Geographical Variations in the Vertical Structure of Geopotential Height Fluctuations

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Manuscript received 31 January 1979, in final form 27 July 1979

ABSTRACT

Temporal (but nonseasonal) fluctuations in the geopotential height field exhibit large regional contrasts in vertical structure, as manifested in the geographical distributions of the correlation between 1000 and 500 mb height, and the ratio of the amplitudes of the fluctuations at those levels. This geographical variability is investigated in order to ascertain its seasonal, frequency and zonal wavenumber dependence and its relation to other indicators of vertical structure: statistics involving the 1000–500 mb thickness, and the structure of the dominant mode in an eigenvector analysis expansion of geopotential height in the vertical. Results are based on operational analyses by the United States National Meteorological Center over a 15-year period.

Particularly striking is the contrast between transient fluctuations over the eastern oceans, which exhibit a highly barotropic structure with strong vertical coherence in the geopotential height field and small temperature variability, and those over the interior of the continents, whose structure is much more baroclinic, with low or negative temporal correlations between 1000 and 500 mb height. Such contrasts show up clearly in station data; they are observed during both winter and summer, and for temporal frequencies ranging from synoptic to interannual time scales. They are largely a reflection of the vertical structure of planetary-scale fluctuations. There is also evidence of smaller scale regional contrasts in vertical structure, some of which appear to be associated with synoptic-scale disturbances.

On the basis of 1000 and 500 mb height data alone it is possible to represent, with a high degree of accuracy, the geographical distribution of the shape of the dominant eigenvector in the expansion of the vertical profile of geopotential height in transient disturbances.

The implications of these results on the design of observing networks and objective analysis procedures are discussed.

1. Introduction

Recognition of the fact that the vertical structure of pressure fluctuations in the atmosphere varies significantly from place to place dates at least as far back as a paper by Ficker (1920) describing an analysis of barometric pressure records from a number of stations in the vicinity of the Alps and the Caucasus mountains. Ficker noticed that the records from the high-altitude stations showed much more temporal coherence with records from lower altitude stations located to the south of the mountain ranges than with stations located comparable distances to the north of the ranges. He attributed these differences to the predominance of what he referred to as "primary pressure waves" on the south side of the mountain ranges and "secondary pressure waves" on the north side, where his "primary waves" correspond to the barotropic component of atmospheric disturbances and his "secondary waves" represent the hydrostatic response of station pressure to temperature changes taking place in a relatively shallow layer located below the level of the mountain stations.

Namias (1947) documented the spatial distribution of the correlation coefficient between 700 mb heights and temperatures over North America, based upon daily station data. His computed values for the winter season ranged from about +0.8 at the weather ships in the North Atlantic and at a number of Pacific coastal stations to <−0.4 over the southern Great Plains. Klein (1951) interpreted these lower values as reflecting the influence of shallow, warm cyclones which frequently form on the lee side of the continental divide.

On the basis of case studies, Hess and Wagner (1948) documented the transformation in the vertical structure of wintertime disturbances passing eastward across the northwestern United States. It was noted that the disturbances approaching the
Pacific coast tend to be characterized by "cold core" lows with weak occluded fronts and amplitude increasing with height; hence, at coastal stations, fluctuations in sea-level pressure are observed to occur in phase with fluctuations in temperature aloft. It was observed that when these same disturbances reach the northern Great Plains they develop strong low-level fronts and a strong westward tilt with height; hence at stations in this region such as Great Falls and Bismarck a much lower level of correlation is observed between the pressure and temperature fluctuations.

Hoinkes (1950) presented maps showing the distribution of the correlation coefficients between 24 h changes in 1) surface pressure and 500 mb height, 2) surface pressure and 1000–500 mb thickness and 3) 500 mb height and 1000–500 mb thickness. The use of 24 h tendencies tends to emphasize high-frequency fluctuations with periods of a few days or less. Both distributions were marked by strong land-sea contrasts. Correlation coefficients for 1) ranged from above +0.7 over most of the eastern Atlantic and parts of the British Isles and Scandinavia to below +0.4 over eastern North America and inland regions of Europe. Large north-south contrasts were noted across the Alps with higher correlations to the south, in agreement with Ficker's earlier study. The correlation coefficients for 2) were characterized by small positive values over the eastern Atlantic and parts of the Mediterranean and negative values ranging down to about −0.4 over Europe and North Africa.

