Waves over topography (CR 10.4), Gill 8.7)

Suppose flow of mean speed U flows over a mountain

$B(x) = B_0 \cos(kx)$.

Then if no boundary as $z \to \infty$.

$z_p(x,0) = B_0 \cos kx$

$z_p(x,z) = B_0 \cos(kx + mz)$

Let $\xi = x - Ut$ be coordinate moving w. fluid, $z_p(\xi, z, t) = B_0 \cos(k\xi + mz + kUt)$

where if $\omega_d = -kU$ is the Doppler-shifted "intrinsic" frequency in the ref. frame moving with fluid,

$$k^2U^2 = \omega_d^2 = \frac{k^2N^2 + m^2f^2}{k^2+m^2} \Rightarrow m^2 = \frac{k^2(N^2 - k^2U^2)}{k^2U^2 - f^2}$$

$z_p(\xi, 0, t) = B_0 \cos(k\xi - k$

$\omega_d = -kU$

- Note $m > 0$ to make the intrinsic vertical phase velocity $\vec{c}_{pi,z}$ negative and intrinsic group velocity positive.
- If $k^2U^2 < f^2$ or $k^2U^2 > N^2$, no wave propagation; response decays exponentially in vertical.
- Waves propagate both momentum and energy upward since $\hat{u}, \hat{p}, \hat{w}$ in phase if $c_{pz} < 0$.

Down ~~Upward~~ Wave momentum flux:

$$\rho_0 \overline{uw} = -\rho_0 k m U^2 H^2 < 0$$

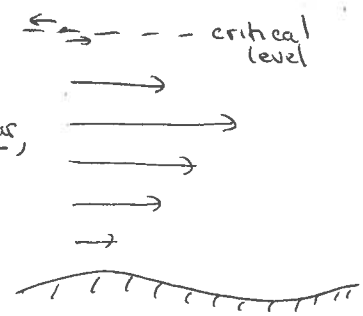
(downward momentum flux from wave absorption region. sustains flow against drag $D = -\rho_0 \overline{uw}$).

$= \int_{B(x)} p' \cdot \hat{n}\, ds$

Upward Wave energy flux

$$\rho_0 \overline{p'w} > 0.$$

Shear (CR 10.6)



Wave propagation is modified by vertical wind shear, variations in U with z. We consider $f=0$ case only

At any level,

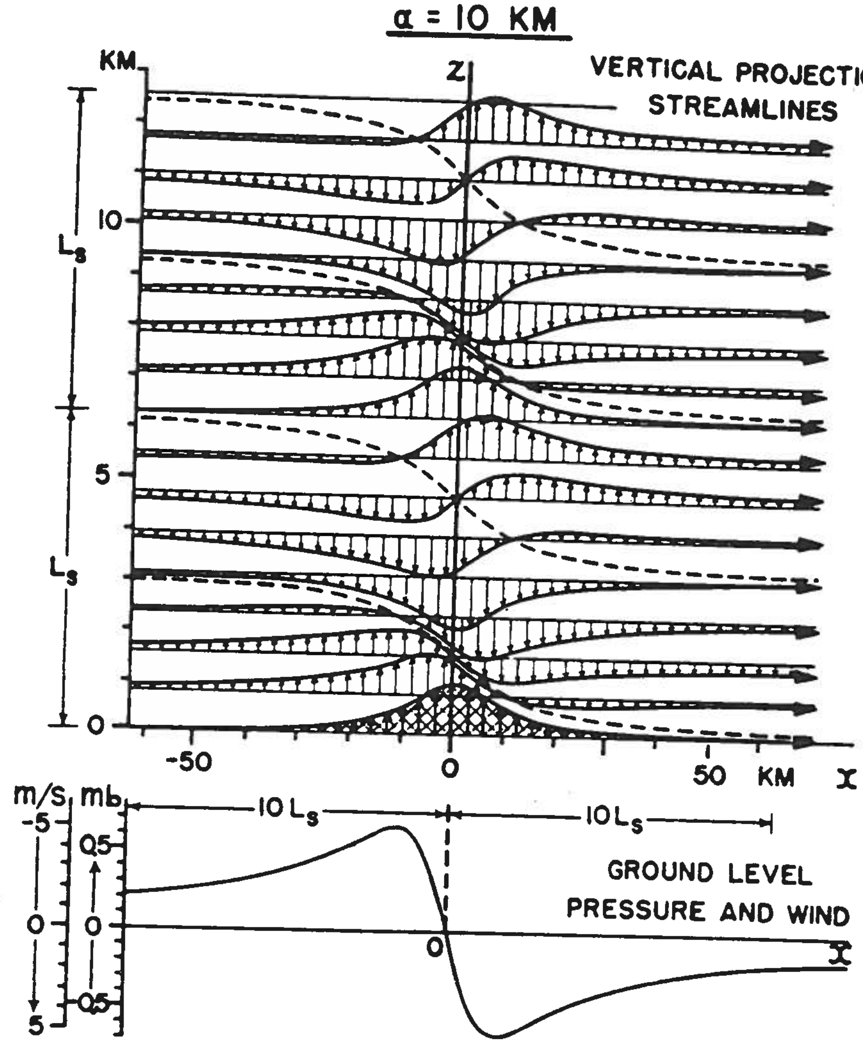
$$k^2U^2 = \omega_d^2 \approx \frac{k^2N^2}{k^2+m^2(z)} \Rightarrow m^2(z) = \frac{k^2(N^2 - k^2U^2)}{k^2U^2} \left[-\frac{1}{U}\frac{d^2U}{dz^2}\right]$$

