

Lecture 13: Windowed Spectral Analysis on Nino SSTA

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13.1 Implementing the pieces in Matlab

Script **nino3.m** applies a set of Hann windows of length $N_w = 240$ (20 years) to the SSTA dataset and calculates the DFT power spectrum for each tapered windowed time series. The window length is chosen long enough to comfortably span several oscillations of 2-6 year period. Windows are overlapped to maximize use of data that would otherwise be downweighted by the taper. The spectral power in each window is concentrated in harmonics $|M| \leq 20$, which correspond to inverse periods less than 1 yr^{-1} . Since the SSTA time series is real, the spectral power is equal between M and $-M$, so we plot only harmonics $0 \leq M \leq 20$. For each window, the spectral power is very noisy; this noise is somewhat reduced when we average it over the windows, since they give the 5 nearly independent estimates of the spectral power.

The average power spectrum is then renormalized into a plot with inverse period in yrs used as the horizontal axis in place of M . Since the inverse period is $M/12$, we have also multiplied the power spectrum by 12 so that the total variance can still be viewed as a visual integral of the power spectrum over the frequency. This rescaling of the power spectrum is called a **power spectral density** or PSD, and has units of spectral power per unit frequency.

A red noise spectrum with autocorrelation e-folding time $\tau = 6.1$ months is superposed. The windowed power spectrum exceeds the red noise prediction by a factor of two for inverse periods of $0.2\text{-}0.3 \text{ yr}^{-1}$ (3-5 year periods). Judging by the fluctuation between adjacent harmonics, this difference is not due to chance. Hence, we infer that SSTA has a 3-5 year band of preferred periods. However, a confidence range based on statistical properties of averaged DFT-derived power spectra would not exclude the red noise fit at 95% confidence for any harmonics.

The main price we've paid for windowing is reduced spectral resolution. Now the fundamental period is 20 years, not 63, so we only get one spectral estimate per 0.05 yr^{-1} of inverse frequency. We could use an even shorter window of 10 years for even more robust spectral estimates, but this would quantize the spectrum even more (and the important 5 year period would be more distorted by the taper).

We have always plotted **two-sided** power spectra, in which the spectral power in only the positive harmonics is shown. Matlab by default plots **one-sided** power spectra, in which the spectral power in the negative harmonics is added to that in the positive harmonics, doubling their spectral power. In a one-sided spectrum, the total variance is equal to the sum of the power in just the zero and positive harmonics.

13.2 Windowed spectral analysis using Matlab's signal processing toolbox

Matlab's signal processing toolbox has a sophisticated function **pwelch** for windowed tapered spectral analysis, which is sometimes called **Welch's method**. Some have slightly better statistical properties than overlapped Hann windows, but for almost all applications, they will give very similar results. You will need to carefully study the documentation for **pwelch** to understand how to use it well. The most important thing you will likely need to modify from the default settings is the window length.

Here we describe how to implement exactly the method we used above, based on overlapping Hann windows and two-sided spectral estimation. The last section of **nino3.m** illustrates how to apply the toolbox functions to the SSTA time series, including syntax, plot customization, and (most useful) adding confidence intervals to the spectral estimates. The 4 harmonics with inverse periods in the range $0.2\text{--}0.35\text{ yr}^{-1}$ all have 95% confidence bounds that barely overlap with the red noise null hypothesis. Since these estimates are quasi-independent of each other, this is unlikely to be a chance occurrence. We therefore accept that there is a statistically significant peak in SSTA (i. e. in El Nino variability) in 3-5 year periods above that expected from red noise.