Lecture 10: Lagged autocovariance and correlation

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Reference: Hartmann Atm S 552 notes, Chapter 6.1-2.

10.1 Lagged autocovariance and autocorrelation

The **lag-p autocovariance** is defined

$$a_p = \sum_{j=1}^{N} u_j u_{j+p}/N, \tag{10.1.1}$$

which measures how strongly a time series is related with itself p samples later or earlier. The zero-lag autocovariance a_0 is equal to the power. Also note that $a_p = a_{-p}$ because both correspond to a lag of p time samples.

The *lag-p autocorrelation* is obtained by dividing the lag-p autocovariance by the variance:

$$r_p = a_p/a_0 \tag{10.1.2}$$

Many kinds of time series decorrelate in time so that $r_p \to 0$ as p increases. On the other hand, for a purely periodic signal of period T, $r_p = \cos(2\pi p\Delta t/T)$ will oscillate between 1 at lags that are multiples of the period and -1 at lags that are odd half-multiples of the period. Thus, if the autocorrelation drops substantially below zero at some lag P, that usually corresponds to a preferred peak in the spectral power at periods around $2P$.

More generally, the autocovariance sequence $(a_0, a_1, a_2, \ldots)$ is intimately related to the power spectrum. Let $\mathbf{S}$ be the power spectrum deduced from the DFT, with components

$$S_m = |\hat{u}_m|^2/N^2, \ m = 1, 2, \ldots, N \tag{10.1.3}$$

Then one can show using a variant of Parseval's theorem that **the IDFT of the power spectrum gives the lagged autocovariance sequence**. Specifically, let the vector $\mathbf{a}$ be the lagged autocovariance sequence (acvs) $\{a_{p-1}, p = 1, \ldots, N\}$ computed in the usual DFT style by interpreting indices $j+p$ outside the range $1, .., N$ periodically:

$$j + p \leftarrow \mathrm{mod}(j + p - 1, N) + 1 \tag{10.1.4}$$

Note that because of the periodicity $a_{n-1} = a_{-1}$, i. e. lags $p \geq N/2$ are best interpreted as negative lags $p - N$. Then

$$\mathbf{a} = N \cdot \mathrm{IDFT}(\mathbf{S}) \tag{10.1.5}$$

10.2 Nino3.4 time series example

Script **nino2.m** computes the SSTA acvs $\{a_p, p = 0, ..., N-1\}$ as a function of lag time $0 \leq p < N/2$. As expected the zero-lag autocovariance a_0 equals the SSTA power. There is a rapid dropoff for lags greater than a few months; the results for longer lags bounce around due to sampling uncertainty. The script continues by plotting the autocorrelation series $r_p = a_p/a_0$ for lags of 0-36 months. The lag-1 autocorrelation $r_1 = a_1/a_0 \approx 0.9$. The autocorrelation drops below $1/e$ for lags about 6 months, then becomes negative for lags of 1-3 years reflecting the 2-6 year periodicity of SSTA.