$$= \frac{N^2}{U^2} - k^2 \to \infty \text{ as } U \to 0$$

A level where $U \to 0$ is called a critical level and represents a barrier to further upward wave propagation where energy is absorbed, or may be reflected downward for a large-amplitude wave.

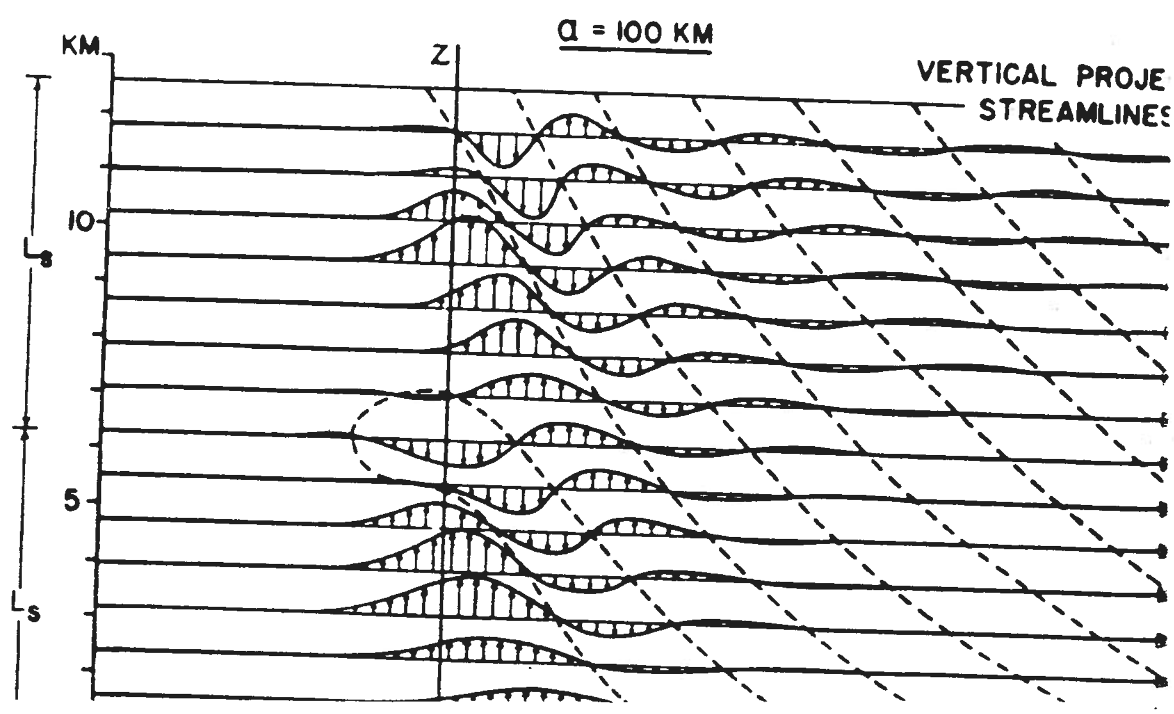
Wave breaking

For a large-enough mtn, or $\bar{\rho}$ decreasing with height, ρ_0 surfaces steepen and overturn, leading to local convective instability, wave breaking + wave dissipation. Occurs at 20-80 km in stratosphere, absorbing upward E, mom. fluxes.



