(\rho w) = 0,
\]

where the symbols have their conventional meanings. If the meridional wind component vanishes identically, scaling considerations dictate that

\[
\frac{u^*}{L} \approx \frac{w^*}{Z},
\]

(where \(u^*\) and \(w^*\) are the amplitudes of the zonal and vertical motion oscillations, respectively), provided that the scale height of the atmosphere is not much less than the vertical scale \((Z/2\pi)\) of the waves.

The zonal amplitude \(u^*\) is estimated by integrating the power spectrum over the frequency range 0.05–0.125 cpd, while \(w^*\) is estimated in a similar manner from the temperature spectrum, under the assumption that the motions are adiabatic.\(^6\) The resulting amplitudes are shown in Table 2, together with the estimates of \(u^*/L\) and \(w^*/Z\) for the upper and lower regimes.

It is apparent from the table that two-dimensional continuity can be satisfied for the lower waves but not for the upper ones, in which the vertical motions are far too small to balance the zonal term. From this we can conclude that meridional velocities accompany the upper waves. Given the proper phase relationship it would require only a 0.2 m sec\(^{-1}\) oscillation in \(v\) with a meridional wavelength of 10\(^o\) latitude to balance the \(u^*/L\) term. An oscillation of this size could not be distinguished from noise in the spectral results.

We conclude that the waves observed below the westerly shear zone satisfy all the criteria for Kelvin waves. The waves occurring in the upper westerlies are not pure Kelvin waves since they require the existence of a small meridional wind oscillation. It is possible that they may be Kelvin waves modified in the presence of strong meridional shear (e.g., see Fig. 6).

### 3. Vertical transport of zonal momentum

Equatorial wave disturbances may transport zonal momentum vertically in two ways:

1) Through a correlation between \(u\) and \(w\) in the waves. This contribution is given by the cospectrum of \(u\) and \(w\), integrated over frequency.

2) Through a correlation between meridional displacement \(\eta\) and \(w\). Lindzen (1970) has shown, in the case of the mixed Rossby-gravity mode, that these two mechanisms oppose each other, with the second one being dominant. In the case of the pure Kelvin wave, it

\(^6\) For further details of this method, see Maruyama (1968) and Section 3 of this paper.
is clear that the vertical transport is entirely due to the first term. It is easily verified that the same is true for $v$ amplitudes of a few tenths of a meter per second, which may occur in the modified Kelvin waves of the upper regime.

We follow the procedure developed by Maruyama (1968) for calculating the vertical flux of zonal momentum by the waves in a particular frequency band. This is based on the assumption that the vertical motions in the waves are adiabatic. The cospectrum of $u$ and $w$ may be expressed as

$$C(u,w) = \frac{k(a-c)}{\frac{\partial T}{\partial z} + \Gamma_d} Q(u,T),$$

where $T$ denotes temperature, $k$ the wavenumber $(2\pi/L)$, $c$ the zonal phase speed, $Q$ the quadrature spectrum, $\Gamma_d$ the dry adiabatic lapse rate, and the overbar signifies a time average. This formulation differs from Maruyama's in that the Doppler-shifted frequency $k(a-c)$ is used in place of frequency with respect to the ground. This modification was made in order to include the advection of temperature by the mean zonal wind which Maruyama neglected.

The above expression was integrated over the frequency band 0.05–0.125 cpd to obtain the total contribution of Kelvin waves to the vertical flux of zonal momentum. For these calculations it was assumed that the disturbances correspond to zonal wavenumber 1 over the entire frequency interval. (This assumption sets a lower limit on the flux.) Lapse rate and mean zonal wind are based on actual data.

The vertical flux of zonal momentum by Kelvin waves was computed for five stations during period A. It is assumed that the calculations at a given station are representative of a zonal average at the latitude of the station. Fig. 9 summarizes the results in meridional cross-section form.

The flux is everywhere directed upward with largest values at the equator. There is a distinct maximum located near 60 mb, which corresponds to the lower maximum of wave activity (Fig. 5) located within the easterly wind regime (Fig. 6). Conspicuously absent is an upper level maximum. This is due to the fact that the upper level waves have smaller vertical motions.

