For low Ro flow $[\zeta] \ll f$, and $\zeta \approx \zeta_g$, the interior flow satisfies

$$\left(\frac{d\zeta}{dt}\right)_E = -\frac{fd}{2h}\zeta = -\frac{\zeta}{\tau_E}$$

i.e. there is an Ekman spindown timescale $\tau_E = \frac{2}{f}\left(\frac{h}{d}\right)$ associated with Ekman pumping.

Atm: $h = 10\,km$
$d = 200m - 1km \Rightarrow \tau_E = 2\times(10-50)\times(10^4 s) = 2\text{-}10$ days

Ocean $h = 1\,km$
$d = 5\text{-}30\,m \Rightarrow \tau_E = 2\times(30\text{-}200)\times(10^4 s) = 6\text{-}40$ days.

L17

Top Ekman Layers

$-f(v - v_g) = \nu\, d^2u/dz^2$
$f(u-u_g) = \nu\, d^2v/dz^2 \quad \Rightarrow$
$(u,v) \rightarrow \vec{u}_g$ as $z \rightarrow -\infty$
$\rho\nu \frac{d\vec{u}}{dz} = \vec{\tau}$ at $z=0$

$\vec{u}_g \approx 0$

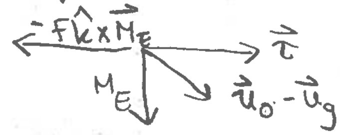
Very similar to before. if $\vec{\tau} = \tau^x\hat{i} + \tau^y\hat{j}$, $\tau = \tau^x + i\tau^y$, $S - S_g = S_0 e^{(1+i)\frac{z}{d}} \Rightarrow \frac{\tau}{\rho\nu} = \left(\frac{dS}{dz}\right)_0 = \frac{S_0}{d}(1+i)$
$\Rightarrow S_0 = \frac{\tau d}{\rho\nu} 2^{-\frac{1}{2}} e^{-\frac{i\pi}{4}}$

$$u = u_g + \frac{2^{\frac{1}{2}}}{\rho f d} e^{z/d}\left\{\tau^x \cos\left(\frac{z}{d} - \frac{\pi}{4}\right) - \tau^y \sin\left(\frac{z}{d} - \frac{\pi}{4}\right)\right\}$$

$$v = v_g + \frac{2^{\frac{1}{2}}}{\rho f d} e^{z/d}\left\{\tau^x \sin\left(\frac{z}{d} - \frac{\pi}{4}\right) + \tau^y \cos\left(\frac{z}{d} - \frac{\pi}{4}\right)\right\}$$

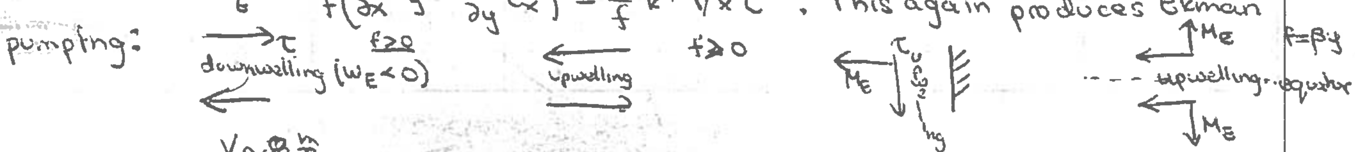
$= \frac{2^{\frac{1}{2}}}{\rho f d}\{\tau^x + i\tau^y\}\{\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\}$

$$\vec{M}_E = \int_{-\infty}^{0} \rho(\vec{u} - \vec{u}_g)\,dz = \frac{1}{f}(\tau^y, -\tau^x) = \frac{1}{f}\hat{k}\times\vec{\tau}$$ (so $f\hat{k}\times\vec{M} = -\vec{\tau}$ as before)



ocean Ekman transport equal+opposite to atmosphere.

Note that $\nabla\cdot\vec{M}_E = \frac{1}{f}\left(\frac{\partial}{\partial x}\tau_y - \frac{\partial}{\partial y}\tau_x\right) = \frac{1}{f}\hat{k}\cdot\nabla\times\vec{\tau}$. This again produces Ekman pumping:



$V \sim 8\,\frac{m}{s}$

Magnitudes: (45N) $\tau_0 = 0.1\,Pa$, $f_0 = 10^{-4}\,s^{-1}$, $h = 20m$, $\Rightarrow M_e = \frac{\tau_0}{f_0} = 10^3\,\frac{kg}{m\text{-}s}$
midlat W-lies

