Barotropic Wave Propagation and Instability, and Atmospheric Teleconnection Patterns

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ABSTRACT

A global barotropic model, linearized about the 300 mb climatological mean January flow, is perturbed by applying a series of localized forcings distributed throughout the tropics and subtropics. Structures which resemble the observed “Pacific/North American” and “East Atlantic” teleconnection patterns noted by Wallace and Gutzler (1981) tend to recur in the responses. Similar patterns are found to result from the dispersion of isolated initial perturbations placed at a variety of locations in the tropics and midlatitudes. It is shown that these structures are related to the most rapidly growing mode associated with barotropic instability of the zonally-varying climatological basic state. In the absence of damping, this mode has an e-folding time of about a week and a period close to 50 days. In localized regions the instantaneous growth rates can be competitive with those of baroclinic instability. These episodes of rapid local barotropic growth are interspersed with intervals in which the local perturbation relaxes as energy disperses throughout the hemisphere. Some of the less unstable modes exhibit a similar structure and time evolution, while others are spatially fixed and grow exponentially in time, without any periodic modulation.

The dominant processes by which the growing disturbances extract kinetic energy from the basic state can be described in terms of two conversion terms: \(-u'v''\partial u'/\partial y\) and \((v'^2 - u'^2)\partial u'/\partial \lambda\), where \(u\) and \(v\) are the velocity components in the zonal (\(x\)) and meridional (\(y\)) directions, respectively, primes refer to the perturbations, and overbars to the basic state. In the fastest growing mode the dominant process contributing to the growth is found to be the second term, which is largest in the Pacific jet exit region where \(u'^2 \gg v'^2\).

Nonlinear initial value calculations for weak damping rates exhibit strong low-frequency oscillations similar in structure and period to those of the most unstable mode. Nonlinear forced solutions show some sensitivity to the polarity of the forcing and they tend to show a larger response in the time average over the first month of the integration than in longer term means. Sensitivity to model resolution, dissipation, and choice of basic state is illustrated by means of selected experiments.

On the basis of these results and recent work by Hoskins et al. (1983), it is suggested that much of the low-frequency variability of the Northern Hemisphere wintertime general circulation is associated with disturbances which derive their energy from the basic state through barotropic instability.

1. Introduction

The tendency for one-point correlation maps for the geopotential height field to show pronounced, geographically dependent, wavelike structures, sometimes referred to as teleconnection patterns, has been known to synopticians for many decades. Observational documentations of these patterns can be found in the works of Walker and Bliss (1932), Sawyer (1970), van Loon and Rogers (1978), Wallace and Gutzler (1981), Namias (1981), Horel (1981) and Blackmon et al. (1983a), among others. During wintertime these dominant teleconnection patterns are best defined at middle and upper tropospheric levels and they are characterized by an equivalent barotropic structure. Although vestiges of these patterns are detectable even in unfiltered daily data, it appears that they are mainly associated with fluctuations with periods longer than a month (Blackmon et al., 1983a,b). Perhaps the most striking example of such a geographically localized pattern is the one that Wallace and Gutzler (1981) referred to as the “Pacific/North American” (PNA) pattern, shown in Fig. 1a. The correlations over the Atlantic sector were described by those authors in terms of “West Atlantic” and “East Atlantic” (EA) patterns. The EA pattern, which is of greater relevance for the present study, is shown in Fig. 1b.
the pattern of geopotential height anomalies during Northern Hemisphere winters in which equatorial Pacific surface waters are abnormally warm (e.g., see Horel and Wallace, 1981), is indeed suggestive of two-dimensional Rossby-wave dispersion from a localized source. However, it is also possible that the teleconnection patterns are a consequence of dynamical processes operating within the atmosphere itself, on timescales longer than that characteristic of Rossby-wave dispersion or the lifetime of baroclinic waves. The latter view is supported by recent results of Lau (1981) who obtained rather realistic looking teleconnection patterns by processing the history tapes derived from an extended run of a GCM in which there was no time-dependent external forcing other than the annual cycle. Related results have also been reported by Cubasch (1981) and Volmer et al. (1983).

The original purpose of the present work was to determine whether some of the observed teleconnection patterns could be simulated by subjecting a global barotropic model, linearized about the climatological mean wintertime basic state to an essentially random distribution of local forcings, and generating spatial correlation statistics, analogous to those displayed in Fig. 1, from the resulting family of responses derived from the model. In the course of the study, it was noted that certain patterns tended to recur in the responses observed in many of the individual experiments with different forcings. Further analysis revealed that these preferred patterns were related to the normal modes associated with barotropic instability of the 300 mb climatological mean basic state. It appears possible that some of the observed teleconnection patterns may be related to this instability phenomenon.

The concept of barotropic disturbances which derive their energy from a zonally varying basic state dates back at least as far as the work of Lorenz (1972) who considered the stability properties of a pure, zonally propagating Rossby wave on a beta-plane, scaled so as to be representative of conditions on a typical upper level synoptic chart. The idealized flow proved to be barotropically unstable, where the perturbations had an e-folding time of the order of two days and a structure conducive to the formation of jet-like features in the zonal flow. Lorenz emphasized the importance of this instability phenomenon in limiting atmospheric predictability. In the present work, we are concerned with the structure and temporal variability of low-frequency fluctuations which derive their kinetic energy from barotropic instability of the observed climatological mean flow. Other contributions to the literature on this subject include the works of Hoskins (1973), Gill (1974) and Baines (1976). These earlier works dealt with idealized flows and were mainly concerned with growth rates, instability mechanisms and criteria for instability. In contrast, in the present work we are concerned with the struc-
ture and temporal variability of low-frequency fluctuations which derive their kinetic energy from barotropic instability of the observed climatological mean flow.

The following section introduces the barotropic model upon which the calculations in this paper are based. Sections 3 and 4 describe the time-dependent linear response of the model to tropical forcing and to initial vorticity perturbations at various locations. Sections 5 and 6 examine the normal mode instability of the climatological mean basic state, based on two different approaches: extended linear time integrations and direct solution of the eigenvalue problem, respectively. (These two sets of calculations, including the underlying model development, were carried out entirely independently, the former by the senior author at ECMWF and the latter by the third author at NCAR.) The energetics of the instability are presented in Section 7. Sections 8 and 9 discuss the results of sample extended nonlinear integrations, based on perturbed initial states and steady forcing, respectively. Section 10 discusses the sensitivity of the results to changes in the basic state and the final section discusses the implications of the results of this study.

2. The barotropic model

The barotropic model is that described by Simmons (1982). It employs a triangularly-truncated spectral representation of fields, and is a global, barotropic version of the multi-level model developed by Hoskins and Simmons (1975). For most of the calculations presented here, a truncation at total wavenumber 21 (referred to as T21) has been used, with a 1 h timestep. Sample solutions have also been computed using higher resolution, with a truncation at wavenumber 42 (T42). Differences will be discussed in cases where they have been found to be significant.

The principal dissipative mechanism in the model is a linear drag. Some aspects of our results have been found to be particularly sensitive to the value of this drag, and calculations will be presented for decay rates of both (5 day)$^{-1}$ and (10 day)$^{-1}$. The model also includes a linear, fourth-order diffusion and, except where otherwise stated, all calculations are for a coefficient of $2 \times 10^{-16}$ m$^3$ s$^{-1}$, which effectively damps the smaller scales at resolution T42, with a decay rate of (0.3 day)$^{-1}$ on the shortest retained scale. It is largely ineffective at resolution T21. Sensitivity to the choice of diffusion coefficient is discussed briefly in Section 9.

The basic state used for all calculations is not a zonal-mean flow as has commonly been adopted for studies of wave dispersion or instability. Instead, a basic flow which varies with longitude as well as latitude is chosen, and the model equations are used to compute precisely that forcing required to keep this basic flow steady. Integrations are then performed in which the balance between the basic state and its associated forcing is disturbed either by adding a perturbation to the forcing, or by adding a perturbation to the initial, zonally-varying flow. Results will be presented from both linear and nonlinear calculations. Linearity is achieved by ensuring a sufficiently small initial perturbation and, in the case of unstable growth, by scaling perturbation amplitudes by a fixed amount at daily intervals.

The basic state used for most results presented here is a 300 mb climatological-mean streamfunction for January. It has been derived from the data of Crutcher and Meserve (1970) and Taljaard et al. (1969), and is illustrated in Fig. 2. The forcing required to maintain this state is essentially as shown by Lau (1979, Fig. 6a). Our choice of the 300 mb level and the January climatology gives a basic state with a larger stationary-wave component than would be the case for lower levels or other seasons. It may be justified partly by observations that show the low-frequency variability of the atmosphere to have its maximum amplitude near the winter tropopause (Blackmon et al., 1977; Lau et al., 1981) and partly by theoretical arguments which indicate that the appropriate level to apply a barotropic model is higher than the traditional 500 mb (Held, 1983). In addition, seasonal and shorter-term means exhibit stationary wave amplitudes which may on occasions be significantly larger than climatological mean values. Use of a climatological basic state might thus underestimate the effect of zonal variations for such situations.