Klein (1967) examined the geographical distribution of the linear correlation between 700 mb height and sea-level pressure making use of about 15 years of gridded data for the entire Northern Hemisphere, poleward of 20°N, stratified by calendar month. The resulting patterns were rather similar to those obtained by Namias (1947) for the correlation between 700 mb height and 700 mb temperature. The patterns were highly reproducible for different calendar months within the same season.³

Sawyer (1970) displayed hemispheric maps of the correlation coefficient between 1) 1000 and 500 mb height and 2) 500 mb height and 1000–500 mb thickness, averaged over a 3-year period. The former correlation coefficient was computed separately for high-pass and band-pass filtered data, where his high-pass filter emphasized fluctuations with periods shorter than two weeks and his band-pass filter emphasized fluctuations in the 20–60 day period range. In agreement with results of the earlier investigations, Sawyer's results are indicative of large geographical variability in the correlation between 1000 and 500 mb height. These geographical contrasts are most pronounced in his band-pass filtered data, with values ranging from over +0.9 over parts of the eastern oceans to around −0.3 in the lee of the Rockies and over portions of central Asia. These contrasts are considerably stronger than those found by Hoinkes for higher frequency fluctuations.

In summary, the studies mentioned above all point to the existence of important geographical variations in the vertical structure of tropospheric disturbances. Over the middle and eastern Atlantic and Pacific Oceans the disturbances appear to be primarily equivalent barotropic with a "cold core" structure in the lower troposphere whereas over the U.S. Great Plains and over parts of central Asia, pressure fluctuations at the earth's surface are more loosely coupled to disturbances in the upper troposphere and there is even some evidence of a negative correlation between them. Contrasts in vertical structure tend to be particularly strong along some of the major mountain barriers such as the Rockies. Sawyer's study indicates that these geographical contrasts are larger for fluctuations with periods on the order of a few weeks or longer than they are for the shorter period fluctuations such as might be associated with baroclinic wave activity.

In order to substantiate and extend the results of these earlier studies, we have undertaken a systematic documentation of the vertical structure of transient disturbances on time scales ranging from days to seasons over the Northern Hemisphere. The primary variables included in this investigation are sea-level pressure, 500 mb height and 1000–500 mb thickness. It is hypothesized that these variables, when examined in suitable combinations, should provide enough information to describe the gross characteristics of the vertical structure of tropospheric disturbances. This hypothesis is subsequently tested in Section 6 by means of eigenvector analysis performed on data for ten pressure levels. Most of the results presented in this paper are based on a 15-year record of operational Northern Hemisphere objective analyses made by the United States National Meteorological Center (NMC).


2. Preliminary data processing

The results presented in the following sections are based on the following data sets:

(i) Twice-daily NMC analyses for the 11 winter seasons beginning with the 1963–66 winter and ending with the 1975–76 winter. The winter season is
defined as the 120-day period beginning 0000 GMT 15 November. The annual cycle was removed from these data by fitting a least-squares parabola to each individual winter’s data at each grid point and subtracting the fitted values from the original data to obtain deviations. These data will henceforth be referred to as unfiltered daily data.

(ii) Time-filtered versions of the deviations in (i) where the band pass filter emphasizes fluctuations

with periods between approximately 2.5 and 6 days and the low-pass filter retains fluctuations with periods longer than about 10 days. The filters are the same as those described in Blackmon (1976).

(iii) Monthly mean data derived from the same NMC analyses for the months December, January and February beginning with the 1962–63 winter and ending with the 1976–77 winter; and for the months June, July and August beginning with the 1963 summer and extending through the 1976 summer. Hence the data set contains 45 winter months and 42 summer months. The seasonal cycle was removed by subtracting the 14- or 15-year mean for each respective calendar month to obtain deviations.

(iv) Seasonal mean data derived from the NMC data set for the same 15 winters and 14 summers as described in (iii).

(v) Selected station data for the period extending from 1 December 1962 to 1 March 1963, with the annual cycle removed by the use of the procedure described in (i).

(vi) Twice daily NMC geopotential height analyses for the three winters 1971–72, 1972–73 and 1973–74 for the 10 levels indicated in Table 1. The time series run from 1 December through the last day of February.

3. Relations between sea level pressure and 500 mb height

a. Winter

Fig. 1 shows the geographical distribution of the standard deviation of 1) 1000 mb height (derived from NMC sea-level pressure analyses) and 2) 500 mb height, based on the monthly mean data set for the 45 winter months described in (iii) of Section 2. Corresponding distributions for the unfiltered, band-pass filtered and low-pass filtered twice-daily data sets described in (i) and (ii) of Section 2 are presented in Blackmon et al. (1977) for the 1000 mb level and Blackmon (1976) for the 500 mb level. The patterns in these distributions are qualitatively similar, especially for the unfiltered and low-pass filtered data sets and for the monthly mean data set. Similar patterns are also obtained for the seasonal mean data set (iv) (not shown). The 1000 and 500 mb patterns shown in Figs. 1a and 1b, respectively, are similar with respect to the major features, but there are some important differences which are a reflection of geographical dependence of the vertical structure of low frequency atmospheric variability.

