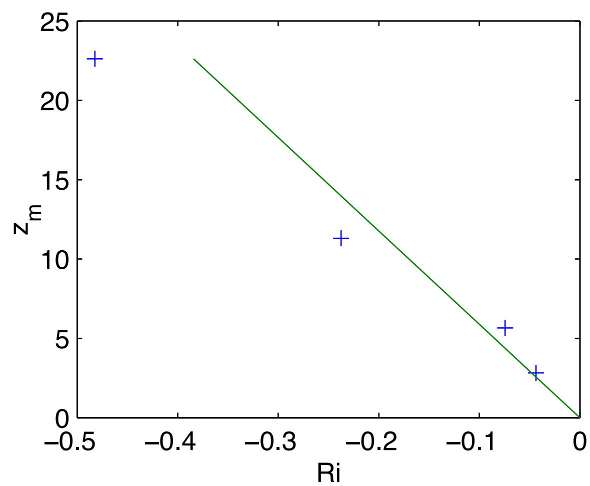
Atm Sci 547     Bretherton

**Homework 3 solutions**

The Matlab script hw3.m implements the calculations for the solutions below.

1. Using the technique suggested in the problem statement for calculating gradients, we obtain the following table, and the associated plot of Ri vs. $z_m$:

| | | | | | |
|---|---|---|---|---|---|
| $z_m$ | [m] | 2.8 | 5.6 | 11.3 | 22.6 |
| $s = du/dz$ | [m s$^{-1}$] | 0.45 | 0.20 | 0.08 | 0.03 |
| $N^2 = (g/\theta)(d\theta/dz)$ | [$10^{-3}$ s$^{-2}$] | -9 | -3 | -1.6 | -0.5 |
| $\mathrm{Ri} = N^2/s^2$ | | -0.04 | -0.07 | -0.24 | -0.48 |
| $L_{\mathrm{est}} = z_m/\mathrm{Ri}$ | [m] | -65 | -76 | -48 | -47 |




(b) One way of estimating $L$ is as the average of all the estimates $L_{est}$ from the different $z_m$'s; this gives $L$ = -59 m. This fit line Ri = $Lz$ is shown on the plot above. This estimate assumes the zero-plane displacement is small, which is reasonable for a Kansas wheat field with stalks perhaps 1 m high.

(c) If $\zeta = z/L$, $d\bar{u}/dz = \phi_M(\zeta)u_*/kz$. At any midpoint level $z_m$, this equation can be solved for $u_*$ in terms of known quantities. We use the lowest midpoint $z_m$ = 2.8 m, where the shear is largest. Here $\zeta = z_m/L$ = 2.8/(-59) = -0.048, so $\phi_M(\zeta) = (1 - 16\,\zeta)^{-1/4}$ = 0.87 and

$$u_* = \frac{kz}{\phi_M}\left(\frac{d\bar{u}}{dz}\right)_m = \underline{0.59 \text{ m s}^{-1}}.$$

Surface buoyancy flux $B_0 = -u_*^3/kL = 8.8\times10^{-3}$ m$^2$ s$^{-3}$.
Surface heat flux $H_0 = \rho c_p \overline{w'\theta'} = \rho c_p(\theta_0/g)B_0$ = <u>331 W m$^{-2}$</u>.
The roughness length can be deduced from the M-O formula for the wind profile:

$$\bar{u}(z) = \frac{u_*}{k}\left\{\log\left(\frac{z}{z_0}\right) - \psi_M\left(\frac{z}{L}\right)\right\} \qquad (1c.1)$$

We apply this formula at the lowest measurement level, $z = 2$ m. The estimate of roughness length is best close to the ground, where stability corrections are small and there is no doubt one is in the surface layer. However, one needs to be sure of any zero-plane displacement (in this problem, we assume this has been accounted for in the definition of $z$). The stability correction, expressed in terms of $x = (1 - 16z/L)^{1/4} = 1.11$, is,

$$\psi_M\left(\frac{z}{L}\right) = \log\left(\left[\frac{1+x^2}{2}\right]\left[\frac{1+x}{2}\right]^2\right) - 2\tan^{-1}(x) + \frac{\pi}{2} \quad = 0.12$$