![Fig. 2. The climatological-mean 300 mb streamfunction for January. The contour interval is equivalent to a 160 m interval in geopotential height computed by applying the geostrophic approximation at 45°N.](image-url)
In the case in which the initial state is perturbed by an additional forcing, this forcing is chosen to model the effect of a localized, anomalous tropical heating such as might be excited by a sea-surface temperature anomaly (or, in a numerical model, by an erroneous parameterization of convective heating). The forcing is as used by Simmons (1982), and is specified by a term \( -fD \) in the vorticity equation, where \( f \) is the Coriolis parameter and the “divergence” \( D \) has the spatial distribution given (apart from subtraction of the zonal mean) by

\[
\left\{ \begin{array}{l}
\cos\left(\frac{\pi(\theta - \theta_d)}{\theta_d}\right) \cos\left(\frac{\pi(\lambda - \lambda_c)}{\lambda_d}\right)^2, \\
|\theta - \theta_c| < \frac{1}{2}\theta_d, \ |\lambda - \lambda_c| < \frac{1}{2}\lambda_d
\end{array} \right. , \tag{1}
\]

where \( \theta \) is latitude and \( \lambda \) longitude. In all calculations we adopt the values \( \theta_d = 30^\circ \), \( \lambda_d = 90^\circ \), and for these values this distribution is shown in the upper right part of Fig. 3 for \( \theta_c = 15^\circ \)N and \( \lambda_c = 135^\circ \)E. The amplitude of the forcing has a maximum value of \( 2 \times 10^{-6} \) s\(^{-1} \), which corresponds approximately to the heating associated with a tropical rainfall anomaly of \( 5 \) mm day\(^{-1} \) (Hoskins and Karoly, 1981).

The other case for which we present results is that in which it is not the basic-state forcing that is locally perturbed, but rather the basic state itself. The perturbation streamfunction is given by the same distribution (1) as used for the tropical forcing, but with the values \( \theta_d = \lambda_d = 45^\circ \). It is also illustrated in Fig. 3, for \( \theta_c = 45^\circ \)N and \( \lambda_c = 45^\circ \)W. Our results in this case extend studies of barotropic dispersion, for example that by Hoskins et al. (1977), to the case of a zonally-varying basic state. They may be of relevance to the behavior of mature baroclinic waves in the upper troposphere since it has long been recognized (e.g., Namias and Clapp, 1944; Rossby, 1945), and more recently quantified in modeling studies (Simmons and Hoskins, 1978, 1980), that this behavior may often be described at least qualitatively by barotropic theory.

3. The linear response to tropical forcing

A series of 240 10-day integrations has been performed using a time scale of 5 days for the linear drag. Each integration was characterized by the latitude \( \theta_c \) and longitude \( \lambda_c \) of the forcing maximum. Values of \( \theta_c \) were varied from \( 15^\circ \)S to \( 30^\circ \)N at intervals of \( 5^\circ \), while values of \( \lambda_c \) spanned complete latitude circles at intervals of \( 15^\circ \) of longitude.

Individual samples from within this family of solutions are much as illustrated in Figs. 17 and 18 of Simmons (1982), the only essential differences in the present case being a generally weaker response resulting from the use of a higher dissipation rate. In particular, inspection of the solutions reveals that the tropical forcing excites wavetrains with amplitudes which are characteristically largest over the northeastern Pacific Ocean and to a lesser extent the northeastern Atlantic, regions which observational studies have shown to be centers of maximum low-frequency variability (Blackmon, 1976; Lau et al., 1981). The patterns of response bear clear similarities to the Pacific North American and East Atlantic teleconnection patterns shown in Fig. 1.

Some specific examples will be presented later in this paper. Here, results are summarized in terms of some statistics of the 240 responses. The mean response is found to be negligibly small, different forcing regions exciting maximum responses of different sign, but maps of standard deviation demonstrate the tendency for individual solutions to exhibit maxima over the North Pacific and Atlantic. An example, computed for day 10, is presented in the upper panel of Fig. 4.

The lower panels of Fig. 4 show the correlation between the perturbation streamfunction at two selected gridpoints (the centers of maximum variance in Fig. 4a) and perturbation streamfunction throughout the hemisphere. Comparison with Fig. 1 shows evident similarities, and although differences in detail may be found, the degree of agreement is comparable to that between observations and the results of the low-resolution GCM integration analyzed by Lau (1981).

The link between the region of forcing and region of largest response is illustrated in Fig. 5. In this figure
the sign plotted at each center of forcing indicates the sign of the largest perturbation excited anywhere on the globe at day 10, and the amplitude of this perturbation is illustrated by the size of the plotted sign. For a representative selection of cases exciting large amplitudes, the center of forcing (which is marked by a heavier sign) is connected with the point of maximum response. For these cases the amplitudes themselves are also indicated by the size of the square drawn at the point of response.

Fig. 5 confirms the existence of regions of preferred response over the northeastern Pacific and Atlantic. Large perturbations over the Pacific are most easily excited by forcing located over Southeast Asia and the tropical northwest Pacific Ocean, and the Atlantic pattern is predominantly excited by forcing similarly located to the southwest of the response region. The sign reversal indicates that the Pacific pattern may be particularly effectively excited by a forcing which has a sign change between the central and extreme western Pacific. Such a pattern is indeed characteristic of rainfall and surface-pressure anomalies associated with the Southern Oscillation (Kiddon, 1975; Quinn et al., 1978; Liebmann and Hartmann, 1982).

A similar series of model runs has been carried out for a 10-day dissipation time. In this case the longitudinal interval between forcing centers was chosen to be 30°. Results are very similar to those discussed above, apart from generally larger responses. The maximum in standard deviation of the height field at day 10 increased from 82 m for the 5-day drag to 133 m with the weaker dissipation.

4. Linear initial-value problems

The response to initial, localized perturbations of the basic state has also been examined by performing a series of 10-day integrations with a 5-day drag. Values of the latitude \( \theta_e \), of the initial perturbation were varied from 15°S to 60°N at intervals of 5°, while the corresponding longitudes \( \lambda_e \), were specified at 30° intervals. We first discuss two particular solutions which exhibit relatively large amplitudes toward the end of the integration period.

Fig. 6 shows the perturbation streamfunction at days 2, 6 and 10 for an initial negative disturbance centered at 30°N on the Greenwich meridian. Over the first two days, this disturbance disperses to give a train of waves which at day 2 is much as would be anticipated from the results of Hoskins et al. (1977), with an eastward movement and weakening of the initial center, a bowing and splitting of the ridge formed immediately downstream of it, and a gener-

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and the zero contour is dotted. (c) As in (b), but for the base gridpoint 51°N, 28°W.
ally equatorward bias to the wave propagation. Dispersion and dissipation continue beyond day 2 such that by day 6 only one major center, located just to the west of the dateline, remains.

A significant change occurs between days 6 and 10. The major center over the Pacific moves slowly eastward without loss of amplitude, with negative centers established to its south, over northeastern Siberia and over western North America. A positive center develops over the southeastern United States. This pattern is characteristic of the forced wave solutions presented by Simmons (1982) and of many of the results presented in this paper. We again note its similarity to the PNA teleconnection pattern.

The solution of a second initial value problem is illustrated in Fig. 7. In this case an initial negative perturbation is centered at 30°N, 120°E. The day 2 pattern shows a more pronounced poleward component of the initial dispersion than in the preceding example, and the largest positive region at this time is located close to the position of the largest disturbance at day 6 in the earlier case. By day 6 the perturbation is very similar in structure to the day 10 result shown in Fig. 6, though its amplitude is larger. By day 10 in this second case, the amplitude of the perturbation has diminished, but there has been little change in the wave pattern except over the Atlantic.

Although the two cases that have been illustrated here were chosen on the basis of the relatively large amplitude of what it is tempting to call their “PNA” pattern, we have observed a general tendency for this pattern to recur again and again. The standard deviation of the day 6 fields for initial perturbations located in the range 30–60°N is shown in the upper part of Fig. 8. The largest amplitude is again located over the northeast Pacific, near the region of maximum amplitude illustrated in Fig. 4a for the forced-wave solutions. The lower plot of Fig. 8 shows the corresponding correlation map for the Pacific point 39°N, 169°W. The distribution of positive and negative centers found for the two individual cases may be seen to be a characteristic feature of the whole sample.

Locating the initial perturbation in the tropics or Northern Hemisphere subtropics is a particularly efficient way of exciting the characteristic response. Fig. 9 presents standard deviations and correlations at day 6 for perturbations located initially in the range 15°S to 30°N. The maximum in standard deviation over the Pacific is about twice as large in relation to the initial amplitude of the perturbation as in the mid-latitude cases shown in Fig. 8. The correlation map for the point 39°N, 169°W is similar to other such maps presented in earlier figures, but more clearly shows the tendency to establish a distinct wave train over the Pacific and North America. Fig. 9 also shows a secondary maximum in standard deviation over the North Atlantic, and the correlation maps for the point 51°N, 28°W is similar to that presented in Fig. 4c for the forced-wave response.

Fig. 10 shows a sample initial value calculation in which the initial vorticity perturbation was located at 15°N, 90°W. The response at day 6 and beyond clearly resembles the east Atlantic pattern (Fig. 1b), but the amplitudes are not as large as those in the Pacific sector in Figs. 6 and 7. Amplitudes are found to be generally larger when a weaker linear drag is used, but patterns of response are unaltered, for reasons discussed in the following section.

5. Normal-mode instability of the climatological basic state

Extending sample linear integrations beyond day 10 has been found to yield an ultimate steady solution for a 5-day drag, but unbounded growth for a 10-day drag. This indicates that the longitudinally-varying climatological basic state used for these calculations is barotropically unstable, with an $e$-folding time between 5 and 10 days in the absence of a linear drag.
Fig. 6. The perturbation streamfunction at days 2, 6 and 10 for an initial negative disturbance centered at 30°N, 0°E. The contour interval is \( \frac{1}{20} \) of the initial maximum amplitude. Positive contours are drawn with solid lines, negative contours with dashed lines, and the zero contour is dotted.