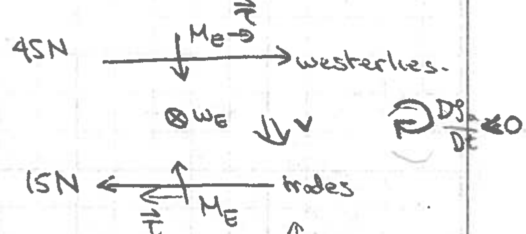
$\Rightarrow v_e = \frac{M_e}{\rho_w h} \sim \frac{10^3}{(10^3)(60)} \sim 1.5\,\frac{cm}{s}$

Integrated across Pac
$W = 10^4\,km = 10^7\,m$

$\Rightarrow M_e\cdot W = 10^{10}\,\frac{kg}{s} = 10\,Sv.$

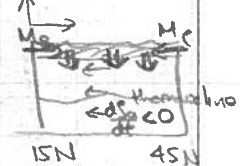
To conserve PV in an ocean column

$$q = \frac{f+\zeta}{h} = \text{const.}$$



Thus, for midocean gyres where $\zeta \ll f$,

$$\left(\frac{1}{f}\frac{Df}{Dt}\right)_E = \left(\frac{1}{h}\frac{Dh}{Dt}\right)_E = \frac{1}{\rho h}\nabla\cdot\vec{M}_E = \frac{1}{\rho h f}\hat{k}\cdot(\nabla\times\vec{\tau})$$



Since $\frac{Df}{Dt} = v\frac{\partial f}{\partial y} = \beta v$,

$$\beta v = \frac{1}{\rho h}\hat{k}\cdot(\nabla\times\vec{\tau}) \quad \text{(Sverdrup transport)}$$

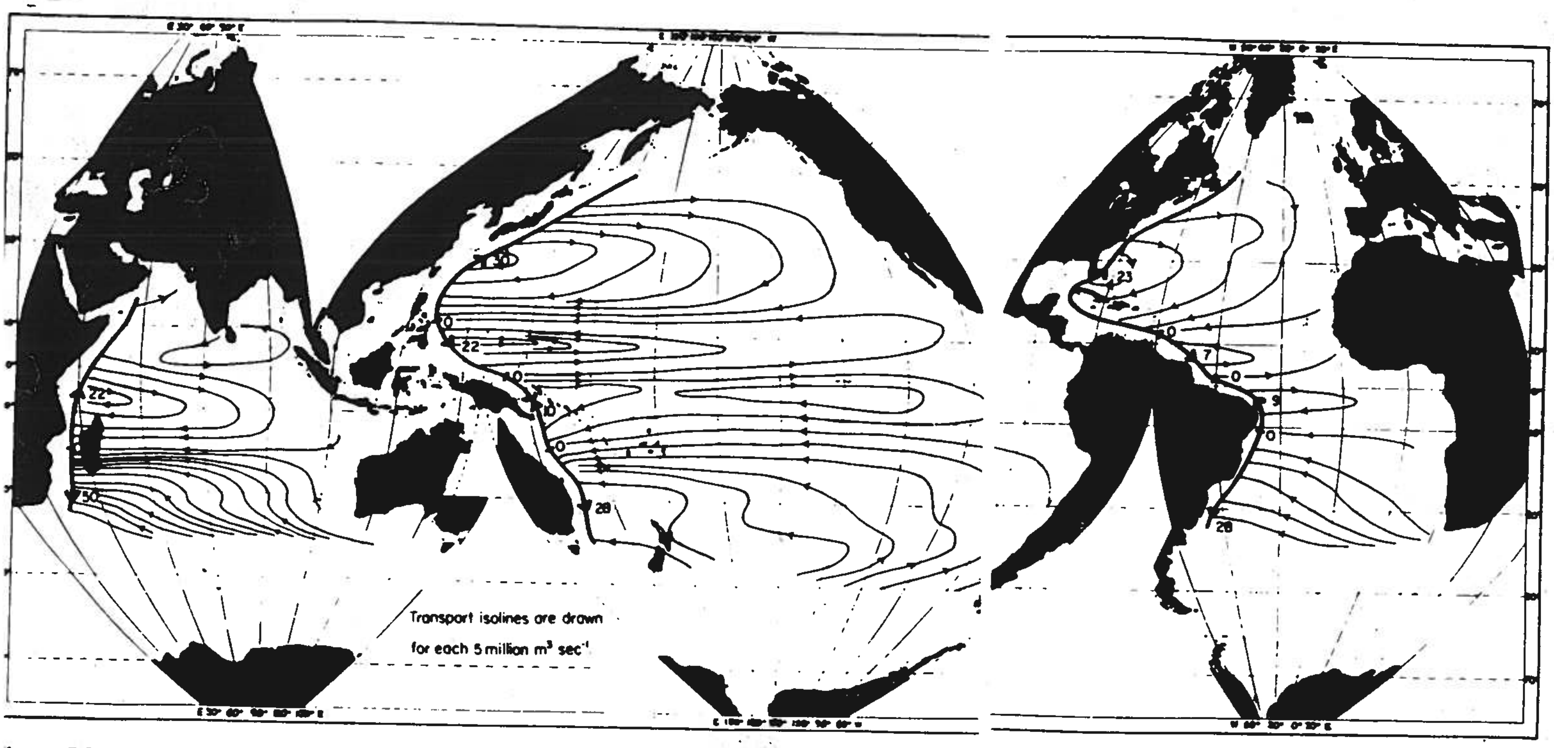
... responsible for most of the upper-ocean currents.