The nature of this dependence will be brought out more clearly in subsequent figures.

Fig. 2 shows the correlation coefficient between sea-level pressure and 500 mb height based on the

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4 The results in these earlier papers are for a slightly different set of winters.
monthly mean data described in (iii). The resemblance to Fig. 9 of Sawyer (1970) is quite striking, even though Sawyer's results are not broken down with respect to season and his calculations are based on time-filtered daily data rather than on monthly mean data. Large positive correlations are found over the oceans poleward of ~30° latitude and over Europe and coastal regions of North America. Negative correlations are found over much of China, central Asia, North Africa and south-central United States. In general, higher correlations tend to be found in regions characterized by mild, maritime winter climates and lower (or negative) correlations tend to be found in regions characterized by colder, drier, continental-type winter climates.

We will now examine the correlation coefficient between sea level pressure and 500 mb height over a range of frequencies by showing, in Fig. 3, distributions analogous to that in Fig. 2, for 1) daily data, which have not been time-averaged or filtered; 2) band-pass filtered daily data; 3) low-pass filtered daily data; and 4) seasonal average data. The daily data are for the 11 winter seasons beginning with the winter of 1965–66 and the seasonal averages are based on the same 15 winter data set as Fig. 2. (The filters and data processing procedures are as described in Section 2.) All four distributions are rather similar to the distribution shown in Fig. 2 but there are some significant differences.

The high-frequency fluctuations exhibit somewhat lower correlation coefficients over the oceans and particularly over the western oceans near 40°N where the contrast between the values in Figs. 2 and 3b is largest. The correlation coefficients gradually increase as one moves eastward across the oceans from the regions of vigorous baroclinic wave development near Japan and the east coast of the United States toward the regions of decaying disturbances near the west coasts of the continents. These results are in accord with a recent study by Lau (1979) in which it was shown on the basis of cross-spectrum analysis applied to this same data set, that transient fluctuations in this range of frequencies undergo a systematic decrease in westward tilt with height as they cross the oceans.

In the seasonal mean pattern in Fig. 3d the belt of low values of the correlation coefficient to the east of the Rockies is split into two centers: one located near the Canada-Alaska border and the other over the Gulf of Mexico. This pattern also shows distinct minima near 50°N along the east coasts of Asia and North America. There is some suggestion of these same features in the monthly mean data in Fig. 2, but they are largely absent in the daily data (Figs. 3a–3c).

Fig. 4 shows the ratio of the standard deviation of 500 mb geopotential height to the standard deviation of 1000 mb geopotential height based on monthly mean data analyzed with a logarithmic contour interval. This ratio provides a measure of how the amplitude of monthly mean geopotential anomalies vary with pressure. Although the pattern in Fig. 4 is rather complicated, certain features have been found to be quite reproducible from one subset of the data to another (e.g., in the statistics for the 15 individual Decembers):

- Relatively low values cover much of the eastern oceans and the European part of the Soviet Union. In these regions the amplitudes of geopotential height fluctuations increase by less than 50% from the 1000 mb level to the 500 mb level. The minima over the low latitude oceans correspond closely to the positions of the subtropical highs in the sea level pressure field.

- High values of the ratio are centered over the southeastern United States and just south of Japan, where the 500 mb height fluctuations are about three times as large as the fluctuations in 1000 mb height. These high values are located near and to the south of the regions of strong baroclinic development described by Lau (1979).

- High values are also found at subtropical latitudes along the west coasts of North America and Africa, which are relatively quiescent regions. The North American feature extends to higher latitudes and exhibits a secondary maximum over British Columbia.

- Low values are present along the west coast of the Gulf of Mexico and over parts of Southeast Asia and the Himalayas. The very low values over
the Himalayas and Tibet may be an artifact of the reduction of pressure to sea level.

Fig. 5 shows the same ratio for the four data sets described in connection with Fig. 3. The unfiltered daily data (Fig. 5a), the low-pass filtered daily data (Fig. 5c) and the seasonal average data (Fig. 5d) all show essentially the same features as described above. The distribution corresponding to the band-pass filtered data (Fig. 5b) is somewhat different: the minima over the eastern oceans are largely absent and the maxima over the east coasts of Asia and North America are less prominent than in the other figures. The minima along the Texas coast and over the highlands of Asia are more pronounced (remember, however, that the values over the Himalayas and Tibet may be artificially low) and the latter area extends much further westward toward the Caspian Sea. In the vicinity of these minima, the 1000 mb height exhibits more high frequency variability than the 500 mb height.
b. Summer

Fig. 6 shows the geographical distribution of the standard deviations of (a) 1000 mb height and (b) 500 mb height, based on the monthly mean data set for the 42 summer months described in (iii) of Section 2. Corresponding distributions for the unfiltered, band-pass filtered and low-pass filtered twice-daily data sets described in (i) and (ii) of Section 2 are presented in Blackmon (1976) for the 500 mb level.