Fig. 7. As in Fig. 6, but for an initial perturbation at 30°N, 120°E.
To investigate this further, the structure, growth rate and period of the most unstable normal mode have been determined by integrating one of the linear initial-value cases for a time period sufficiently long (in practice 200 days) to allow the perturbation to converge to the form of the fastest growing normal mode. Before discussing details, it is appropriate to outline the general form of an unstable mode for what might be called the "two-dimensional" barotropic instability of a zonally-varying basic state. As in corresponding baroclinic calculations (e.g., Frederiksen 1979), the mode does not have a regular sinusoidal variation with longitude and a regular zonal phase velocity. Rather, it is of the form

$$e^{\sigma t}[A(\lambda, \theta) \sin \omega t + B(\lambda, \theta) \cos \omega t], \quad (2)$$

FIG. 9. As in Fig. 8, but for initial perturbations centered between 15°S and 30°N. The lower panel (c) is the correlation map for 51°N, 28°W.
in amplitude modified by an overall exponential growth. Viewed synoptically, the perturbation may appear locally as predominantly standing or traveling depending on the relative magnitudes of the functions $A$ and $B$.

For the January climatology at 300 mb used as a basic state in the bulk of the computations presented here, time integrations of the barotropic model indicate that the fastest-growing mode has a period $2\pi/\omega$ of 45 days and an $e$-folding time $\sigma^{-1}$ of 6.8 days in the absence of linear drag. It is shown in Appendix A that addition of a linear drag with rate $\kappa$ simply modifies the $e$-folding time to $(\sigma - \kappa)^{-1}$, so that an $e$-folding time of 22 days is implied for a 10-day drag. Removal of the fourth order diffusion slightly reduces the $e$-folding time to 6.6 days in the zero drag case.

The structure of the fastest growing normal mode is illustrated in Fig. 11. The exponential growth factor $e^{\sigma t}$ has been suppressed in these plots, which show the resulting perturbation at selected times within the last half-period of the 200-day integration used to determine its structure. Concentrating attention first on day 178 (upper left) we note the weak positive center over the dateline near 35°N. During the next few days of the integration this center drifts slowly eastward and grows in amplitude to give the pattern shown at day 183 (upper right). Over the same time interval there is a decay of the relatively zonal perturbation over the Atlantic, whose structure qualitatively resembles the observed correlation patterns in that sector of the hemisphere. Beyond day 183 the Pacific center is the dominant feature on the map, growing more or less in situ, followed by negative and positive centers downstream over North America and the western Atlantic. A negative center also develops near Iceland, and becomes the dominant feature as the Pacific center weakens beyond day 191 (middle right). Thus by day 200 (bottom right) the pattern is almost the complete reverse of that shown for day 178. Over the Pacific, the negative center to the south of what was the major positive center has moved westward to the dateline, from which position it will move eastward and amplify beyond day 200 as the normal mode pattern repeats itself with reversed sign. A synoptically interesting view of the mode is obtained by giving it a realistic amplitude, and superimposing it on the climatological basic state. Such maps will not be presented here, as a similar sequence will be presented later (in Fig. 19) for a corresponding nonlinear solution.

With an $e$-folding time of the order of a week or more for the most unstable normal mode it might be thought that this barotropic instability would be of much less importance than baroclinic instability as a source of variability in middle latitudes. However, it must be noted that this $e$-folding time refers to the growth of a global, low-frequency mode. Locally in

where $\sigma$ is the growth rate and $\omega$ the frequency of the normal mode. At any point in space the perturbation in general undergoes a sinusoidal oscillation

FIG. 10. As in Fig. 6, but for an initial perturbation at 15°N, 90°W.
FIG. 11. The streamfunction of the most unstable normal mode at selected days within one-half cycle of its oscillation. The contour interval is arbitrary.
space and time, growth rates may be much larger, with episodes of rapid growth in regions in which the basic state is quite strongly unstable (see Section 7), alternating with intervals in which the local disturbance relaxes as energy disperses into other regions. As an example, the Pacific perturbation illustrated in Fig. 11 grows by a factor of 3 between days 178 and 183, upon which growth must be superimposed such that the overall mode growth is at a rate of \((6.8 \text{ day}^{-1})\) in the absence of dissipation. Since baroclinic modes growing on a longitudinally dependent basic state, like those of Frederiksen (1982), also exhibit locally enhanced growth rates, it is questionable whether the barotropic modes ever grow as rapidly as the baroclinic modes. Nevertheless, the barotropic modes might still be the more important contributors to atmospheric variability on the time scale of weeks by virtue of their much lower frequencies: in contrast to the baroclinic modes, whose wavetrains undergo substantial phase propagation resulting in local sign reversals in the wind and geopotential height fluctuations every few days, the barotropic modes evolve more gradually so that at some locations monotonic growth may be sustained throughout intervals of a week or longer, as demonstrated in Fig. 11.

The quite complicated behavior outlined above may be summarized in terms of the amplitude and phase of the normal mode. The mode structure (2) may be written

\[ e^{-it} R(\lambda, \theta) \cos(\omega t - S(\lambda, \theta)). \]  

(3)

Fig. 12 presents the amplitude \( R \) and phase \( S \) for the case discussed above. The amplitude is denoted both by contours and by the magnitude of the arrows on this figure. The direction of the arrows on the printed page denotes the phase, and the sign convention is such that a clockwise rotation of the arrows implies a passing of time. If there are two regions in which arrows point in opposite directions, then a positive perturbation in one of the regions occurs in conjunction with a negative perturbation in the other.

Close examination of Fig. 12 shows it to reveal the essential details deduced from the synoptic maps of Fig. 11. Plots in this format will be discussed subsequently for less unstable modes and in the context of studies of the sensitivity of results to the choice of basic state. For the time being we note, on the subject of sensitivity, that for the present basic state repetition of the normal-mode calculation with the linear diffusion set to zero results in a very similar structure, an \( e \)-folding time of 6.6 days and a period of 46 days. Repeating the calculation with resolution T42, and linear diffusion included, we again find a very similar structure and \( e \)-folding time (7 days). The period of the mode, however, is increased to 53 days. Further results concerning the sensitivity of this period will be described later.

The structure of the most unstable normal mode at that point in its cycle when it has largest amplitude over the northeast Pacific is clearly very similar to the structure characteristically excited toward the end of the 10-day integrations in the forced-wave and initial-value problems discussed earlier. More generally, a knowledge of the normal-mode characteristics is helpful in interpreting the longer term behavior of a number of integrations extended beyond day 10. For experiments with a 5-day drag, there is a gradual decay of perturbations in the initial value problems at a rate of \((19 \text{ day}^{-1})\). For the case shown in Fig. 7, the perturbation appears to be in approximately normal-mode form by day 10, and has a gradually decaying amplitude. Thus by day 18 the perturbation is found to be of negligible amplitude over the Pacific, but to possess a significant negative center over the Atlantic. By day 28 this center has decayed, and a new negative center has developed over the Pacific. Its amplitude is between one-third and one-quarter that of the corresponding disturbance (which had the opposite sign) at day 10. This behavior is in agreement with the normal-mode structure and period shown in Fig. 11. The decay in amplitude is rather more than expected for the pure least-damped normal mode, indicating that other, more rapidly decaying modes contribute partly to the day 10 structure.

The full normal-mode instability problem outlined in Appendix A is such that the effect of linear drag with rate \( \kappa \) is to modify the amplitude of all modes by a factor of \( \exp(-\kappa t) \). Assuming the modes to form a complete set, an initial perturbation may be expanded in terms of them. Since all modes are damped at the same rate, a change in drag would then change
only the amplitude of the linear response to the perturbation, and not the pattern. In particular, a change from a 5-day to a 10-day drag would be expected simply to increase the amplitude of the day 10 response by a factor of \( e \). This has been confirmed by numerical integrations for the examples presented in the preceding section.

An example of a longer term integration for a forced-wave problem is shown in Fig. 13. In this case the linear drag rate has been adjusted to a value of \((6.8 \text{ days})^{-1}\) to give a long-term response consisting of a forced, stationary-wave component and a neutral free mode with a 45-day period. Fig. 13 shows the perturbation at day 15, when its amplitude is close to its maximum, at day 40, when the perturbation is globally of small amplitude, and at day 60, by which time the phase of the free mode is such that the day 15 pattern is reproduced. This relatively simple behavior indicates that in this case at least, for which the forcing was located at 10°N, 120°E, the structure of the stationary-wave component is very close to that of the normal mode at that point in the latter’s cycle when its center over the Pacific sector reaches its peak positive value.

The above result is consistent with the steady, long-term solution found for the same forcing when a 5-day drag is used. The structure of this solution (not shown) closely resembles the normal-mode form shown for day 191 in Fig. 11, apart from a much weaker negative center over northeastern Siberia. The amplitude of its Pacific maximum is similar to that attained at day 10, and some 30% weaker than peak values reached between days 15 and 25 of the integration. This behavior is not illustrated here since similar examples will be presented in Section 9 in the context of nonlinear integrations.

6. Solutions of the eigenvalue problem

As a test of the validity of the normal mode structure and behavior deduced from the extended linear time integrations described in the previous section, solutions to the eigenvalue problem have been obtained by a matrix technique. These solutions include not only the fastest growing barotropically unstable mode, but also more slowly growing modes which appeared not to be strongly excited in the numerical integrations described in Sections 3 and 4, but which might conceivably be of some importance in the real atmosphere. An outline of the technique is given in Appendix A. It is important to note that this calculation was programmed quite independently of others presented here. Results for the fastest growing mode should differ a little from those discussed in the preceding section, mainly because of small differences in the climatological basic state, as discussed in Appendix A.

