GFD – The study of large-scale geophysical [or related] fluid flows, aiming at simple and fundamental approaches to their understanding.

Distinguishing characteristics

- Stratification
- Rotating reference frame

Other complications

- Topography
- Compressible/forced/dissipative

Central strategies

- Scaling of equations to find dominant balances appropriate for particular space/time scales of motion, e.g. 'synoptic scale':

| | A (midlat. cyclonic storms) | O (Gulf stream eddies) |
|---|---|---|
| Length | 1000 km | 50 km |
| Height | 10 km | 1 km |
| Velocity | 10 m/s | 0.5 $\frac{m}{s}$ |
| Earth rotation rate $\frac{2\pi}{1\,\text{day}}$ | $\sim 10^4$ s | $10^4$ s. |

- Idealization of fluid characteristics (e.g. two incompressible fluid layers of different density as a representation of stratification) or geometry
- Separation into steady and wavelike flow components; Fourier analysis and linear dispersive wave theory
- Ideas of ~~4~~ momentum conservation & vorticity
- Laboratory + numerical simulations.

Our task

Discuss basic eqns (including thermo, planetary rotation)
↓
Scaling analysis, including some commonly applicable simplifications
↓
Two layer flow ⇒ linear shallow water eqns. (SWE)
... a realizable idealization that can be thoroughly understood.
↓
Return to continuously stratified case.

Momentum eqns of classical fluid dynamics

{ } = "Units of"

$\{p\} = \text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}\,\frac{\text{m}}{\text{s}^2}}{\text{m}^2} = \frac{\text{kg}}{\text{m-s}^2}$

$$\frac{D\vec{u}}{Dt} = \underbrace{-\frac{1}{\rho}\nabla p}_{\text{PGA}} \underbrace{- \nabla\phi}_{\substack{\text{conservative body force}\\ \text{due to potential } \phi(\vec{x})\\ \text{e.g. } \phi = gz}} + \underbrace{\vec{F}(\vec{u}, \text{etc.})}_{\substack{\text{Other forces (e.g. friction)}\\ \text{rotation}}}$$

$\{\rho\} = \frac{\text{kg}}{\text{m}^3}$ $\{\phi\} = \frac{\text{m}^2}{\text{s}^2}$.

$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{u}\cdot\nabla$ is the "material derivative". (following the flow).

Mass continuity

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \nabla\cdot\vec{u} = 0.$$

For an incompressible fluid, $D\rho/Dt = 0$ so $\nabla\cdot\vec{u} = 0$.
For a compressible fluid, supplemented by eqns which determine $\rho$.

Eqn of state

$\rho(p, T, S)$, where $T$ is temperature and $S$ is density-affecting constituents. (water vapor for air; salinity for sea-water).

Scalar transport eqn

$$\frac{DS}{Dt} = F_S$$ ← source term

Thermodynamic equation: $T\dfrac{Ds}{Dt} = Q_{rev}$ where specific entropy $s(p, T, S)$.

## Simplifications

Thermodynamic equation: We will stick with adiabatic flow ($Q_{rev} = 0$), so $s$ is conserved following the fluid. For either air or water, this can be reduced to

$$\frac{D\rho_\theta}{Dt} = 0 \quad \Leftrightarrow \quad \frac{D\theta}{Dt} = 0 \text{ for air}$$

Equation of state:

Air: We neglect the slight dependence of the density of moist air on water vapor mass fraction q (rarely > 1%) :

$$\rho = \frac{p}{R_d T(1+.61q)} \approx \frac{p}{R_d T}$$

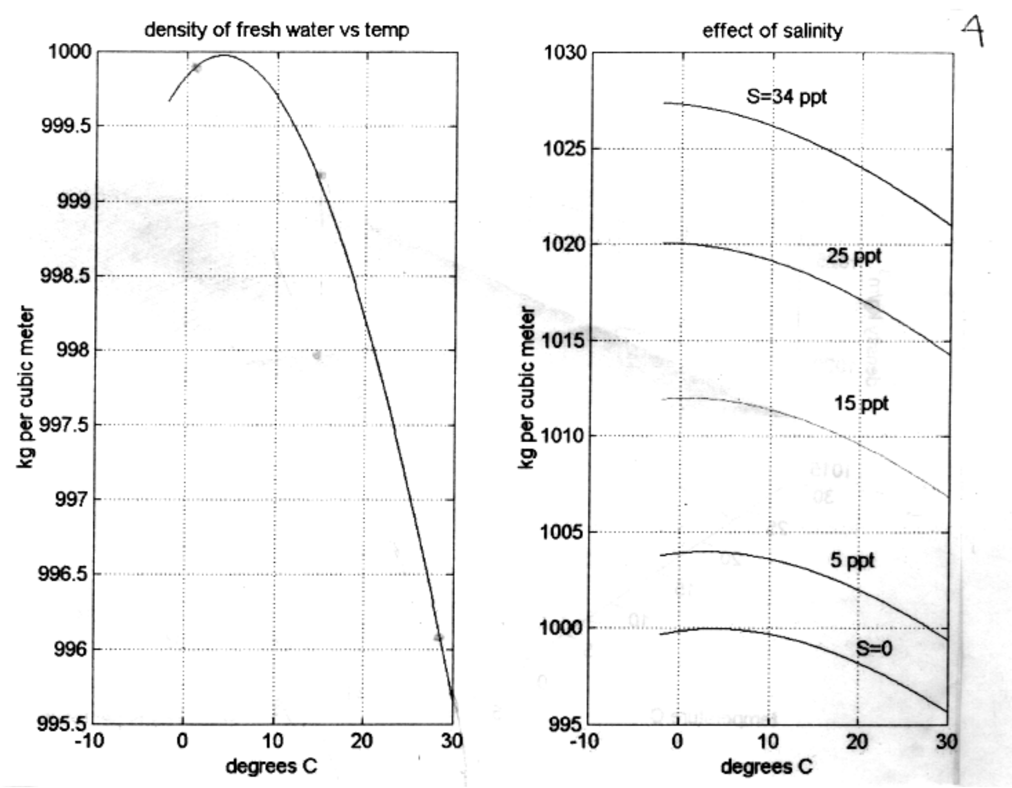
Water: Equation of state is quite nonlinear in $T$ and salinity $S$ (density of fresh water is maximum at 4 C, but for typical ocean salinity $S_0 = 3.5\%$, ρ increases down to freezing temp of -2 C). However, for estimating the relative roles of temperature and salinity, we will use a simplified form

$$\rho = \rho_\theta + c_s^{-2}(p - p_0), \quad \rho_\theta = \rho_0\left[1 - \alpha(T - T_0) + \beta(S - S_0)\right]$$

where for a reference temperature of $T_0$ = 10 C and pressure $p_0$ = 1 bar, $c_s = 1450\,\mathrm{m\,s^{-1}}$,

$\rho_0 = 1028$ kg m$^{-3}$, $\alpha = 1.7\times10^{-4}\,\mathrm{K^{-1}}$ and $\beta = 0.76$.

In low and mid-latitudes the temperature dependence dominates in the upper ocean, but in the deep ocean and high-latitudes ('thermohaline circulation'), the salinity dependence also become equally important.




Density of fresh and salt water vs. temperature at pressure 1 bar