α = 10 KM
KM
VERTICAL PROJECTION
STREAMLINES
Z
10
5
0
L_s
L_s
-50
0
50 KM
x
m/s
mb
-5
0.5
0
0
5
-0.5
10 L_s
10 L_s
GROUND LEVEL
PRESSURE AND WIND
0
x

(b)



a = 100 KM
KM
Z
VERTICAL PROJE
STREAMLINES
10
5
L_s
L_s

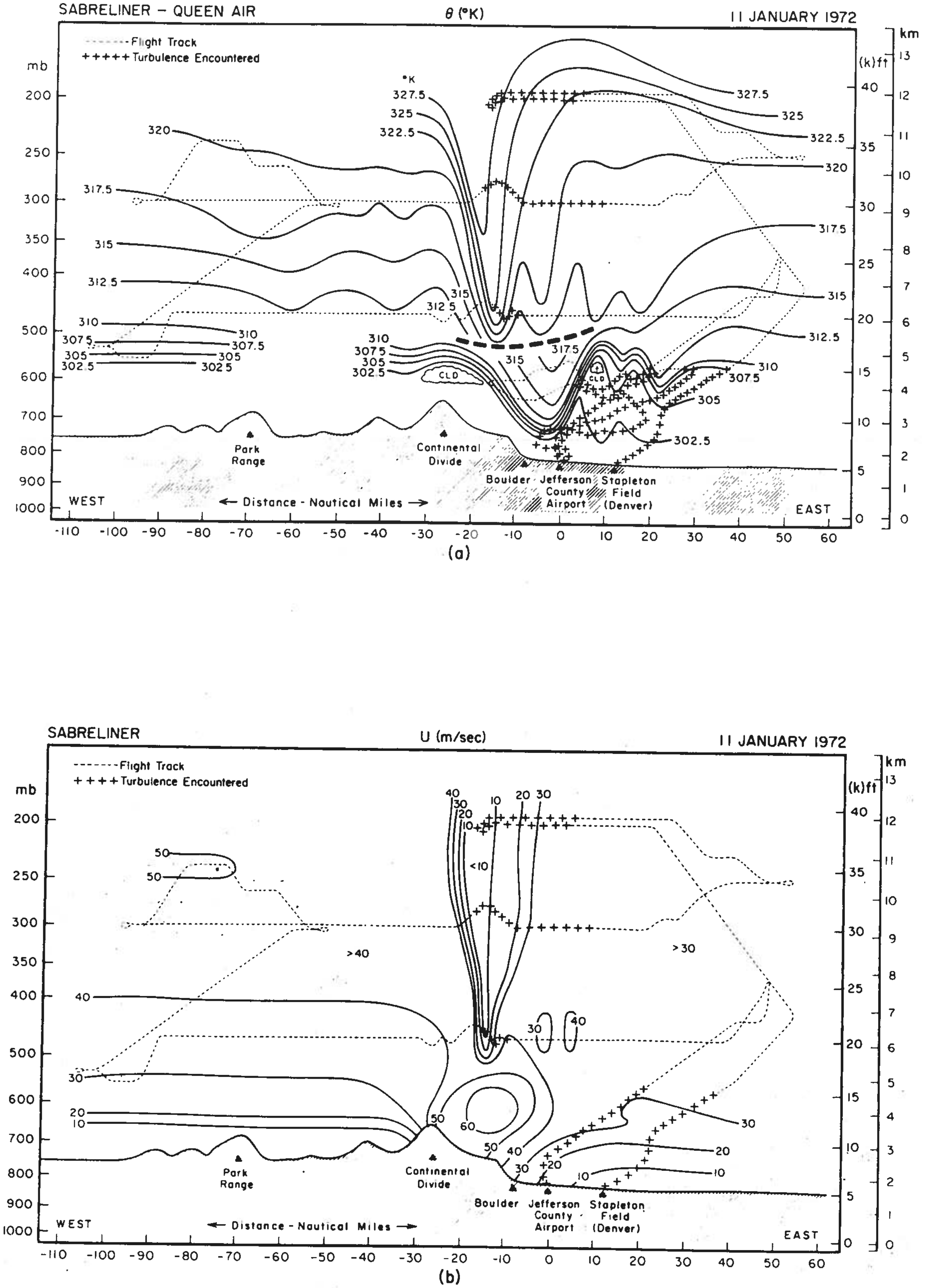


Fig. 8.13. Observations of (a) potential temperature (in degrees Kelvin) and (b) velocity (in meters per second) in a section across the Rockies in Colorado on January 11, 1972. The dashed lines show aircraft tracks with periods of significant turbulence shown by plus signs. The heavy dashed line separates observations made a few hours apart—it is quite possible that at a given time, the upper dip in the potential temperature contours was directly above the lower one. [From Lilly 1978, Figs. 7 and 9).]