![Fig. 13. The response at days 15, 40 and 60 to a forcing centered at 10°N, 120°E for a 6.8-day drag. The contour interval is 120 m.](image-url)
time of 7.3 days and a period of 40 days, and is shown in Fig. 14a. Comparison with Fig. 12 shows that it is indeed virtually identical in structure to the fastest growing mode determined from the linear integrations.

The second mode, with its 9.8-day e-folding time, is sufficiently unstable to be of some practical interest. The temporal behavior of this solution represents a special case of that described by (2) in the sense that the period $2\pi/\omega$ is infinite, so that at any given location the perturbation displays a pure exponential growth. Hence the vectorial representation of the phase is purposely omitted in Fig. 14b and the polarity of the mode at individual gridpoints is indicated by conventional positive and negative contours, where the sign convention is arbitrary. Pure exponential growth was also found for some of the normal modes deduced from the linear integrations (see Section 10). It may be significant that a response resembling this mode was not obtained in any of the integrations described in Sections 3 and 4. Perhaps such modes are relatively difficult to excite because they lack the local periodic surges in growth rate characteristic of the oscillating modes discussed in the previous section.

The third mode has an e-folding time of about 13 days and a period near 120 days. Its structure, shown in Fig. 14c, resembles that of the first mode with respect to the strong “center of action” near 45°N, 165°W and the wavetrain trailing off downstream from it, with maximum amplitude of the north–south dipole pattern occurring over eastern North America in this case. The fourth mode has an e-folding time of about 29 days and a period of about 17 days. Like the first and third modes it comprises a strong center of action in the northeast Pacific and a north–south
dipole structure in the North Atlantic sector, these oscillating in phase quadrature with one another (Fig. 14d). The structures of the first, third and fourth modes are sufficiently similar that it is difficult to distinguish between them in interpreting the results of the numerical integrations in Sections 3 and 4. Given sufficient time the first mode always becomes the dominant one, but the patterns at days 6 and 10 often show elements that could just as well be interpreted in terms of the other modes.

The higher modes include further variants on the mode 1 pattern. A contrasting pattern is that associated with the mode shown in Fig. 15; it is characterized by a distinctive, eastward propagating zonal wavenumber 4 pattern, with maximum amplitude along the poleward flank of the Southern Hemisphere summertime jet stream. It has a period of about 4 days and is very weakly decaying in this case with linear diffusion, but no drag. Its strong northwest-southeast phase tilt, most evident in the South-Atlantic sector, is suggestive of a poleward flux of westerly momentum out of the jet stream, and as such it resembles the classical barotropically-unstable mode for a zonally-uniform basic state (Kuo, 1949). In agreement with this interpretation, Branstator (1983) shows that the meridional gradient of potential vorticity of the climatological mean state changes sign close to 60°S at almost all longitudes, with largest negative values between 30°W and 60°E. Tupaz et al. (1978) obtained a barotropically unstable normal mode with a qualitatively similar structure in an idealized basic state with a zonally varying easterly jet.

Frederiksen (1982) has recently carried out an investigation of the normal modes of a two-layer quasigeostrophic model with climatological mean, zonally varying (Northern Hemisphere wintertime) forcing analogous to that included in the present study. The resulting normal modes are dominated by eastward propagating wavetrains with substantial phase shifts between upper and lower levels, which is indicative of the important role played by baroclinic instability. In experiments with an enhanced static stability parameter, Frederiksen obtained modes with north-south dipole structures somewhat more similar to those described in this section; however, one has the overall impression that the modes described by Frederiksen are characterized by faster growth rates, shorter periods, smaller horizontal scales, and more wavelike longitudinal structures than the dominant modes associated with the purely barotropic instability examined here.

7. The energetics of the normal mode

As in the classical, or one-dimensional, barotropic instability of a zonal-mean basic state, the energy source for the normal-mode growth discussed in the previous two sections is the kinetic energy of the basic state. It is shown in Appendix B that the growth of the kinetic energy $KE$ associated with deviations from the basic state is given by

$$\frac{\partial KE}{\partial t} = CK_x + CK_y,$$

with

$$CK_x = -\frac{1}{a} (u'^2 - v'^2) \left[ \frac{1}{\cos \theta} \frac{\partial u_b}{\partial \lambda} - v_b \tan \theta \right]$$

$$CK_y = -\frac{1}{a} u' v' \left[ \cos \theta \left( \frac{\partial u_b}{\cos \theta} \right) + \frac{1}{\cos \theta} \frac{\partial v_b}{\partial \theta} \right]$$

Here $u_b$ and $v_b$ are respectively the zonal and meridional velocity components of the basic state, and $u'$ and $v'$ are the corresponding velocities of instanta-
neous deviations from the basic state, $a$ is the radius of the earth, and the overbar denotes an area-weighted integration over latitude $\theta$ and longitude $\lambda$. Dissipative terms are not included.

Eq. (4) may be simplified to a good approximation. Defining local Cartesian coordinates by

$$x = (a \cos \theta) \lambda, \quad y = a \theta,$$

it may be shown, either by scaling arguments or by direct calculation of all terms of the complete equation (see below), that (4) is given to good accuracy by

$$\frac{\partial KE}{\partial t} = -(u'^2 - v'^2) \frac{\partial u_b}{\partial x} - u'v' \frac{\partial u_b}{\partial y}. \quad (5)$$

Here the second term on the right-hand side is of familiar form since in the case of a zonally-uniform basic state ($\partial u_b/\partial x = 0$) it represents the total barotropic exchange of kinetic energy between the mean and eddy form, with eddy energy growing when the meridional eddy transfer of zonal momentum ($u'v'$) is in the direction of a weakening westerly component of the basic flow (Kuo, 1951).

The first term on the right-hand side describes an additional energy transfer between the basic state and the perturbation. Local contributions to this globally-integrated conversion term depend on the anisotropy of the disturbance as reflected in the different magnitudes of the zonal and meridional velocity perturbations, and on the zonal gradient of the basic zonal velocity. A generation of eddy energy is implied by zonally-elongated eddies ($u'^2 > v'^2$) in regions of divergence ($\partial u_b/\partial x < 0$), and by meridionally-elongated eddies ($v'^2 > u'^2$) in regions of confluence ($\partial u_b/\partial x > 0$). In the climatological wintertime mean, divergence occurs over the central North Pacific and North Atlantic. Predominantly zonal perturbations in these regions are a feature both of the normal-mode structures shown in Figs. 11 and 14, and of the observed low-frequency variability of the atmosphere (Blackmon et al., 1977; Hoskins, et al., 1983; Wallace and Blackmon, 1983; Blackmon et al., 1983).

Eq. (5) may be written as

$$\frac{\partial KE}{\partial t} = \mathbf{E} \cdot \nabla u_b, \quad (6)$$

with

$$\mathbf{E} = -(u'^2 - v'^2, u'v'). \quad (7)$$

The significance of the vector $\mathbf{E}$ is discussed extensively by Hoskins et al. (1983), who show that it may be considered as an effective flux of easterly momentum, or as the horizontal components of an extended Eliassen-Palm flux. $\mathbf{E}$ is plotted vectorially in Fig. 16 for the fastest-growing normal mode considered previously. It is computed as an average over one cycle of the mode, where the exponential growth factor (the inclusion of which would introduce an arbitrariness which is discussed later) is suppressed, or equivalently a linear drag with a rate of $6.8 \text{ day}^{-1}$ can be regarded as included.

Also shown in Fig. 16 are contours of the zonal component of the January climatological mean basic state used for the normal mode calculation. The sense of the representation of $\mathbf{E}$ is such that arrows pointing from weak to strong values of the zonal velocity imply a positive contribution to the net growth of eddy energy. Fig. 16 thus indicates eddy growth due to the strong zonal component of $\mathbf{E}$ that occurs downstream of the jet maxima over the western Pacific and Atlantic Oceans. A significant meridional component occurs only to the northeast of the Pacific jet. It is weak in comparison with the zonal components mentioned above but is also an effective contributor to wave growth since it occurs in a region of particularly strong meridional gradient of the basic zonal flow.

The above picture is confirmed by computations of the two complete conversion terms comprising the right-hand side of (4). Maps of the local contributions to the two terms are shown in Fig. 17. That for the $\mathbf{CK}x$ term involving the zonal variation of the basic state, shown in the upper panel, confirms that the dominant contributions to this term indeed come from the divergent regions of the Pacific and Atlantic jet streams. The contribution from the more familiar $\mathbf{CK}y$ term is shown in the lower panel. As expected,

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2 Hoskins et al. (1983) refer strictly to $\mathbf{E}$ as a pseudo vector, since it does not transform as a vector under a rotation of the coordinate system.

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**FIG. 16.** The "extended Eliassen-Palm flux" $\mathbf{E}$ averaged over one cycle of the most-unstable mode, and superimposed on isotherms of the zonal velocity component of the climatological basic state. The contour interval is $10 \text{ m s}^{-1}$, and jet maxima are indicated by the symbol J. The scaling of the arrows denoting the flux vector is arbitrary.
netic energy of the perturbation at a minimum, and a dominant contribution to $CK_{x}$ from the weakening disturbance in the Atlantic sector. $CK_{y}$ is close to zero. Over the 13 days starting from day 181, both conversions are dominated by Pacific contributions which have distributions similar to those shown in the mean maps of Fig. 17. $CK_{y}$ develops a day or so ahead of $CK_{x}$, and reaches a maximum value on day 187. The Pacific contribution to $CK_{x}$ is a maximum one day later, by which time the Atlantic perturbation is beginning to grow again, so that $CK_{x}$ as a whole is largest at about day 191, the time of maximum kinetic energy of the perturbations. $CK_{y}$ at this time is decreasing rapidly to zero. The Atlantic contribution to $CK_{x}$ peaks at day 196, by which time the Pacific contributions are negligibly small. The overall dominance of the Pacific sector is seen in the peaking of the net conversion of basic state energy on day 188, three days before the kinetic energy maximum.