In view of the relatively weak frequency dependence of the patterns in Figs. 3 and 5, we have chosen to describe the gross features of the summertime sea-level pressure and 500 mb geopotential height relationships through the use of monthly mean data alone. Fig. 7 shows the geographical distribution of the correlation coefficient between sea-level pressure and 500 mb height for the summer season, based on monthly mean data for 42 summer months. The gross features of the pattern bear considerable similarity to those in Fig. 2: Strong positive correlations are present over the eastern oceans and weak or negative correlations are observed over land areas characterized by continental climates during both seasons. However, there are some important seasonal differences:

- Correlation coefficients over the western third of the United States are positive during winter but negative during summer with a pronounced minimum along the California coast. A similar region of negative correlation is found along the northwest coast of Africa during summer.
- During summer there exists a region of significant negative correlations centered over the Canadian prairies to the west of Hudson Bay.
- Correlation coefficients over China are more positive during summer than during winter.

Fig. 8 shows the ratio of the standard deviation of 500 mb height to the standard deviation of 1000 mb height for the summer season, using the same plotting conventions as in Fig. 4. The seasonal differences in this field are quite large:

- The regions where the ratio is large move poleward during summer to latitudes around 50°N. The local maxima over British Columbia, Hudson Bay and eastern Siberia are also present as secondary features in the wintertime distribution (Fig. 4).
- The small region of relatively low ratios over the Gulf of Mexico during winter expands and moves poleward during summer. There appears to be an analogous seasonal change over the region south of Japan. Belts of relatively low values extend northeastward from these regions across the Atlantic and Pacific Oceans. These belts correspond closely to the axes of the oceanic high pressure belts in the sea level pressure patterns, which are indicated by the dashed lines in Fig. 8. A similar relationship was found to exist during the winter season.

In summary, the correlation coefficient between 1000 and 500 mb height and the ratio of the amplitude of the height fluctuations at the two levels both display strong regional contrasts that are not related to one another in any obvious way. The patterns in the two fields appear to be remarkably independent of frequency, particularly for frequencies lower than those of migrating synoptic-scale systems. The wintertime and summertime patterns show some similarities, but there are some significant seasonal differences, particularly in the pattern of amplitude ratios which shifts poleward during summer.

4. Evidence based on station data

In order to demonstrate that the regional differences in vertical structure described in the previous section are not merely a response of the NMC objective analysis scheme to inhomogeneities in the spacing of radiosonde stations, we will show in this section some sample time series of 1000 and 500 mb height for a pair of stations located in regions where the transient fluctuations exhibit a strongly contrasting vertical structure. These same time series will also serve to illustrate, in a more concrete form, the importance of these contrasts.

Fig. 9 shows sample time series of 1000 and 500 mb height, unfiltered and low-pass filtered, for Shanwell, U.K. (56.4°N, 2.9°W) and Bismarck, North Dakota.
Fig. 5. As in Fig. 4 but based on (a) unfiltered daily data, (b) band-pass filtered daily data, (c) low-pass filtered daily data and (d) seasonally averaged data.

(46.8°N, 100.8°W) for the 1962–63 winter. The Shanwell time series are strongly correlated in the vertical (the correlation coefficients are 0.81 for the unfiltered series and 0.95 for the low-pass filtered series, in close agreement with the results in Figs. 3a and 3c). In contrast, the Bismarck time series for this particular winter show evidence of a weak or negative correlation in the vertical (the corresponding correlation coefficients are −0.28 and −0.64, respectively; considerably more negative than the 11 winter average results in Figs. 3a and 3c). Hence, we conclude that the regional contrasts in 1000 and 500 mb height statistics described in the previous section are indicative of real geographical variability in the structure of atmospheric disturbances. A more detailed analysis of station data, including data for weather ships and Southern Hemisphere stations, is now in progress.
5. Relations involving 1000–500 mb thickness

Making use of the product moment formula for the correlation coefficient, together with the trigonometric identity relating the length of one side of a triangle to the lengths of the two other sides and the cosine of the opposing angle, it is readily shown that fluctuations in 1000 mb height, 500 mb height and 1000–500 mb thickness are related as indicated in Fig. 10 where the lengths of the three sides of the triangle are proportional to the standard deviations (denoted by $\sigma$) of the three parameters in question and

\[
\begin{align*}
\cos \alpha &= r(z_{1000}, z_{500}) \\
\cos \beta &= r(z_{500}, z_{500} - z_{1000}) \\
\cos \gamma &= r(z_{1000}, z_{500} - z_{1000})
\end{align*}
\]

where $r$ refers to the correlation coefficient between the two parameters indicated in parentheses,
$z_{500}$ and $z_{1000}$ are 500 and 1000 mb height, respectively, and $z_{500} - z_{1000}$ is the thickness of the 1000–500 mb layer. Similar relationships were used by Klein (1951) in his study of the variance associated with day-to-day changes in sea-level pressure.