Magnitudes

$$\nabla\times\tau = \frac{\partial\tau}{\partial y} = \frac{0.1\ \text{Pa}\ \frac{kg}{m\text{-}s^2}}{2000\times10^3 m} \sim 0.5\times10^{-7}\ \frac{kg}{m^2\text{-}s^2}$$

$$\Rightarrow v = \frac{1}{\rho h\beta}\nabla\times\tau = \frac{0.5\times10^{-7}}{(10^3 \frac{kg}{m^3})(10^3 m)(1.5e\text{-}11 \frac{1}{m\text{-}s})}$$

$$\Rightarrow M_{sv} = \rho h v = 5\times10^3 \frac{kg}{m\text{-}s} = 0.5\ \frac{cm}{s}$$



igure 5.3.4 The Sverdrup transport and the implied western boundary currents in the oceans corresponding to the annual mean wind-stress field t... rdrup transport is required to satisfy the normal-flow condition on oceanic eastern boundaries. Figures represent the volume transports in vo... ps (millions of m³ s⁻¹). (Reprinted courtesy Welander, 1959.)

GFD

Real Geophysical Ekman Layers

Atmosphere

Turbulence

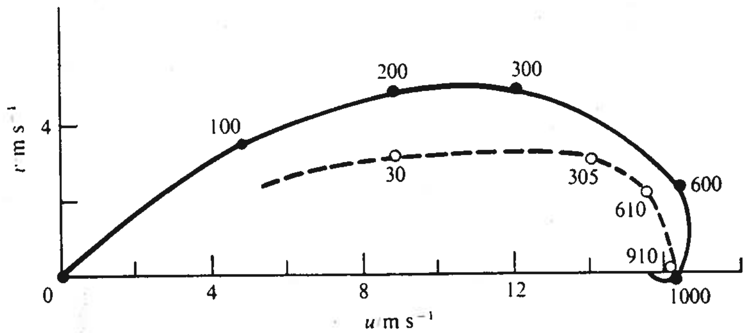


Fig. 9.2. Wind hodograph for the lowest kilometre as measured by Dobson (1914) (dashed line) compared with Ekman spiral (full line). Figures on curves are heights in metres.

Ocean

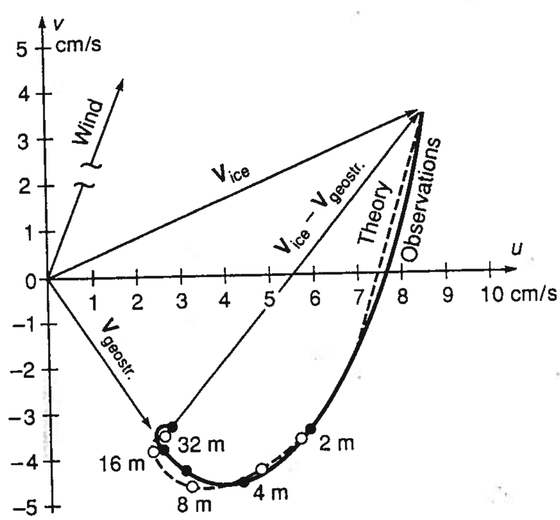


Figure 5-7 Comparison between observed currents below a drifting ice floe at 84.3°N and theoretical predictions based on an eddy viscosity $\nu = 2.4 \times 10^{-3}$ m^2/s. (Reprinted from *Deep-Sea Research*, 13, Kenneth Hunkins, Ekman drift currents in the Arctic Ocean, p. 614, copyright 1966, with kind permission from Pergamon Press Ltd, Headington Hill Hall, Oxford 0X3 0BW, UK.)