The local contributions to the conversion terms do not, of course, directly yield local growth of eddy energy because of the existence of additional flux terms which distribute the energy while contributing nothing to global-mean values. A distinct picture of the nature of the fastest-growing normal mode nevertheless emerges from studying the synoptic maps of Fig. 11 and the contributions to the net energy conversion. In this picture, the central North Pacific appears as the dominant region of local barotropic instability. Perturbation growth is rapid in this region, and energy is dispersed downstream to give centers over the locally stable regions of North America and the western subtropical Atlantic Ocean. A disturbance centered near Iceland also forms, and subsequently amplifies due to the local instability of the central North Atlantic region. Energy is then dispersed southeastward over eastern Europe and Asia.

A final comment to be made here is that there is an ambiguity in the relative contributions of the Pacific and Atlantic regions, and in the details of the local patterns of conversion. As noted earlier, all averages over one cycle of the mode have not taken into account the exponential growth factor. In the absence of dissipation the $\epsilon$-folding time of the most unstable mode is substantially shorter than its period, and if the growth factor were to be taken into account, conversions would be dominated by the contribution from the last few days of the cycle, when amplitudes were largest. Since there is not an unambiguous starting and finishing point to the cycle, results would be ambiguous. Averaging over an ensemble of random phases gives results equivalent to those presented here (Frederiksen, 1982), although in situations more general than that of an isolated normal mode, the relative importance of the contributions from the Pacific and Atlantic sectors will depend crucially on the manner in which the perturbation is excited, as has been seen.
in the 10-day integrations for forced-wave and initial-value problems presented in Sections 3 and 4. In fact, in some cases it may be possible to trigger normal modes other than the fastest growing one because, for a limited period of time, they have the most rapid growth rates in the localized regions near to which the forcing has been applied.

8. Nonlinear initial-value problems

In order to investigate the nonlinear modification of normal-mode behavior a set of seven extended integrations has been performed using a 10-day drag. For such a rate of damping, the $\varepsilon$-folding time of the most unstable mode considered in the preceding section is 22 days. With such a weak instability it might be expected that a small nonlinear modification of the mean state would be sufficient to inhibit wave growth. Different initial perturbations were used in an attempt to determine whether different nonlinearly stable states could be set up.

All seven integrations, in fact, appeared to exhibit a very similar long-term behavior, although in one case to be discussed below a substantial time was required before it began to resemble the apparently asymptotic behavior found beyond day 100 in all the other integrations. A typical solution illustrating this latter behavior was obtained by applying the standard forcing at $10^\circ$N, $120^\circ$E over the first 10 days of integration, and then setting it to zero. The resulting mean state shown in Fig. 18a evidently differs little from the forced climatological basic state shown in Fig. 2, and an instability calculation shows the modified state to remain linearly unstable, with an $\varepsilon$-folding time of about 8 days in the case of zero drag. Deviations about the modified mean state do not continue to grow, however, but rather exhibit a regular oscillation, with a period of $\sim$50 days and a horizontal structure broadly similar to that illustrated in Fig. 11 for the growing mode on the climatological basic state. The standard deviation shown in Fig. 18b has a structure which clearly resembles the normal-mode amplitude presented in Fig. 12, with maxima again over the North Pacific and Atlantic Oceans.

A synoptic view of the oscillation at four instants, 12 days apart, during one of the regular 50-day cycles is shown in Fig. 19. Comparing the upper and lower left-hand panels, which represent approximately opposite phases of the oscillation one can see a strongly contrasting structure in the eastern Atlantic sector, with a blocking ridge and split flow on day 159 and a more zonal flow with a single midlatitude jet one-half cycle later on day 183. The right-hand panels, which represent the times when the oscillation is in phase-quadrature with the left-hand ones, emphasize the contrasting structure in the Pacific. The day 171 pattern is characterized by a foreshortening of the Asian jet stream with most of the diffuence occuring to the west of the dateline. It is at this time that heights are above normal in the vicinity of the strong maximum in Fig. 18b. One-half cycle later on day 195 the jet has lengthened to its maximum extent so that the maximum diffuence occurs well to the east of the dateline, and a high-latitue ridge has developed over Alaska and the Bering Sea.

The corresponding time variation of the perturbation at the point (39$^\circ$N, 161$^\circ$W) of maximum standard deviation is indicated by the light curve in the upper graph of Fig. 20. The oscillation is evidently not purely sinusoidal, a result which appears to be

Fig. 18. Mean total streamfunction (upper, contour interval 160 m) and standard deviation (lower, contour interval 30 m) for days 100–200 from the nonlinear initial-value problem specified in the text.
consistent with the weak linear instability of the mean state and suggestive of the nonlinearity of the oscillation. The heavy curve also shown in this graph was derived by applying the same forcing, but with the opposite sign. It is apparent that by day 100 the behavior of the two solutions is very similar, apart from an unimportant phase difference.

The nonlinear results described above were obtained with resolution T42. When the same experiment was conducted with the standard T21 resolution, the behavior between days 100 and 200 was quite different, as illustrated in Fig. 20b. The solution denoted by the heavy curve is generally similar to that discussed above, but the lighter curve represents an oscillation of distinctly smaller amplitude. The corresponding mean deviation from the climatological basic state, as indicated by the mean level of the curve, also differs markedly from that for the case shown in Fig. 18.

This quite different behavior is, however, only transient (although long-term) in character. Beyond day 250, the amplitude of the oscillation in the anomalous case gradually increases, and Fig. 20c shows that the two T21 solutions are extremely similar beyond day 500. By this stage both means and standard deviations are negligibly different from the higher resolution results discussed earlier.

It is evident from this case that the long-term nonlinear integrations can exhibit a pronounced sensitivity to resolution. Further examples are discussed in the following section.

9. Some nonlinear forced solutions

A series of cases has been integrated nonlinearly in order to determine the long-term response to the localized forcing prescribed earlier in Section 2. At-
tention has been concentrated on forcings which excite a large response in the Pacific region.

Results for days 15 and 50 are shown in Fig. 21 for two cases in which a 5-day drag was used. With this level of dissipation an essentially steady solution is found by day 50. The two cases both had forcing centered at 10°N, 120°E, and differed only with respect to the sign of the forcing. In linear calculations the responses would thus be identical but for their sign.

The two solutions for day 15 show, in agreement with Simmons (1982), that nonlinearity can enhance the short-term anticyclonic response over the North Pacific and suppress the cyclonic response, the nonlinear anticyclonic response illustrated in Fig. 21a being some 10% larger than the corresponding linear response, which is not shown. However, beyond day 15 there is a more substantial decrease in wave amplitude than found in the linear calculation. The long-term nonlinear anticyclonic response, typified by the day 50 maps shown in Fig. 21c, is some 15% weaker than in the linear case, and the nonlinear cyclonic response (Fig. 21d) is weaker still.

The two cases shown in Fig. 21 also exhibit some marked differences in wave pattern. The linear solution for the 6.8-day drag shown in Fig. 13 has a maximum perturbation close to 165°W and a similar result is found for a 5-day drag. In these nonlinear solutions, the major anticyclonic response occurs some 15° further west, while the corresponding cyclonic response is shifted eastwards by a similar amount. The anticyclonic response (Fig. 21a, c) is characterized by a more pronounced PNA type wave train, and a weak pattern in the Atlantic sector reminiscent of the east Atlantic pattern, whereas the cyclonic response (Figs. 21b, d) is weaker over North America and its north–south “seesaw” over the Atlantic sector occurs somewhat further poleward.

Repetition of the above calculations with resolution T42 gives very similar results, but calculations for a 10-day drag have been found to be more sensitive to resolution. Results will thus be presented here for both T21 and T42 truncations. Fig. 22 shows 100-day mean responses for two cases, with the lower resolution results shown in the upper panels. The left-hand panels refer to the case of standard forcing centered at 10°N, 120°E, and should be compared with the left-hand panels shown in Fig. 21 for the 5-day drag. It is evident that the reduction in drag has little impact on the wave pattern apart, not surprisingly, from the somewhat larger distant response in the case of weaker dissipation. Use of higher resolution gives a slightly larger response, but there are no major differences in these mean maps.

The right-hand panels of Fig. 22 show the corresponding response to a forcing centered at 20°N, 150°E, and reveal much more sensitivity to resolution. The anticyclone centered over the Aleutian Islands is substantially stronger in the T42 integration, and is in fact of sufficient amplitude to give a pronounced splitting of the total flow. Large differences in the wave pattern over the Atlantic Ocean and Europe also occur, with much more amplitude again found at higher resolution.