Making use of the relationships in Fig. 10, it is possible to infer some of the characteristics of the thickness fluctuations on the basis of the results presented in the previous section. For example, in the vicinity of the British Isles, the 1000 and 500 mb height fluctuations are roughly comparable in size and positively correlated with one another ($\alpha \approx \pi/2$); hence the thickness fluctuations should be small, as indicated in Fig. 10a. In contrast, over parts of central Asia, where 1000 and 500 mb height are negatively correlated, thickness fluctuations should be large and negatively correlated with fluctuations in 1000 mb height, as indicated in Fig. 10b.

### a. Winter

Fig. 11 shows the correlation coefficient between 500 mb height and 1000–500 mb thickness for the winter season, based on the monthly mean data. Positive correlations prevail throughout the hemisphere, but there are significant geographical variations. The lowest values are found over the eastern oceans in the vicinity of the subtropical highs in the sea-level pressure field and over the European part of the Soviet Union. The overall pattern bears some similarity to that for the $\sigma(z_{500})/\sigma(z_{1000})$ amplitude ratio in Fig. 4. The correlation coefficient between 1000 mb height and 1000–500 mb thickness, shown in Fig. 12, displays a much stronger geographical variability with values ranging from above +0.5 over the northern oceans to below −0.5 over much of Asia.

![Fig. 10. Relationships between standard deviations of 1000 mb height, 500 mb height, 1000–500 mb thickness, and the correlation coefficients between those variables over (a) regions with maritime climates and (b) regions with continental climates. See text for details. Note: Triangles are not drawn to scale.](image-url)
The ratio of the standard deviation of 1000–500 mb thickness to that of 500 mb height, based on data for the same 45 winter months, is shown in Fig. 13. The patterns are very similar to those in Fig. 2 with thickness variability exceeding 500 mb height variability over regions characterized by “continental climates,” and much more barotropic fluctuations prevailing over the regions characterized by “maritime climates,” particularly the eastern oceans. The corresponding ratio of the standard deviation of 1000–500 mb thickness to that of 1000 mb height is shown in Fig. 14. The resulting pattern is obvi-
summer season, based on the monthly mean data. As in winter, positive correlations prevail throughout the hemisphere but there are significant geographical variations, most of which bear a strong relation to features in the pattern of \(\sigma(z_{500})/\sigma(z_{1000})\) amplitude ratio. The correlation between 1000 mb height and 1000–500 mb thickness, shown in Fig. 16 exhibits strong regional contrasts with predominantly positive values at high latitudes, and negative values equatorward of 40°N. The corresponding amplitude ratios shown in Figs. 17 and 18 are strongly related to the patterns shown previously.

c. Further interpretation

Although the distributions shown in this section are in some sense entirely redundant with those presented in Section 3, they reveal more clearly certain aspects of the vertical structure of low-frequency atmospheric variability. In fact, it is evident from Fig. 10 that these distributions are no more or no less fundamental than those selected for presentation in Section 3. A knowledge of relationships involving the thickness field should be helpful in interpreting low-frequency temperature variability in terms of fluctuations in the geopotential height field.

6. Results of eigenvector analysis

In this section we will consider the extent to which the statistics for the 1000 and 500 mb height fields presented in the previous sections are representative of the vertical structure of the first mode of an eigenvector analysis expansion based on twice-daily data for the 10 pressure levels indicated in
Table 1 for three winter seasons (1971–72, 1972–73 and 1973–74; the months of December, January and February).

Previous investigations of the eigenvectors (or empirical orthogonal functions) associated with the vertical structure of the fluctuations in the middle- and high-latitude troposphere have mostly been based on covariance matrices that are spatially averaged. Analyses by Obukhov (1960), Holmström (1963) and others indicate that the first or dominant mode, which typically accounts for more than 70% of the total variance, is in phase throughout the troposphere. Its amplitude increases monotonically with height from near zero at 1000 mb to a maximum near the tropopause level (250 mb), above which it decreases rapidly with height. On the basis of the results presented in the previous sections, it can be inferred that the structure of the first mode should vary considerably from place to place. For example, over England one should expect to see substantial amplitudes at the 1000 mb level, in phase with those at higher levels, whereas over central Asia one should expect to see a phase reversal between the 1000 and 500 mb levels.