This is relatively small; ignoring it and using a pure log-layer would create about a 10% error in the estimated roughness length $z_0$. We now solve (1c.1) for the only unknown:

$$\log\left(\frac{z}{z_0}\right) = k\frac{\bar{u}(z)}{u_*} + \psi_M\left(\frac{z}{L}\right) = \frac{(0.4)(5.81)}{0.59} + 0.12 = 4.04$$

$$\Rightarrow \quad z_0 = \underline{0.035\text{ m}}.$$

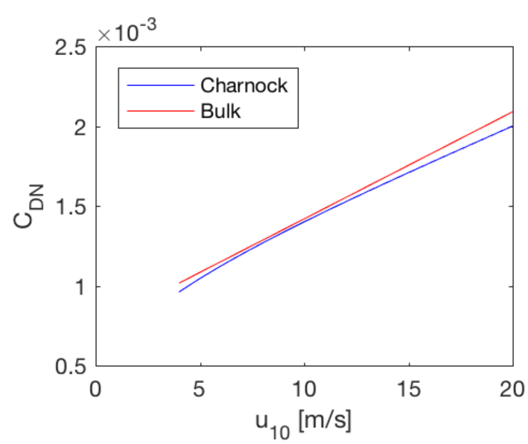
This does seem reasonable for a field of wheat stubble; it is about 10% of 35 cm, which would be a reasonable height for the roughness elements (cut wheat stalks). One cannot estimate the thermal roughness length from the given data; one also would need the surface skin potential temperature $\theta_0$. To see this, consider the M-O formula for the $\theta$ profile:

$$\theta_0 - \bar{\theta}(z) = \frac{\theta_*}{k}\left\{\log\left(\frac{z}{z_{T0}}\right) - \psi_H\left(\frac{z}{L}\right)\right\}$$

There are two unknowns in this equation, $\theta_0$ and $z_{T0}$ ($\theta_*$ is determined from the heat flux and roughness length). If we isolate them on the left side, we see that

$$\theta_0 + \frac{\theta_*}{k}\log\left(\frac{z_{T0}}{z}\right) = \bar{\theta}(z) - \frac{\theta_*}{k}\psi_H\left(\frac{z}{L}\right)$$

The different heights in the $\theta$ profile give independent estimates of the LHS, but cannot individually determine $\theta_0$ and $z_{T0}$. The same $\theta$ profile can be created with a high surface temperature and a low thermal roughness length, or with a lower surface temperature and a higher thermal roughness length.




2. The neutral drag formula tells us that

$$u_*^2 / u_{10}^2 = C_{DN} = (0.75 + 0.067u_{10}) \times 10^{-3},$$

We can use this as a first guess for a direct determination of $C_{DN}$ from Charnock's formula. We iterate:

$$u_*^2 = C_{DN}^i\, u_{10}^2$$

$z_0^i = a_c\, u_*^2/g, \quad (a_c = 0.016).$

$C_{DN}^{i+1} = k^2/\log^2(z_{10}/\, z_0^i)$

As shown in the Matlab script, five iterations are sufficient to get convergence of $C_{DN}$ to within a relative error of $10^{-4}$ at all 10 m wind speeds of 4-20 m/s. A plot of the $C_{DN}$ from the two approaches is shown above.

3. (a) Applying the given bulk transfer coefficient formula with $u_{10} = 10$ m s$^{-1}$, we obtain:

$$C_{DN} = 1.42\times10^{-3},$$

$$\text{friction velocity } u_* = |C_{DN}u_{10}^2|^{1/2} = \underline{0.38 \text{ m s}^{-1}},$$

$$\text{surface stress } \tau = \rho_0 u_*^2 = \underline{0.17 \text{ Pa}} \ (= \text{kg m}^{-1}\text{s}^{-2}).$$

(b) Latent heat flux $H_L = \rho L_v C_{qN} u_{10}(q_0^* - q_{10}) = \underline{20 \text{ W m}^{-2}}$.
Sensible heat flux $H_S = \rho c_p\, C_{HN} u_{10}(\text{SST} - \theta_{10}) = -47$ W m$^{-2}$

Buoyancy flux

$$B_0 = (g/\rho c_p \theta_{10})(\, H_S + [c_p\theta_{10}/L_v]\, H_L = \underline{-1.3\times10^{-3} \text{ m}^2\text{s}^{-3}}$$

Using the neutral approximation to the friction velocity from prob. 2a, the Obukhov length is

$$L = -u_*^3/(kB_0) = -(0.38 \text{ m s}^{-1})^3/(-0.4\times1.3\times10^{-3} \text{ m}^2\text{s}^{-3}) = \underline{103 \text{ m}}.$$

(c) Let $\zeta = z_{10}/L = 0.10$. From the top curve with $z/z_0 = 10^5$ in Garratt Fig. 3.7, reproduced in the Lecture 6 notes, we can estimate $C_D/C_{DN} = 0.93$, i. e. the surface stress is reduced by <u>7%</u> by the colder SST.