Sensitivity to resolution is even more marked in the time dependence of these solutions with the 10-day drag. The time series shown in Figs. 23a, b are for points at the maximum mean (T21) response. The heavier curves represent T42 resolution, and the lighter curves T21. It is clear that in both these cases the T42 responses at these points ultimately become quite steady in time, whereas the T21 responses show pronounced variability which is extremely regular with a 14-day period in one case and more irregular with (as confirmed by a longer integration) a 90-day
FIG. 21. Perturbation streamfunction at days 15 (upper) and 50 (lower) forced with positive (left) and negative (right) sign from 10°N, 120°E for a 5-day drag. The contour interval is 40 m.

quasi-periodicity in the other case. It is interesting that the former case shows such a similar time-mean structure for the two truncations and yet such a difference in temporal variability. More generally, the T21 solutions exhibit a wide range of behavior. For example, repeating these two cases with forcings of opposite sign is found to yield one extremely steady solution, and another solution with a small-amplitude, but highly irregular, variability in time.

A tendency for spurious oscillations to occur in low-resolution spectral models has been noted by Puri and Bourke (1974), and their results in fact led us to perform the higher resolution experiments described above. Their results also suggest that use of a higher value of diffusion coefficient might lead to a better result at T21 resolution. This is confirmed by the lighter curve in Fig. 23c which was obtained from a T21 integration using a diffusion coefficient four times larger than the standard value adopted here. There is clearly much closer agreement with the heavier T42 curve than in the standard diffusion case shown in Fig. 23b. This better agreement is dramatically confirmed by the map of the mean perturbation obtained using the higher diffusion, shown in Fig. 24, which more closely resembles the time mean map for the T42 integration (Fig. 22d) than that for the T21 integration with the weaker diffusion (Fig. 22b).

The above result, although encouraging, should not be interpreted as indicating that the spurious behavior of low-resolution models can be eliminated simply by increasing the diffusion. As an illustration of the problems that still remain, we note that the nonlinear initial-value problems reported in the previous section exhibit marked long-term oscillations at both T21 and T42 resolutions, indicating that for a 10-day drag an oscillating solution should be expected at both
resolutions for a sufficiently weak forcing, since in the limit of large time and small amplitude the forcing would serve only to excite the linearly unstable mode and subsequent nonlinear oscillations. This has been confirmed by repeating the experiment described in Figs. 22 and 23 with the forcing amplitude reduced by a factor of $\sqrt{2}$. In this case the T42 integration exhibits a strong oscillation with a period of a little under 60 days. This oscillatory behavior is partly captured by a T21 integration with the lower value of diffusion coefficient, but missed by the integration with higher diffusion, which incorrectly (in this particular case) tends again to a steady, stationary long-term response (now shown).

To summarize, the results discussed in this section indicate that although nonlinearity can modify some detailed aspects of the response to localized forcing, the long-term means still exhibit the preferred patterns found in the linear solutions. Temporal variability can evidently differ from case to case for a dissipation rate for which the basic state is unstable, and for some of these cases we have found substantial differences between the solutions given by lower and higher resolution models.

10. Sensitivity to the basic state

We conclude our presentation of results from this barotropic model with a summary of results of experiments designed to investigate the sensitivity of linear solutions to the choice of basic state. For this purpose, different states were generated in three ways. The first was to use 100-day mean 300 mb streamfunctions for the winters of 1980–81 and 1981–82 from the operational analyses produced daily at ECMWF. Second, states were derived by artificially modifying the stationary-wave component of the standard climatological basic state. Finally, basic
and for the January climatology modified by multiplying its wave component by a factor \((1 + 0.5 \cos \lambda)\) to increase its amplitude around the Greenwich meridian.

Additional sensitivity studies have concentrated on examination of the normal mode stability properties of the various basic states. Of 13 basic states examined, e-folding times varied from 4.5 days (for the January climatology with a 50% enhancement of its stationary wave component) to 24 days (for the corresponding case in which this stationary-wave amplitude was reduced by 50%). The shortest period found was 27.6 days, while in 4 of the 13 cases the normal mode structure took the special form of a spatially-fixed pattern which simply amplified in time. It is highly unlikely that this result, which corresponds to an infinite period for the mode, would occur by chance. Rather it appears that it is preferred behavior for the most unstable mode to adopt a structure that can remain fixed in space, although as noted previously this type of mode may be more difficult to excite.

Three examples of normal mode structure are given in Fig. 26 (right), together with maps of the basic states (left) used in their calculation. For the upper two cases, these basic states are the computed long-term response of the climatological flow to forcing of negative (upper) and positive (middle) polarity located at 20°N, 150°E. The lower maps are for the observed mean state for 1981–82.

These particular cases were chosen to illustrate the variety of structures that can be found. In the upper example, the mode develops with largest amplitude over the North Pacific, and its attainment of maximum amplitude in this region is followed by a weaker states were derived from some of the steadily-forced solutions obtained using a 5-day drag.

The mean state for the winter of 1981–82 is shown in Fig. 25, together with some statistics of the day 10 response to a family of forcings centered in the range from 15°S to 30°N, with a longitudinal interval of 30° between forcings, and a 5-day drag. Comparing the standard deviation with that shown in Fig. 4 reveals a weaker Pacific response and a slightly larger Atlantic response for the 1981–82 basic state than for the climatological mean pattern. The correlation map for the Pacific point (39°N, 169°W) is much less suggestive of the PNA teleconnection pattern than other such maps presented in Sections 3 and 4, while conversely the map for the Atlantic point (51°N, 28°W) more clearly resembles the East Atlantic pattern shown in Fig. 1. A similarly enhanced Atlantic response is also found for the 1980–81 basic state.

**Fig. 23.** Deviation of streamfunction \((m)\) from the climatological mean as a function of time, for resolution T42 (heavy lines) and T21 (light lines) with a 10-day drag; (a) at 39°N, 169°E, for forcing from 10°N, 120°E; (b) at 34°N, 169°E, for forcing from 20°N, 150°E; (c) as in (b), but for a T21 integration with a higher diffusion coefficient.

**Fig. 24.** The mean response for days 100–200 to forcing from 20°N, 150°E computed using a 10-day drag and a T21 resolution with a higher diffusion coefficient. The contour interval is 40 m.
maximum, of opposite sign, close to the North Pole, and later by a yet weaker maximum over the North Atlantic. Only the weakest hint of a PNA pattern is seen for this mode.

The other examples are two of the cases that exhibit a fixed spatial structure for the normal mode. The mode shown in the middle panel has significant amplitude over much of the hemisphere, and a wave pattern for which a link may be made between individual centers and the corresponding centers in the pattern shown in Fig. 12 for the climatological basic state. The most unstable mode for the 1981–82 basic state (lower panel) is exceptional in the sense that it has only small amplitude over the Pacific. It may be that in this case there is a more weakly unstable mode growing with a structure similar to that shown in the upper plot of Fig. 26, since the two basic states are not greatly dissimilar over the Pacific. The existence of such a mode is also suggested by the pattern of the Pacific response excited in the forcing experiments summarized in Fig. 25.

11. Discussion

a. Interpretation of teleconnection patterns

In using the traditional normal mode nomenclature and technique in the preceding sections, we do not mean to imply that in the atmosphere or in a complex general circulation model such structures typically begin as small perturbations and evolve through several growth cycles to become major circulation features. We envision a less clearly defined situation in which finite-amplitude, low-frequency circulation anomalies resulting from a number of different forcing mechanisms (e.g., tropical heating
FIG. 26. Three basic states (left), and the structures of the corresponding most-unstable normal modes (right), plotted in amplitude and phase form for the upper (periodic) mode, and by positive and negative contours for the lower two (spatially-stationary) modes.
anomalies, dispersion of eddy energy excited by baroclinic instability, interactions involving orography) are continuously present. We hypothesize that those anomaly patterns resembling characteristic normal-mode structures are in a favored position relative to other patterns, because they are able to extract energy from the climatological mean flow, even if they assume the normal-mode form for only part of the growth cycle. Therefore in long-term statistics such as one-point correlation maps one might expect to see evidence of these normal-mode structures in fluctuations with periods of weeks or longer, even if one rarely sees them in a pure form on synoptic charts.

These same normal modes may be present with disproportionately large amplitudes in the atmospheric response to steady-state anomalous forcing. A consequence is that the observation of wave trains following approximate "great circle" routes does not necessarily imply a tropical source for the perturbation, and even in the case of tropical forcing the position of the wavetrain does not directly imply the position of the source. An example of the latter may be seen by referring back to the left-hand plots of Fig. 21 in which the Pacific/North American wavetrain appears to emanate from the central Pacific a little west of the dateline, whereas the forcing is in fact centered at 120°E.

Although tropical forcing is not the only way of exciting the preferred wave patterns, it can nevertheless be an effective source of excitation for them. Just as there are preferred regions of response in the extratropics, there are regions of the tropics which may excite a particularly large midlatitude response. One of the most effective of these regions, and one where there is normally a substantial amount of convective activity, is Southeast Asia and the tropical northwest Pacific. Forcing from this region can readily excite a large response over the extratropical northeast Pacific, and a similar, though weaker, response may be excited by forcing of opposite sign over the tropical central Pacific. Thus a tropical anomaly comprising an eastward or westward shift of convective heating between the dateline region and regions lying further to the west, near 120°E, may be a particularly efficient way of triggering a response which resembles the PNA pattern. As has been noted, just such a redistribution of convective heating is implied by rainfall anomalies associated with the Southern Oscillation.

