

## Question 1

When Earth's climate is in equilibrium the planet is in global energy balance, with absorbed solar radiation equal to outgoing longwave radiation:

$$Q(1 - \alpha) = A + BT_e.$$

Let  $Q = 338 \text{ W m}^{-2}$ ,  $A = 203.3 \text{ W m}^{-2}$ , and  $B = 2.09 \text{ W m}^{-2} \text{ }^\circ\text{C}^{-1}$ .  $T_e$  is the annual global mean temperature in  $^\circ\text{C}$ . This is sometimes called a "zero-dimensional climate model". Assume that the following simple relationship between albedo and temperature exists:

$$\alpha = \begin{cases} 0.6; & \text{ice covered: } T_e \leq -30^\circ\text{C} \\ 0.3 + 0.3 \left( \frac{10 - T_e}{40} \right); & \text{partially ice covered: } -30^\circ\text{C} < T_e \leq 10^\circ\text{C} \\ 0.3; & \text{ice free: } 10^\circ\text{C} < T_e. \end{cases}$$

1. Show that for this climate model three equilibrium solutions exists, and find the corresponding values of  $T_e$ . (Hint: sketch by hand or with a computer the emitted longwave radiation and absorbed solar radiation as a function of  $T_e$  across the range  $-40$  to  $20^\circ\text{C}$ .)
2. Keeping the value of  $A$  fixed, find the minimum value of  $B$  (but not the trivial case of  $B = 0$ ) beyond which one equilibrium climate exists.

## Question 2

A doubling of  $\text{CO}_2$  produces an increase in direct radiative forcing of about  $1.8 \text{ W m}^{-2}$  at the surface. Take the zero-dimensional climate model of question 1, with  $F = A + BT_e$  and  $S = Q(1 - \alpha)$ .

1. Calculate the climate sensitivity parameter,

$$\lambda = \left( \frac{dF}{dT} - \frac{dS}{dT} \right)^{-1}$$

and find the change in equilibrium temperature  $T_e$  for a doubling of  $\text{CO}_2$ . Assume  $\alpha$  is independent of  $T_e$  here.

- Now assume that the albedo is a function of temperature. Show that in this case the new climate sensitivity parameter,  $\lambda^*$ , is given by

$$\lambda^* = \frac{\lambda}{1 + Q\lambda\frac{d\alpha}{dT}}$$

For the case where  $\alpha = \alpha_0 + \alpha_1 T$  with  $\alpha_0 = 0.3$  and  $\alpha_1 = -0.005^\circ\text{C}^{-1}$ , find the value of  $\lambda^*/\lambda$ , and determine the new change in  $T_e$  for a doubling of  $\text{CO}_2$ .

- Some researchers have used values for  $A$  and  $B$  of  $202 \text{ W m}^{-2}$  and  $1.45 \text{ W m}^{-2} \text{ }^\circ\text{C}^{-1}$ , respectively. Does this choice make the climate more or less stable (bigger or smaller  $\lambda$ ) in general? Briefly explain physically why this is true.

**The remaining questions require the use of the one-dimensional energy balance model in MATLAB. See additional handout for instructions.**

### Question 3

#### Varying $D$

- Run the model with the standard parameter set. Note the maximum poleward heat flux, the mean global temperature ( $\bar{T}$ ), and the pole-to-equator temperature difference ( $\Delta T_{p-e}$ ). What happens when there is no meridional heat transport ( $D = 0$ )? Estimate  $\bar{T}$  and  $\Delta T_{p-e}$ , and briefly describe the changes to the climate.
- Try values of  $D = 0.24 \text{ W m}^{-2}\text{ }^\circ\text{C}^{-1}$  and  $D = 0.64 \text{ W m}^{-2}\text{ }^\circ\text{C}^{-1}$ . Are the changes to the climate consistent with your expectations? For example, note changes to  $\bar{T}$  and  $\Delta T_{p-e}$ . Compare the maximum poleward heat flux in these integrations with that using  $D = 0.44 \text{ W m}^{-2}\text{ }^\circ\text{C}^{-1}$ . Explain why the changes to the heat flux are not simply proportional to the changes in  $D$ .
- Fossilized remains of crocodiles dating back to the Eocene (53-37 million years ago) have been found on Ellesmere Island (latitude is now 80N). Assuming crocodiles can survive when the mean annual temperature is  $10^\circ\text{C}$  and that Ellesmere has not shifted much from its present location (which is true), find the rough value of  $D$  necessary for the crocodiles to have survived. What is the poleward heat flux required for this? Make a guess about what might change  $D$  in this way (Don't worry—no one else knows either).

### Question 4

#### Varying $Q$

The sun's luminosity is not constant in time. It has been gradually increasing. Models of solar evolution suggest that the sun's intensity (i.e.  $Q$ ) has increased by roughly 10% over the last  $10^9$  years. Assume that this trend is linear and will continue. Start with the standard initial temperature profile for each part of this exercise.