In order to test the validity of these inferences, eigenvector analysis was performed on the NMC data described above, interpolated onto a 10° latitude by 10° longitude grid. The procedure for calculating the eigenvectors $F_k$ was as follows:

(i) $C(i,j)$, a covariance matrix between geopotential heights at pressure levels $p_i$ and $p_j$, was estimated for each grid point. Each element in $C(i,j)$ was then pressure-weighted by the factor $(\Delta p_i \Delta p_j)^{1/2}/925$. The $\Delta p_i$ used are indicated in

![Diagram](image)

**Fig. 18.** As in Fig. 14 except for summer.

Table 1. Pressure levels $p_i$ and weighting factors $\Delta p_i$ used in eigenvector analysis.

<table>
<thead>
<tr>
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<th>7</th>
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<td>100</td>
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<tr>
<td>$\Delta p_i$</td>
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<td>175</td>
<td>175</td>
<td>150</td>
<td>100</td>
<td>75</td>
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</tbody>
</table>

Table 1 and 925 is the total pressure interval which is taken to be 1000–75 mb.

(ii) The eigenvectors were then determined by first multiplying the pressure-weighted covariance matrix by a trial vector. The product provides a second trial vector. The multiplication is repeated and the process continues iteratively until successive products converge to a single vector. The method follows exactly that outlined by Lawley and Maxwell (1963, pp. 45–51). The resulting eigenvectors are multiplied by $(25/\Delta p)^{1/2}$ in order to make them orthogonal when weighted with respect to pressure, as in Eq. (1).

(iii) Finally, following Holmström (1963), the eigenvectors $F_k(p)$ were normalized such that

$$\frac{1}{925} \int_{-75}^{75} F_k^2(p) dp = 1, \quad p = 1000,$$

with $dp$ approximated by $\Delta p_i$ of Table 1.

For each grid point the covariance matrix was estimated by averaging over the three winter months of the corresponding winter seasons; therefore, they represent covariability in time only. In order to minimize the computer resources required for this analysis the annual cycle was not removed from the data. Inspection of the 1000–500 mb covariances computed before and after the removal of the annual cycle indicated that the contribution of the annual cycle to the covariance patterns is very small.

Fig. 19 shows the horizontal distribution of the percentage variance explained by the first mode, $k = 1$, in the eigenvector analysis expansion. With the exception of a few small regions, the percentage is in excess of 75% and it reaches values as high as 95% over the British Isles. The next two figures describe how the vertical structure of this dominant first mode varies from place to place during the Northern Hemisphere winter season.

Fig. 20 shows a comparison between the distribution $[F_i(1000)/F_i(500)]$ and $[r(z_{1000}, z_{5000})/\sigma(z_{1000})]$ where the latter statistic is based on exactly the same data set as the covariance matrix that was used for the eigenvector analysis. The strong similarity between the patterns in the two figures confirms our expectation that the 1000 and 500 mb data alone can be used to infer with considerable accuracy the horizontal distribution of certain prop-
Fig. 19. Percentage variance explained by the dominant mode in an eigenvector analysis expansion of geopotential height at ten pressure levels for three winter seasons.

Fig. 20b. Correlation coefficient between 1000 and 500 mb heights multiplied by the ratio of the standard deviation of 1000 mb height to the standard deviation of 500 mb height, based on monthly mean data for the same three winters used for (a).

properties of the dominant mode in the full vertical empirical orthogonal function expansion of geopotential height in the vertical, in the 1000–100 mb layer. A clearer interpretation of this result is given in Fig. 21 which shows the actual shape of this dominant mode at four selected locations in the Northern Hemisphere for the three individual winters used in the expansion. Over the eastern oceans this first mode contains a strong barotropic component with substantial 1000 mb height fluctuations, in phase with those at upper tropospheric levels; this result is highly reproducible from one winter to another. In contrast, at the two continental locations shown in this figure, the amplitude of the first mode either becomes negligibly small at the 1000 mb level, or there is a phase reversal with 1000 mb height fluctuations being out of phase with geopotential height fluctuations in the upper troposphere; the vertical structure in these regions varies somewhat from winter to winter. Hence, Fig. 20 can be interpreted as giving the geographical distribution of a certain “shape parameter” which describes most of the important regional differences in the vertical structure of day-to-day geopotential height fluctuations at individual stations. Furthermore, the figure confirms that the distribution of the “shape parameter” can be inferred from the 1000 and 500 mb height statistics presented in the previous sections.

7. Vertical structure of planetary-scale versus synoptic-scale disturbances

In this section we will attempt to determine the horizontal scales of the transient disturbances responsible for the strong regional contrasts in vertical structure statistics described in the previous sections. For this purpose the unfiltered daily data described in Section 2a were expanded in terms of zonal harmonics on each latitude circle. The respec-
tive sine and cosine functions for each synoptic map were then summed over groups of zonal wave numbers \((k)\) in order to obtain time series of 1000 and 500 mb height for planetary-scale \((k = 0–3)\) and synoptic-scale \((k \geq 5)\) components of the geopotential height field. Statistics analogous to those in Section 3 were then computed for these planetary-scale and synoptic-scale time series. It is important to emphasize that these twice-daily time series were not subject to any filtering or other pre-processing in the time domain other than the removal of the least-squares best-fit parabola for each of the 11 winter seasons; except for the breakdown in terms of zonal wavenumber, they are entirely analogous to the series that form the basis for the figures in Section 3.