The preferred responses found in the family of forced and initial-value problems appear to be intimately linked to the barotropic instability of the basic state. This relationship is most obvious for those responses that resemble the Pacific/North American teleconnection pattern: both forcing and dispersion excite a structure close to that of an unstable normal mode of the climatological basic state at that point in the mode's cycle when its amplitude is most con-

b. Low-frequency variability in the context of the general circulation

A second major conclusion is that the normal modes associated with the barotropic instability of the zonally varying time-mean state may account for an appreciable fraction of the observed low-frequency variability of the geopotential height field during the winter season and, in particular, for the large variability over the Pacific and Atlantic sectors. Whether such perturbations are actually unstable, neutral or damped in the real atmosphere is difficult to determine, but in any case, it seems reasonable to expect that structures resembling the normal modes should be present with disproportionately large amplitudes in comparison to other structures which have no energy source to offset the continuous "spindown" by boundary-layer processes and thermal damping. The same barotropic instability process could conceivably play an important role in restricting the amplitude of the stationary waves to the observed range of values and in limiting the sharpness of the jetstream.

Furthermore, it would appear that the energy conversions described in Section 7 should play an important role in the general circulation, as viewed in the context of the energy cycle depicted in Fig. 27, in which kinetic and available potential energy are partitioned into components associated with the climatological mean state, denoted by the $M$ subscripts, and components associated with transient fluctuations, denoted by the $T$ subscripts. [In the conventional, zonally averaged kinetic energy cycle formu-

\[ \begin{align*}
G_M & \quad C_A & \quad G_T \\
A_M & \quad C_M & \quad A_T \\
K_M & \quad C_K & \quad K_T \\
B_M & \quad D_M & \quad B_T & \quad D_T
\end{align*} \]

**Fig. 27.** Schematic kinetic energy cycle with energies partitioned between time mean (M) and transient (T) components of the general circulation. Heavy arrows denote conversions associated with baroclinic instability while lighter arrows denote conversions associated with barotropic instability. See text for further details.
lated by Lorenz (1955), the climatological mean wintertime stationary waves are grouped on the right-hand side, with the transient eddies.] In this alternative formulation the basic state is viewed as being maintained not only by differential heating between low and high latitudes, but also by longitudinal heating contrasts, which are included as part of the $G_M$ term, and by mechanical forcing by the mountains, which is indicated schematically by the $B_M$ term. The barotropic energy conversions discussed in Section 7 are indicated in the figure by the light arrow labeled $C_K$, which for amplifying disturbances points from left to right. The corresponding conversions associated with baroclinic instability and the subsequent countergradient flux of westerly momentum associated with mature and decaying baroclinic waves are indicated by the darkened arrows $C_A$, $C_T$ and $C_K$, which circulate clockwise in the figure. Hence, barotropic and baroclinic instability are seen to be in direct competition with respect to their contributions to the conversion $C_K$ so that the net globally averaged conversions in Fig. 27 must be viewed as a residual. It might not be possible to distinguish between the two processes in observational data were it not for the fact that they involve a different range of frequencies, baroclinic instability generally being associated with fluctuations with periods shorter than a week and barotropic instability being associated with characteristically longer time scales. This view of competing processes is in contrast to the general impression that one gets from the conventional Lorenz formulation, which tends to emphasize the role of baroclinic instability. Therefore, it is of some interest to compare the relative importance of the two processes in the modified formulation.

A quantitative description of the kinetic energy cycle is beyond the scope of this paper, but it is possible to make a few qualitative statements as to the relative importance of barotropic and baroclinic processes.

1) In the integral over all frequencies $\overline{u'^2 - v'^2}$, but in the Pacific jet exit region, the $u'^2$ term dominates (e.g., see Lau et al., 1981, Figs. II.B.3-4). Therefore it would appear that decaying baroclinic waves and low-frequency fluctuations which extract kinetic energy from the climatological mean flow make roughly equal and opposite contributions to the conversion $C_K$. The tendency for cancellation between the two contributions is evident in the time-filtered, extended Eliassen-Palm flux distributions presented by Hoskins et al. (1983).

2) Despite the apparent dominance of disturbances generated by baroclinic instability in the energy cycle as a whole, the dominance of the low-frequency contribution to the geopotential height field (e.g., see Blackmon, 1976; Wallace and Blackmon, 1983) suggests that most of the transient energy $K_T$ and $A_T$ is associated with disturbances for which the barotropic conversion $C_K$ is a significant energy source. (The dominance of the low frequencies may be more a reflection of the difference in dissipation rates for low- and high-frequency fluctuations than of the relative magnitudes of the barotropic and baroclinic conversions: the low-frequency fluctuations tend to be larger in horizontal scale and their kinetic energy is largely confined to the upper troposphere whereas the higher frequency fluctuations extend into the range of scales where dissipation becomes significant and they attain large amplitudes within the planetary boundary layer where dissipation is effective).

It is evident that the low-frequency transients associated with barotropic instability are capable of shifting the jet streams and thereby influencing the positions of the stormtracks along which baroclinic waves develop. The baroclinic waves, in turn, influence the time evolution of the low-frequency transients in complicated ways and the picture is complicated further by the interactions between the low-frequency transients and the orographic forcing, which can give rise to a transient forcing term $B_T$. However, it is remarkable that despite all these complicated interactions the signatures of the two instability mechanisms are evident in time-filtered general circulation statistics.

C. Implications for numerical modeling

If barotropic instability is, in fact, an important source of low-frequency variability, it follows that it should make a significant contribution to the error growth during the course of medium range forecast integrations. In support of this argument, it is perhaps worth noting that beyond day 5, forecast error maps from the ECMWF model often show evidence of equivalent barotropic features with horizontal structures qualitatively similar to those derived from initial-value calculations with the barotropic model, particularly when they are averaged over sequences of consecutive days. For example, compare the day 6 and day 10 charts in Figs. 6 and 7 of this paper with Fig. 20 of Wallace et al. (1983). In view of the rather slow overall growth rates associated with barotropic instability, there is reason to hope that with improved models, this component of the error could, in principle, be reduced substantially in the first 10 days of the forecast, even in the face of the predictability constraints discussed by Lorenz (1972).

A number of other implications of relevance to the design and use of numerical, forecast and general circulation models are suggested by our results. Insofar as inaccurate orographic and thermal forcing in these models may be regarded as a perturbation of the true climatological forcing of the stationary waves, our findings indicate that systematic errors will
tend to be largest over the northeastern Pacific and Atlantic. Such is indeed the case in the operational ECMWF forecast model (Hollingsworth et al., 1980, Bengtsson and Simmons, 1983), and in this context the barotropic model used here has also proved useful in giving a better understanding of the relationship between local, erroneous, orographic forcing and the mean evolution of large-scale forecast errors (Wallace et al., 1983). A further consequence of our results is that if the systematic error in a model climatology is such as to give an underestimation of the stationary wave pattern then this model is also likely to underestimate the response to anomalous boundary conditions, for example anomalies in sea-surface temperature.

Another result of these barotropic experiments may also be of relevance to GCM sensitivity studies. It has been recognized for some years that care must be taken to establish the statistical significance of the results of anomaly experiments (Chervin et al., 1974). One possible approach to this problem is to perform a number of medium term (say 30-day) experiments starting from different real synoptic situations. Compared with the alternative strategy of carrying out a single much longer-term integration, this approach has the advantage of lessening the chance of an erroneous response due to a spurious drift of the unperturbed model climate away from that of the atmosphere. Our results suggest, however, that this experimental strategy may tend to result in an overestimate of the long-term response, since the extended-range barotropic integrations involving the localized forcing of an initially undisturbed state tend to exhibit a substantially larger response between days 10 and 30 than in the longer-term mean.

The results obtained using different resolutions may also have consequences for more complete atmospheric models. Some quite different intensities of both mean response and temporal variability have been found between spectral truncations at total wavenumbers 21 and 42. While these differences may in some instances be effectively removed by an appropriate choice of horizontal diffusion, it is by no means clear that a single choice of diffusion can be found which satisfactorily enables the range of higher resolution behavior to be reproduced at lower resolution.

d. Remaining problems

As in all such studies involving a relatively simple model, conclusions must be tempered by the recognition of the limitations of the model employed to deduce them. In the present case the barotropic nature of the model, while permitting an economical program of extensive experimentation, appears to be its outstanding restriction. In view of the contrasting normal-mode structures obtained by Frederiksen (1982) in a two-layer primitive equation model (see Section 6), it is important to determine whether modes predominantly associated with barotropic instability of the type examined here can be identified in models which allow for the possibility of baroclinic energy conversions.

It should also be of interest to determine how the forced response and barotropic instability are influenced by interactions with the shorter-term transients excited by baroclinic instability. The latter may be viewed as included in the present barotropic calculations only in the crudest possible way through the linear drag, and a very marked sensitivity to the magnitude of this drag has been demonstrated in many of our results.

Scope clearly exists for other studies. A more complete investigation of the barotropic normal-mode instability problem is required, in particular to derive necessary conditions for the “two-dimensional” instability and to determine factors influencing the period of the instability. Other studies range from investigation of the associated nonlinear oscillations to investigation of the nonlinear response in cases in which the zonally-varying basic state is maintained by an interactive forcing. In reality, growth of an anomaly to significant amplitude will modify the normal orographic, thermal and short-period transient wave forcing of the climatological state, a forcing which has been held fixed in the present calculations, and determined solely by the climatological state itself. There is thus a need for extension of these calculations to a model with a physically more realistic basic-state forcing.