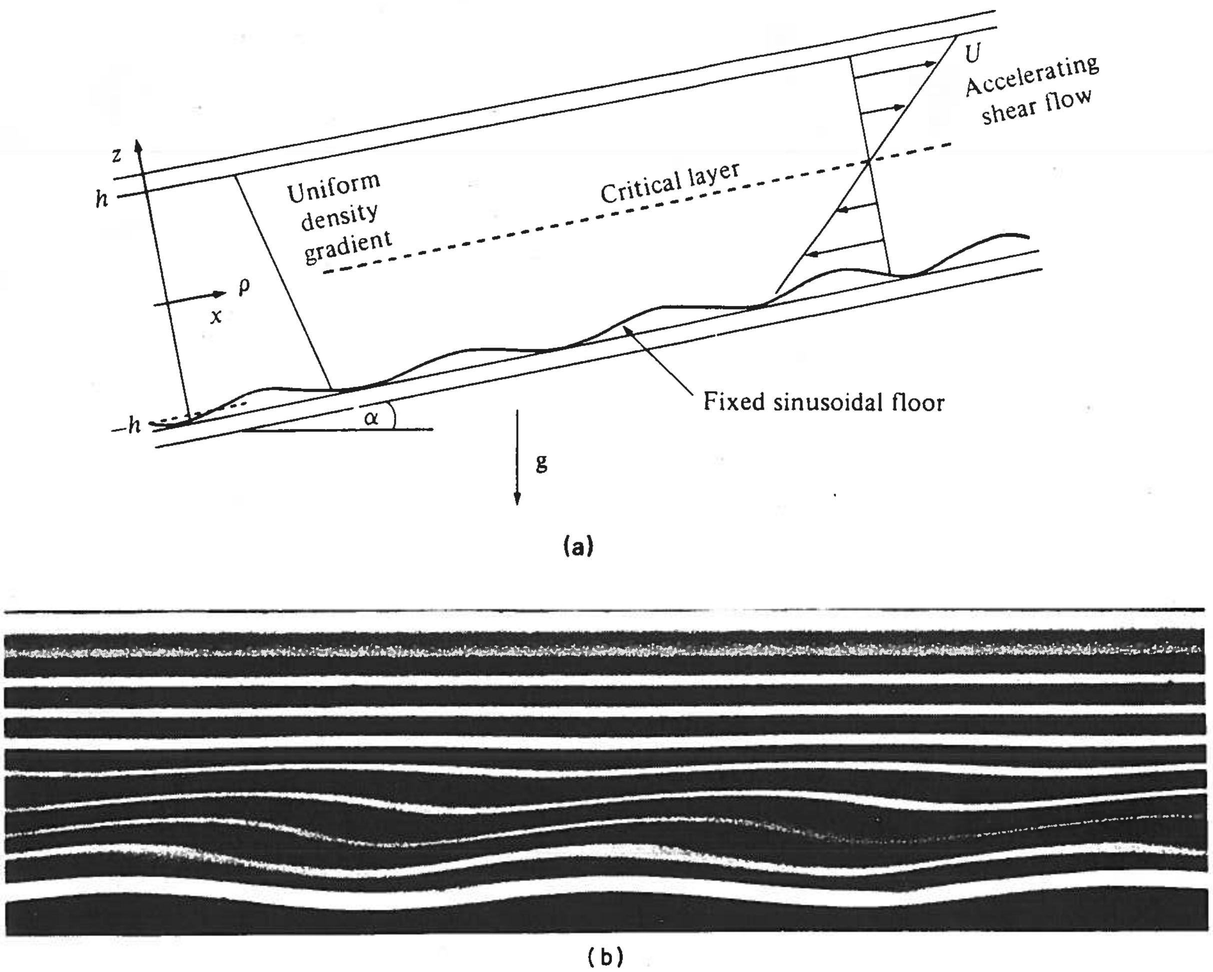


Fig. 8.15. An experimental demonstration of critical layer absorption of internal waves. (a) The apparatus—an inclined tube with a corrugated floor. (b) Waves observed in an accelerating shear flow over a corrugated floor of amplitude 0.5 cm and wavelength 25 cm. The tube contains stratified fluid with $N = 2.626\ \mathrm{s}^{-1}$ and has been tilted through 5.2°. [From Thorpe (1981, Figs. 1 and 4e).]