Fig. 22 shows the geographical distribution of the correlation coefficient between 1000 and 500 mb height for the (a) planetary-scale and (b) synoptic-scale disturbances in the winter season. From a comparison between this figure and Fig. 3a, it is immediately apparent that the dominant pattern of geographical variability in the temporal correlation between 1000 and 500 mb height is associated with planetary-scale disturbances. The corresponding planetary-scale pattern in the monthly mean data described in Section 2c (not shown) bears an even stronger resemblance to its counterpart in Fig. 2. These results are confirmed by Fig. 23 which shows the correlation coefficient between 1000 and 500 mb height, based on 30-day mean data for the months of December–February for 17 winter seasons. The strong similarity between the patterns in this figure and those in Fig. 2 indicates that the disturbances

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5 This figure, kindly supplied to us by Glenn H. White, was generated as part of a different investigation, and hence the data set and the data handling procedures are slightly different from those used in this paper. The figure is based on data from 17 consecutive winter seasons beginning with January 1962 and ending with December 1977. The first and second harmonics of the mean annual cycle are removed from the time series. The correlation coefficient is based on 30-day mean data, computed at 10-day intervals over December, January and February. Hence the procedure is roughly comparable to that used in generating Fig. 2.
responsible for these patterns penetrate far into the stratosphere. Since the 50 mb height field is dominated by planetary waves \((k = 0-3)\), the pattern of 1000–50 mb height correlations should be largely a reflection of the geographical variability of the vertical structure of planetary waves. The pattern associated with the synoptic-scale disturbances, shown in Fig. 22b, bears a strong resemblance to the pattern associated with the band-pass time-filtered fluctuations shown in Fig. 3b. Hence, it seems reasonable to interpret these patterns in terms of the "signature" of rapidly moving and developing synoptic-scale disturbances, including baroclinic waves.

Fig. 24 shows the geographical distribution of the ratio of the standard deviation of 500 mb height to that of 1000 mb height for the (a) planetary-scale and (b) synoptic-scale disturbances. Comparison of the figure with Fig. 5a reveals that both planetary scale and synoptic-scale disturbances contribute to the patterns described in Section 3.

8. Application to the design of global observing systems and objective analysis schemes

Because of the large geographical variability in the vertical structure of tropospheric disturbances, the minimum requirements for observing systems may be much more stringent in some regions of the globe than in others and the utility of a particular component of the observing system may vary greatly from region to region. A quantitative treatment of this subject would require the calculation of multiple-regression statistics, taking into account the error characteristics of the various components of the observing system. However, on the basis of the results presented in the foregoing sections, it is possible to make some qualitative statements concerning the relative utility of remote temperature soundings versus reference level data for inferring the upper tropospheric geopotential height field in various regions of the Northern Hemisphere. For the purposes of this qualitative discussion, we will assume that the gross features of the patterns based on the monthly mean statistics are representative of the geographical variability of the
vertical structure of disturbances over the full range of time scales.

As a specific example, it is interesting to compare the problem of defining the 500 mb height field over the eastern United States versus that over the eastern Atlantic and Pacific Oceans during the winter season. Over the eastern United States 500 mb height is much more strongly correlated with 1000–500 mb thickness (Fig. 11) than with 1000 mb height (Fig. 2) and the amplitude of the thickness fluctuations is two or three times as large as the amplitude of the fluctuations in 1000 mb height (Fig. 14). In this region thickness data usually plays the dominant role in the determination of the 500 mb height field. In contrast, over the eastern oceans 500 mb height is much more strongly correlated with 1000 mb height than with thickness and the amplitude of the thickness fluctuations is smaller than the amplitude of the fluctuations in 1000 mb height. Thus in these regions the reference level (1000 mb) data ordinarily play the dominant role in the determination of the 500 mb height field. Under conditions of strong baroclinic wave development, which, though infrequent, are of greatest importance for forecasting, the situation could be quite different.