1. Find the increase in the value of  $Q$  (to within  $1 \text{ W m}^{-2}$ ) required to eliminate ice from the earth (assume ice exists when the temperature is below  $-10^\circ\text{C}$ ). How long would it take before there is no ice left (assume nothing else changes)?
2. Find the decrease in  $Q$  (to within  $1 \text{ W m}^{-2}$ ) required to cause complete glaciation, and hence show that the model would predict a snowball earth prior to about  $1.0 \times 10^9$  years ago.

While there is evidence of extensive (maybe total) glaciations over a half-billion years ago, these periods appear to have been broken by times that were relatively warm. Speculate what assumptions made in the climate model might not have applied to the earth's climate  $10^9$  years ago. [Note that the snowball earth has a much weaker pole to equator temperature difference. This would lead to a much weaker Hadley circulation and jet stream.]

## Question 5

### No albedo feedback

This exercise is designed to illustrate the different climate sensitivities with and without albedo feedback. Use the standard parameter set for this exercise, except for the albedo parameterization.

1. The modern ice line is at about  $72\text{N}$  (or  $x = 0.95$ ). Click on the no ice-albedo feedback button on the graphical user interface to fix the ice line at this value. Fixing the albedo in this way turns off the feedback between the temperature and the albedo. Show that the model is now much less sensitive to changes in  $Q$ . That is, find the values of  $Q$  required for complete glaciation/deglaciation as in exercise 3. Assume that  $-10^\circ\text{C}$  still denotes where it is cold enough for ice to form. Remember to unset the alternative albedo parameterization when you are finished.

## Other ideas to think about

These are some other ideas to explore with the simple EBM. Try at least one of them. A detailed investigation is not required, but I hope you have some fun just playing around with the model. Simply write a few sentences about what you tried. If any other ideas occur to you too, feel free to explore those instead.

1. **Sensitivity to initial conditions.** The above exercises have all started off with warm initial conditions for the model. However the model has multiple equilibrium solutions for some range of  $Q$ . To show this, begin with the default model. Map out the variations in the ice-line as  $Q$  varies over the range  $290 \text{ W m}^{-2}$  to  $420 \text{ W m}^{-2}$ . Now instead start off with a cold initial temperature profile by clicking on the cold start button. Map out the ice line variations over the same range. This will show the hysteresis loop in the climate and is a measure of how hard it is to get out of a completely glaciated climate.

This can be used to illustrate the faint sun paradox. Because the sun was known to have been weaker in the past, the earth should have had cold “initial conditions”. Yet snowball earth conditions are thought to have been rare and discontinuous.

## 2. Varying $A$ and $B$

- (a) Alternative sets of longwave parameters have sometimes been used. For example try implementing values of  $A = 211.2 \text{ W m}^{-2}$  and  $B = 1.55 \text{ W m}^{-2}$  in the model (holding everything else at the standard values). Describe briefly how the resulting climate is different from that using the standard parameters.
- (b) For this choice of  $A$  and  $B$ , and using  $D = 0.52 \text{ Wm}^{-2}\text{C}^{-1}$  (in order to get back a climate more closely resembling the modern one), find the decrease in  $Q$  (to within  $1 \text{ W m}^{-2}$ ) required for complete glaciation. Explain why a smaller decrease in  $Q$  is now required to produce a snowball earth.

3. **CO<sub>2</sub> increases.** A crude way of introducing CO<sub>2</sub> forcing into to the model is to adjust the model parameter  $A$  by  $\Delta A$ , where

$$\Delta A = -k \ln \frac{\text{CO}_2}{360}$$

with  $k = 3 \text{ W m}^{-2}$  and  $\text{CO}_2$  is the concentration of  $\text{CO}_2$  in ppmv. Thus, an increase in atmospheric  $\text{CO}_2$  causes a decrease in the longwave emissions to space. During the ice ages, records from ice cores show  $\text{CO}_2$  varied between 200 ppmv and 280 ppmv. A doubling of  $\text{CO}_2$  from today’s values would take it up to around 720 ppmv. Explore what changes these values would make to the climate. Note any effect that the ice albedo feedback or  $D$  has. You may find it instructive to estimate the climate sensitivity parameter from  $\lambda \sim \Delta T_{\text{global}}/\Delta A$ .

4. **Spatial variations in  $D$**  Lindzen and Farrell (*J. Atmos. Sci.*, 1977) suggested that since the Hadley Cell was more effective at redistributing heat than extratropical weather systems,  $D$  should vary with latitude (i.e.  $D = D(x)$ ). They suggested using a larger value of  $D$  within the tropics. This can be crudely represented by increasing  $D$  by a factor of 10 (equatorwards of about  $30^\circ$ , say). An implementation that will achieve this is the following:

$$D = 0.45[1 + 9 \exp(-(x/\sin 30^\circ)^6)]$$

(push the “simulate Hadley Cell” button). Describe the resulting climate. Examine and try to explain the effect on the decrease in  $Q$  required for complete glaciation.