

Homework Set 3

1. (Adapted from Arya, p. 180) The following measurements of mean wind and potential temperature were taken around noon during the 1968 Kansas field program:

z (m)	2	4	8	16	32
u (m s ⁻¹)	5.81	6.70	7.49	8.14	8.66
θ (K)	307.20	306.65	306.28	305.88	305.62

We wish to calculate the surface fluxes of heat and moisture using the Monin-Obukhov relations $\phi_h = \phi_m^2 = \{1 - 16z/L\}^{-1/2}$ for an unstable surface layer. To proceed:

- (a) Calculate the gradients of u and θ by differencing between successive heights. In the surface layer, the profiles tend to vary roughly logarithmically with height, so it is better to difference using $\ln(z)$ as the height coordinate. Note that $du/dz = z^{-1} du/d(\ln z)$ and similarly for θ . Numbering the levels 1 (2 m) to 5 (32 m), the estimated gradient of u between levels 1 and 2 would be

$$\left(\frac{\Delta u}{\Delta z}\right)_{21} = \frac{1}{z_m} \left(\frac{\Delta u}{\Delta \ln z}\right)_{21} = \frac{1}{z_m} \left(\frac{u_2 - u_1}{\ln z_2 - \ln z_1}\right)$$

at a height z_m such that $\ln z_m = (\ln z_1 + \ln z_2)/2 = (\ln 2 + \ln 4)/2$, i. e. at $z_m = 2.82$ m. This works out to a gradient $(\Delta u/\Delta z)_{12} = 0.45$ s⁻¹. Neglecting virtual effects on buoyancy, use your calculated gradients to find Ri vs. z .

- (b) From this data, estimate the Obukhov length L (note that $Ri = z/L$ in an unstable surface layer).
- (c) Using the data from the lowest two heights and your L from (b), calculate the friction velocity, the sensible heat flux, and the surface roughness length z_0 . Does the implied roughness length seem appropriate for a field of wheat stubble? Take the air density = 1.2 kg m⁻³ and $C_p = 10^3$ J kg⁻¹ K⁻¹. Can we also estimate the thermal roughness length z_T from the given data?
2. By simultaneously solving Charnock's formula (Eq. 5.8 of notes) and Eqs 5.9-10 for the neutral drag coefficient over the ocean, calculate and plot how C_{DN} varies with 10 m wind speed u_{10} over the range $4 < u_{10} < 20$ m s⁻¹. Also plot the approximation for C_{DN} given in Eq. 5.11; they should agree pretty well.
3. An oceanographic research ship is stationed 100 km off the California coast. At a height $z_R = 10$ m above sea level, it measures a wind speed of 10 m s⁻¹, an air temperature of 286.9 K (i.e. 287 K if adiabatically displaced to the sea surface), a water vapor mixing ratio of 7.4 g kg⁻¹, and an air density of 1.2 kg m⁻³. The ocean surface temperature is 284 K and the saturation mixing ratio for salt water at the sea surface is 7.9 g kg⁻¹. We wish to find the surface fluxes. We proceed iteratively, starting by assuming a neutral boundary layer ($\zeta_R = z_R/L = 0$), inferring the fluxes and L , then updating:
- (a) Using the bulk aerodynamic approach, with $C_{DN} = (0.75 + 0.067u_{10}) \times 10^{-3}$, find the neutral surface stress and friction velocity.
- (b) Using neutral bulk formulas with $C_{qN} = C_{HN} = 1.3 \times 10^{-3}$, calculate the surface latent heat flux, sensible heat flux, virtual heat flux, buoyancy flux, and Obukhov length L . Use L to make a new guess at ζ_R ; it won't be perfect, but it is good enough for this purpose.
- (c) Using this ζ_R in Garratt Fig. 3.7 (shown on page 6 of the Lecture 6 notes; use the $z/z_0 = 10^5$ curve), estimate by what percentage C_D is decreased from C_{DN} .