Homework Set 2

1. Consider a clear convective boundary layer driven by strong surface heating. Such boundary layers are observed to be *well-mixed*, i. e. the ensemble-averaged $\overline{\theta}$ and $\overline{q}$ do not vary with height. Assume that both the mean wind and the geostrophic wind are zero. Reference BL density is $\rho_R = 1.2$ kg m^{-3}; reference BL virtual potential temperature is $\theta_{vR} = 300$ K, BL depth $h = 1$ km.

   (a) Consider the ensemble-averaged equations for $\overline{\theta}$ and $\overline{q}$. Assume S_θ and S_q are negligible. The well-mixed assumption implies that the tendencies of $\overline{\theta}$ and $\overline{q}$ must be height-independent within the BL. Deduce that the vertical fluxes of θ and q must vary linearly with height in the BL, and hence that the buoyancy flux $B = (g/\theta_{vR})\overline{w'\theta_v'}$ = $(g/\theta_{vR})\left(\overline{w'\theta'} + 0.61\theta_{vR}\overline{w'q'}\right)$ must vary linearly with height.

   (b) If the surface latent and sensible heat fluxes are both 300 W m^{-2}, what is the surface buoyancy flux B_0? (Note $L_v = 2.5\cdot10^6$ J kg^{-1} and $C_p = 10^3$ J kg^{-1} K^{-1})

   (c) Take the TKE equation and vertically integrate it over the BL, assuming:
      i) there is no vertical flux of TKE through the ground and the BL top.
      ii) the pressure work term $\overline{w'p'}$ at the surface and above the BL are zero.
      iii) TKE dissipation rate $D = -V^3/h$, where the BL depth h is a characteristic large-eddy size and V is a characteristic convective large-eddy velocity.
      iv) the BL-averaged TKE tendency is negligible.

   Show that the vertically-integrated shear and transport sources of TKE are zero, and that

$$V = \left(\int_0^h B(z)dz\right)^{1/3}.$$

   (d) In clear convective BLs the 'entrainment' buoyancy flux at the BL top is observed to be approximately $-0.2B_0$. Estimate V and D this case for the parameters in (b), using the result from (a) that $B(z)$ is a linear function of z. You should find $V = 1.5$ m s^{-1}.

   (e) At mid-height in the BL, most of the fluxes are carried by the large eddies. Suppose we idealize the vertical motion, buoyancy, temperature and moisture patterns associated with these eddies as being sinusoidal in the horizontal, with amplitudes V, δb, $\delta\theta$, δq:

$$w' = V\sin(\pi x/h),\ b' = \delta b\sin(\pi x/h),\ \theta' = \delta\theta\sin(\pi x/h),\ q' = \delta q\sin(\pi x/h),$$

   Show that the corresponding buoyancy flux would be $B(z = h/2) = V\delta b/2$. Argue from (a) and (e) that $B(z = h/2) = 0.4B_0$ and use this approach to estimate δb. Assume also that the moisture flux does not vary with height (i. e. it is equal to its near-surface value), and use the same approach to estimate δq. Lastly, back out the upward potential temperature flux at $h/2$ from the buoyancy and moisture fluxes, and deduce a typical large-eddy horizontal temperature perturbation $\delta\theta$.

   (f) Use the idealization of a Kolmogorov energy cascade with a turbulence dissipation rate $-D$, $D = V^3/h$, to argue that a typical turnover velocity for eddies of half-wavelength l is $(|D|l)^{1/3} = V(l/h)^{1/3}$. A aircraft flies at 100 m s^{-1}, measuring vertical velocity 25 times per second. Estimate the typical change in w between successive samples by assuming this is roughly the turnover velocity of eddies whose half-wavelength is the distance Δx between samples. Consistent with earlier parts of the problem, assume $h = 1$ km and $V = 1.5$ m s^{-1}.

2. Following arguments made in lecture, fill in the steps of the following dimensional argument that interprets Obukhov length L as a height at which buoyancy effects become important to the structure of flux-carrying eddies near the ground. Assume that the near surface wind (and hence the near-surface momentum flux) are in the x-direction. We are given the surface momentum and buoyancy fluxes

$$\overline{u'w'} = -u_*^2, \quad \overline{w'b'} = B_0 > 0 \ .$$

We assume that in the relevant eddies, the perturbations u', w' and b' near the ground are strongly correlated.

(a) Using these ideas, argue that near $z = 0$, a characteristic eddy updraft velocity is u_* and that a characteristic eddy updraft buoyancy is B_0/u_*.

(b) Assuming that the updraft accelerates due to its buoyancy. At what height will it achieve an updraft velocity of $2u_*$? Show this height is proportional to $-L$.