The maximum observed flux is $5.8 \times 10^{-8}$ m$^2$ sec$^{-2}$ at Canton Island. This is more than three times as large as the maximum value obtained by Maruyama (1969) during another period of descending westerlies. The difference may be partially explained by Maruyama's use of ground-based rather than Doppler-shifted frequency, and the fact that his six-month sampling interval may have been longer than the period of vigorous wave activity.

For purposes of comparison, a similar computation was made on Balboa during the period B, another period of descending westerlies. The results, shown in Table 3, are very similar to those described above.

### 4. Role of Kelvin waves in the zonal momentum balance

The magnitude of the density-weighted flux divergence was computed for the layers bounded by 100–80 mb, 80–70 mb, etc. The results for period A are shown in Fig. 10. It is evident that there is a convergence of momentum flux in the descending westerly shear zone and a divergence in the easterly shear zone. This is in qualitative agreement with the momentum balance requirements.

In Table 4 the contribution of the convergence of the vertical flux of zonal momentum to the zonal acceleration is compared with the observed zonal accelerations at Canton Island during period A and Balboa during period B. The overall agreement is quite good, considering the uncertainties involved in the flux computations. From this we conclude that the vertical transport of zonal momentum due to Kelvin waves is large enough to

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**Table 3. Upward flux of relative westerly momentum due to waves in the 0.050–0.125 cpd frequency band, together with mean zonal wind for Balboa during period B (10 January–23 April 1966).**

<table>
<thead>
<tr>
<th>Pressure (mb)</th>
<th>80</th>
<th>70</th>
<th>60</th>
<th>50</th>
<th>40</th>
<th>30</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux ($10^{-8}$ m$^2$ sec$^{-2}$)</td>
<td>4.2</td>
<td>2.8</td>
<td>2.4</td>
<td>3.8</td>
<td>2.9</td>
<td>1.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Zonal wind (m sec$^{-1}$)</td>
<td>-9</td>
<td>-11</td>
<td>-16</td>
<td>-21</td>
<td>-16</td>
<td>-9</td>
<td>-2</td>
</tr>
</tbody>
</table>
account for the zonal wind changes associated with the quasi-biennial oscillation.

5. Long-term relation of Kelvin waves to the mean zonal flow

In order to verify the observation by Wallace and Kousky (1968b) that Kelvin waves are strongly enhanced during those rather brief periods when westerly wind regimes associated with the quasi-biennial oscillation are making their descent, an analysis of Kelvin-wave activity over the period 1963–66 was undertaken. Power spectra were computed at each observing level for overlapping 104-day periods (see Fig. 1) at Ascension Island. (Canton Island would have been preferable, but there were too many missing data during 1964–65.) The resulting spectral estimates, averaged for the 0.050–0.125 cpd frequency band, are shown in time-height section form in Fig. 11, superimposed on the zonal wind analysis for Canton Island.

As expected, there are distinct power maxima which accompany the descent of westerly regimes, with maximum power located in the easterlies, below the shear zone. The power in this frequency band is enhanced by about a factor of 2–4 during periods of descending westerlies. Also evident in the figure are power maxima situated at high levels in both westerly regimes. These correspond to the upper wave regime discussed in Section 2.

6. Discussion

Two fundamental questions concerning Kelvin waves in the equatorial stratosphere remain unanswered: What is their energy source? What is the mechanism
by which the waves are absorbed by the zonal flow in westerly shear zones? The results of previous sections lend some insight into the second question.

It is apparent that absorption of the waves takes place in a thin layer within the westerly shear zone. The mean zonal wind in this layer is near zero and may even be slightly easterly, while the phase speed of the waves is $>12 \text{ m sec}^{-1}$ and probably closer to $30 \text{ m sec}^{-1}$ westerly. This rules out the possibility of "critical layer absorption" as proposed by Lindzen and Holton (1968).

The authors have observed that very strong vertical shears are created when the easterly phase of a Kelvin wave passes below a westerly shear zone. Despite the large static stabilities typical of the equatorial stratosphere, the Richardson number in such situations frequently approaches 0.25 which is commonly accepted as the criterion for the onset of clear air turbulence. Perhaps this, or some other form of nonlinear interaction between the waves and the zonal flow, is responsible for the breakdown of the waves in regions of strong westerly shear.

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