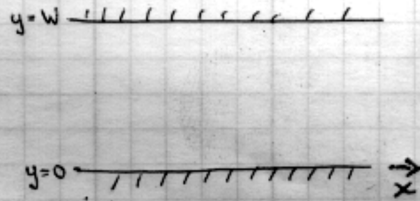
Flow in a channel; Kelvin Waves (CR6.2, Gill 10.4)

In a channel, $v=0$ at $y=0,W$. Governing LSWE:



with $v = \mathrm{Re}\, \tilde{v}(y) e^{i(kx-\omega t)}$.

$$\frac{\partial}{\partial t}\left\{-\frac{\partial^2}{\partial t^2} - f^2 + gH_0\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right\} v = 0$$

$\Rightarrow$

$$-i\omega\left\{gH_0\frac{d^2\tilde{v}}{dy^2} + (\omega^2 - f^2 - gH_0k^2)\tilde{v}\right\} = 0 \qquad \tilde{v}(0)=\tilde{v}(W)=0.$$

Possibilities: (I) $\omega = 0$ (leads to geostrophic balanced mode with $\tilde{v} \propto \frac{\partial\tilde{\eta}}{\partial y} = 0$ at $y=0,W$)

(II) $gH_0\frac{d^2\tilde{v}}{dy^2} + (\omega^2 - f^2 - gH_0k^2)\tilde{v} = 0$

$\tilde{v} = \sin l_n y$, $\omega^2 = f^2 + gH_0(k^2+l_n^2)$, $l_n = \frac{n\pi}{W}$ (Standing Poincaré wave)

(III) $\tilde{v} = 0$ everywhere!

$$\Rightarrow \frac{\partial u}{\partial t} \cancel{-fv} = -g\frac{\partial \eta}{\partial x}$$

$$\cancel{\frac{\partial v}{\partial t}} + fu = -g\frac{\partial \eta}{\partial y} \qquad \text{Geostrophic balance in } y$$

$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}}\right) = 0$$

$$\begin{bmatrix}\eta\\ u\end{bmatrix} = \mathrm{Re}\, e^{i(kx-\omega t)}\begin{bmatrix}\tilde{\eta}\\ \tilde{u}\end{bmatrix}(y)$$

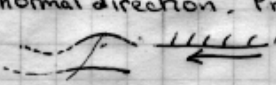
$$\Rightarrow -i\omega\tilde{u} = -ikg\tilde{\eta} \qquad \text{or } \tilde{u} = \frac{kg}{\omega}\tilde{\eta}$$

$$-i\omega\tilde{\eta} + ikH\tilde{u} = 0 \qquad \Rightarrow -i\omega + ikH\frac{kg}{\omega} = 0 \Rightarrow \omega^2 = k^2c_0^2$$

Two roots: $\omega = kc_0$ (R-travelling Kelvin wave), $\frac{d\tilde{\eta}}{dy} = -\frac{f}{c_0}\tilde{\eta} \Rightarrow \tilde{\eta} = \hat{\eta}e^{-y/R}$

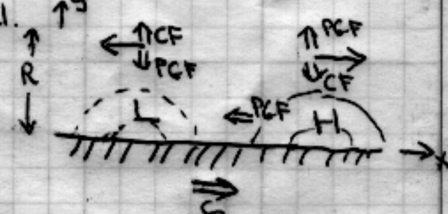
$\omega = -kc_0$ (L-travelling Kelvin wave), $\frac{d\tilde{\eta}}{dy} = \frac{c_0}{f}\tilde{\eta} \Rightarrow \tilde{\eta} = \hat{\eta}e^{+y/R}$

Kelvin waves are non-dispersive + like gravity waves in propagation direction, supported by geostrophic balance in normal direction. Propagate with wall on their R. (if $f>0$)



For a R-moving Kelvin wave, $\tilde{u} = \frac{g}{c_0}\tilde{\eta}$ is in phase with $\tilde{\eta}$.

Fluid "leans" on the wall to counteract Coriolis accel.



Equatorial Kelvin wave (Gill 11.3) 28

A Kelvin wave in which the fluid in each hemisphere "leans" on the other hemisphere ; equator takes place of wall

LSWE on β-plane

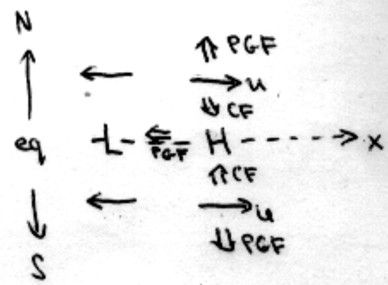
$$\frac{\partial u}{\partial t} - \beta y v = -g\frac{\partial \eta}{\partial x}$$
$$\frac{\partial v}{\partial t} + \beta y u = -g\frac{\partial \eta}{\partial y}$$
$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

Look for mode with $v=0$, $\begin{pmatrix} u \\ \eta \end{pmatrix} = \mathrm{Re}\begin{pmatrix} \hat u(y) \\ \hat\eta(y) \end{pmatrix} e^{i(kx-\omega t)}$

$$-i\omega\hat u = -ikg\hat\eta \quad \Rightarrow \hat u = kg\hat\eta/\omega \text{ as before} \qquad (1)$$
$$\beta y\hat u = -g\frac{\partial\hat\eta}{\partial y} \quad \text{(geostrophic balance in } y) \qquad (2)$$
$$-i\omega\hat\eta + ikH\hat u = 0 \quad \Rightarrow -i\omega + ikH\left\{\frac{kg}{\omega}\right\} = 0 \text{ as before} \qquad (3)$$
$$\text{so } \omega = \pm c_0 k,\ \hat u = \pm g\hat\eta/c_0$$

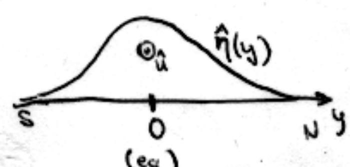
Plug into (2):

$$\pm\frac{\beta y \cancel{g}\hat\eta}{c_0} = -\cancel{g}\frac{\partial\hat\eta}{\partial y}$$
$$\Rightarrow \frac{d\hat\eta}{\hat\eta} = \mp\frac{\beta y}{c_0}dy$$
$$\hat\eta = \eta_0 e^{\mp\frac{\beta y^2}{2c_0}}$$



Only the top soln, $\frac{\omega}{k} = +c_0$, which is E-ward propagating is bdd (and → 0) as $y \to \pm\infty$, and is the equatorial Kelvin wave

Very similar to wall-trapped K-wave, but meridional scale is $R_{eq} = \left(\frac{c_0}{\beta}\right)^{\frac{1}{2}}$.



Oceanic equatorial Kelvin waves play a key role in the dynamics of El Nino

Atmospheric Kelvin waves are often coupled to enhanced tropical thunderstorm activity and rainfall, especially in the central Pacific ITCZ during boreal summer. The Madden-Julian Oscillation, though not really a Kelvin wave, also has related features (equatorial trapping, E-ward propagation) despite a slow propagation speed of 5-10 $\frac{m}{s}$ in the Indian/West Pac ocean. Equatorial Kelvin waves propagating up into stratosphere help excite Quasi-Biennial Osc (QBO).