From an inspection of a series of maps analogous to those in Figs. 3a and 5a but computed for individual summers and winters (not shown), we have found that year-to-year differences in the vertical structure of transient disturbances at a given location are much smaller than the regional differences within a given season of a given year. Hence, in the design of optimum interpolation schemes for data assimilation in numerical prediction models, the treatment of geographical variability of the various weighting functions deserves higher priority than efforts to “update” the weighting functions on the basis of recent data. As a highly simplified example of the way in which the geographical variability might be taken into account, we consider the relationship for estimating 500 mb height

\[ z_{500} = z_{500}^0 + a_1 z_{1000} + a_2 L' , \]

where \( z^0 \) is some suitable “first guess” 500 mb height field, \( z_{1000} \) is a correction based on the sea level pressure field, determined from a dense network of data from ships and land stations, and \( L' \) is a correction based on the satellite observed radiance field in a wavelength band in which the radiation emanates from the lower stratosphere, just above the level of the highest stratiform cloud decks, and \( a_1 \) and \( a_2 \) are weighting functions which vary with latitude and longitude. We would expect \( a_1 \) to be quite large and positive over the eastern oceans and small or even negative over some continental regions. The other coefficient \( a_2 \) would probably be negative throughout middle and high latitudes, though it might also display significant geographical variability. The

![Fig. 24. As in Fig. 5a but including only disturbances with zonal wavenumbers (a) 0, 1, 2, 3 and (b) 5 and above.](image)

“improved first guess” derived from such a regression equation could serve as input to more conventional “optimum interpolation” schemes. It might be particularly helpful in the analysis of the planetary waves, because of their strong vertical coherence.

9. Physical Interpretation

The large observed geographical variability of the vertical structure of transient disturbances in the earth’s atmosphere does not appear to be due to
any single, obvious dynamical mechanism. Planetary-scale waves account for most of this variability, at least during winter; but synoptic-scale disturbances account for some of the more localized features. The dynamical processes responsible for this variability appear to operate on time scales ranging from a week to a season with relatively little frequency dependence. Some of the major features of the geographical distribution of vertical structure are present in similar locations throughout the year while others exhibit significant seasonal differences.

We can tentatively identify some of the more localized features in the geographical patterns of vertical structure with distinctive regional characteristics of the synoptic climatology of the Northern Hemisphere; e.g.,

1) Baja California is a favored site for the occurrence of wintertime, cold-core cutoff lows which are relatively intense at the 500 mb level but very weak at the earth’s surface. These systems might account for the localized maximum in \( \sigma(z_{500})/\sigma(z_{1000}) \) over this region in Figs. 4, 5, and 24b.

2) During heat waves along the west coast of the United States the “thermal low” which normally covers the southwest desert region during summer temporarily extends northward into coastal Oregon and Washington. The return of cooler air at the end of such an episode is usually marked by a pronounced drop in 500 mb height together with a sharp rise in sea-level pressure. This phenomenon may be responsible for the local minimum in \( r(z_{1000}, z_{500}) \) over this region in Fig. 7. It appears as though the coast of Morocco might have a similar synoptic climatology, in this respect.

3) Incursions of shallow, modified polar air penetrate into particularly low latitudes over Indochina and over the lowlands of Texas and Mexico, just to the east of the Rockies during the winter season. These so-called “cold surges” are marked by pronounced rises in sea-level pressure accompanied by more modest geopotential height changes at the 500 mb level. This phenomenon may be at least partially responsible for the low ratios of \( \sigma(z_{500})/\sigma(z_{1000}) \) over these regions in Figs. 5a, 5b and 24b.

The geographical contrasts in vertical structure noted in the previous sections are sharpest for the low-frequency planetary waves. At this point we have only a few pieces of indirect evidence concerning their nature and cause. The strong correspondence between the patterns of the correlation coefficient between 1000 and 500 mb height and the mean sea-level pressure pattern (e.g., the regions of low or negative correlations near and to the south of the cold continental anticyclones, the regions of high positive correlations near and to the south of the Aleutian and Icelandic lows, and the relationship of some of the patterns to the subtropical oceanic anticyclones) suggests that most of the transient variability with periods longer than a week is related to fluctuations in the quasi-stationary planetary wave pattern forced by flow over topography and by longitudinal heating contrasts. Are these fluctuations a manifestation of some sort of dynamically determined vacillation cycle involving the quasi-stationary planetary waves, or are they essentially a passive response to temporal variations in the forcing (e.g., changes in cloudiness or snow cover, or in the mass flux out of the tropical monsoons)? The observed lack of frequency dependence of the vertical structure of fluctuations with periods longer than a week is supportive of the latter interpretation, but further analysis of the synoptic climatology of low-frequency planetary waves will be needed to resolve this question in a definitive manner.

Acknowledgments. We would like to thank Dr. Ngar-Cheung Lau for much helpful computing assistance. His data set (to be published as an NCAR technical note) provided the basis for most of our daily data calculations. Mr. Roy Jenne and his associates facilitated the consolidation of monthly mean data.

This work was supported by the National Science Foundation under Grant ATM 78-07369 (Climate Dynamics Program, Climate Dynamics Research Section, Division of Atmospheric Sciences).

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