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APPENDIX A

The Matrix Eigenvalue Problem

The absolute velocity $\xi$ and corresponding streamfunction $\psi$ are written as sums of basic-state values $\xi_b$ and $\psi_b$, and perturbation values $\xi'$ and $\psi'$. The perturbation vorticity equation may then, in the absence of linear drag, be written as
\[ \frac{\partial \zeta'}{\partial t} + J(\psi_b, \zeta') + J(\psi', \zeta_b) = -\eta \nabla^2 \zeta', \quad (A1) \]

where

\[ J(\psi, \zeta) = \frac{1}{a^2 \cos \theta} \left( \frac{\partial \psi}{\partial \lambda} \frac{\partial \xi}{\partial \theta} - \frac{\partial \psi}{\partial \theta} \frac{\partial \xi}{\partial \lambda} \right), \]

\( a \) is the radius of the earth and \( \eta \) the coefficient of horizontal diffusion.

We pose a solution of normal-mode form, with a spectral representation triangularly truncated at total wavenumber \( N \). Then if the perturbation vorticity is given by

\[ \xi' = \text{Re} \left\{ \sum_{m=0}^{N} \sum_{n=-m}^{m} \xi_n^m P_n^m(\sin \theta) e^{i(mk-\Omega t)} \right\}, \quad (A2) \]

where \( P_n^m \) is an associated Legendre function, the corresponding streamfunction is

\[ \psi' = \text{Re} \left\{ a^2 \sum_{m=0}^{N} \sum_{n=-m}^{m} n^{-1}(n+1)^{-1} \right\}
\times \xi_n^m P_n^m(\sin \theta) e^{i(mk-\Omega t)}. \quad (A3) \]

Substituting expressions (A2) and (A3) into (A1), multiplying by a complex conjugate spherical harmonic and integrating over the sphere, we obtain an equation which may be written in the form

\[ \Omega \xi_n^m = \sum_{r=0}^{N} \sum_{s=-r}^{r} C_{ns}^{mr} \xi_s', \quad (A4) \]

Here \( C_{ns}^{mr} \) is an \( M \times M \) matrix of complex values which depend on the basic state, and \( M = \frac{1}{2}(N + 1)(N + 2) \). Thus \( \Omega \) is an eigenvalue of the matrix \( C \), and for a growing mode the growth rate \( \sigma \) is given by \( \text{Im}(\Omega) \), and the frequency \( \omega \) by \( \text{Re}(\Omega) \). The order of the corresponding real matrix whose eigenvalues are to be found may be reduced from \( (N + 1)(N + 2) \) to \( (N^2 + 2N - 1) \) by noting that the zonal coefficients \( \xi_0^m \) are necessarily real, and that the spectral form of the governing equations guarantees conservation of global-mean vorticity and angular momentum: \( \xi_0^0 = \xi_1^1 = 0 \).

Inclusion of a linear drag in the model adds a term \(-\kappa \xi'\) to the right-hand side of (A1). It may readily be demonstrated that the only impact of this term is to cause a change in eigenvalue from \( \Omega \) to \( \Omega - i \kappa \). The frequency and structure of modes are thus unaltered by inclusion of the linear drag, which simply reduces the growth rate of an unstable mode from \( \sigma \) to \( \sigma - \kappa \).

The matrix eigenvalue problem (A4) has been solved independently of the remainder of the calculations presented in this paper. The elements of the matrix \( C_{ns}^{mr} \) have been computed by using Fourier transforms and Gaussian quadrature as described by Branstator (1983), and eigenvalues and eigenvectors determined using a series of EISPACK subroutines. The results presented in Section 6 used the same parameter values as in the initial-value calculations of the most unstable mode. The basic states differed only in the interpolation procedures used to derive them from the climatological data of Crutcher and Meserve (1970) and Taljaard et al. (1969). Apart from rounding error, the only other source of difference between the two calculations of the most unstable mode is time-truncation error in the initial-value calculations, but the period and growth rate of the mode are sufficiently slow for this to be unimportant.

**APPENDIX B**

**The Perturbation Energy Equation**

The relative vorticity of the basic state is denoted by \( \xi_b \), and that of the perturbation by \( \xi' \). \( U_b \) and \( V_b \) denote the zonal and meridional velocity components of the basic state multiplied by \( \cos \theta \), and \( U' \) and \( V' \) are the corresponding perturbation velocities, also scaled by \( \cos \theta \). In the absence of dissipation, the vorticity perturbation satisfies the equation

\[ \frac{\partial \xi'}{\partial t} = -\frac{U_b}{a(1 - \mu^2)} \frac{\partial \xi'}{\partial \lambda} - \frac{V_b}{a} \frac{\partial \xi'}{\partial \mu} - \frac{U'}{a(1 - \mu^2)} \frac{\partial \xi_b}{\partial \lambda} - \frac{V'}{a} \left( 2\Omega + \frac{\partial \xi_b}{\partial \mu} \right), \quad (B1) \]

where \( \Omega \) is the earth's rotation rate and \( \mu = \sin \theta \). With \( \psi' \) as in Appendix A, basic relations deriving from the nondivergence of the velocity field are

\[ U' = -\frac{1}{a} \left( 1 - \mu^2 \right) \frac{\partial \psi'}{\partial \mu}, \quad V' = \frac{1}{a} \frac{\partial \psi'}{\partial \lambda}, \quad (B2) \]

\[ \frac{1}{(1 - \mu^2)} \frac{\partial U_b}{\partial \lambda} + \frac{\partial V_b}{\partial \mu} = 0, \quad (B3) \]

\[ \frac{1}{(1 - \mu^2)} \frac{\partial U'}{\partial \lambda} + \frac{\partial V'}{\partial \mu} = 0. \quad (B4) \]

The perturbation vorticity is given by

\[ \xi' = \frac{1}{a} \left[ \frac{1}{(1 - \mu^2)} \frac{\partial V'}{\partial \lambda} - \frac{\partial U'}{\partial \mu} \right]. \quad (B5) \]

The corresponding equation for the perturbation kinetic energy is obtained by multiplying (B1) by \( \psi' \), and integrating globally with respect to \( \lambda \) and \( \mu \). The latter operation is denoted by an overbar. Then integration by parts gives

\[ \overline{\psi' \frac{\partial \xi'}{\partial t}} = \psi' \frac{\partial \nabla^2 \psi'}{\partial t} = -\nabla \psi' \cdot \frac{\partial \nabla \psi'}{\partial t} = -\frac{\partial KE}{\partial t}, \]

where \( KE \) is the required kinetic energy. Integration by parts also shows the integrated products associated with the last two terms of (B1) to vanish, since

\[ \overline{V' \psi'} = \psi' \frac{\partial \psi'}{\partial \lambda} = 0 \]
and
\[
\frac{1}{(1 - \mu^2)} \frac{\partial}{\partial \lambda} (U \psi') + \frac{\partial}{\partial \mu} (V \psi') = 0,
\]
by virtue of (B2) and (B4). Integration by parts involves the first two terms of (B1), and use of (B3) then gives
\[
\frac{\partial KE}{\partial t} = \frac{1}{(1 - \mu^2)} \left( V_b U' - U_b V' \right) \frac{\partial U'}{\partial \lambda}.
\]
or using (B5)
\[
\frac{\partial KE}{\partial t} = \frac{1}{a} \left[ \frac{1}{(1 - \mu^2)} (V_b U' - U_b V') \right].
\]
Further integration by parts shows that (B6) may be written as
\[
\frac{\partial KE}{\partial t} = \left[ \frac{U'' V'}{a} \left( \frac{\partial}{\partial \mu} \left( \frac{U_b}{1 - \mu^2} \right) + \frac{1}{(1 - \mu^2)^2} \frac{\partial V_b}{\partial \lambda} \right) \right]
- \frac{1}{a} \left[ \frac{V_b}{(1 - \mu^2)^2} \frac{\partial U'}{\partial \lambda} - \frac{V''}{2} \frac{\partial U_b}{\partial \lambda} - \frac{1}{2} U' \frac{\partial}{\partial \mu} \left( \frac{V_b}{1 - \mu^2} \right) + \frac{U'' U_b}{(1 - \mu^2)} \frac{\partial V'}{\partial \mu} \right].
\]
When velocities scaled by \cos\theta are replaced by actual velocities, the first bracketed group of terms on the right-hand side of (B7) is directly in the form of the conversion term \(CKy\) in Eq. (4). The second group is reduced to the form of \(CKx\) by:

1) Interchanging \(1 - \mu^2\)^{-1}\partial U'/\partial \lambda and \(\partial V'/\partial \mu\) between the first and last terms of the group (using B4).
2) Integration of these terms by parts.
3) Further use of (B3) to remove the explicit dependence on \(\partial V_b/\partial \mu\).

The integrals \(CKx\) and \(CKy\) have been evaluated by summing terms evaluated on the regular 5.625° \times 5.625° latitude/longitude grid on which the results of experiments were archived. Derivatives of the basic-state velocities were obtained using second-order finite differences and velocities evaluated (from their spherical harmonic representation) on a 2.8125° \times 2.8125° grid. As a measure of the accuracy of this calculation, and indeed as a further basic check on the coding of the barotropic model, we note that averaging over one cycle of the most unstable mode in the case of zero dissipation, the net rate of energy conversion implies a growth rate of (6.6 day)^{-1} for the mode, a value identical to two significant figures to that computed directly using the initial value technique.

A comment should also be made on the interpretation of the kinetic energy equation with the perturbation energy defined, as here, by
\[
KE = \frac{1}{2}(u^2 + v^2).
\]
This interpretation is unambiguous if the overbar denotes not only a spatial average, but also an average over a period of time for which \(u' = v' = 0\), as for example in the conversions shown in Fig. 17. At an instant in time, the total kinetic energy will include not only an eddy contribution of form (B8), but also terms involving the product of basic and perturbation velocities. Eq. (4) remains valid as an equation describing the growth of a measure of disturbance amplitude, but care must be taken in interpreting it.

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