Assume \( T = 273^\circ K \)

\( T_v = 273 + 2.66^\circ C \)

Virtual temp is warmer than actual temp by a few degrees at rest.

**Derivation of hysometric**

\[
\frac{dp}{dz} = -\rho g
\]

\[
dp = -\rho g dz = -\rho \left( \frac{P_2}{R_D T_v} \right) dz
\]

\[
\Rightarrow -R_D T_v \frac{dp}{P} = g dz
\]

Integrate \( \frac{2}{P_1} \) \( \frac{P_2}{P} \)

\[
\int_{z_1}^{z_2} g dz = -R_D \int_{P_1}^{P_2} \frac{P_2}{T_v} \frac{dp}{P} = R_D \int_{P_1}^{P_2} \frac{P_2}{T_v} \frac{dp}{P_2}
\]
So generally geopotential height $z$ is $z = z_{0} - \frac{1}{g} \int_{0}^{z} \sigma \, dz$. Let's define something called geopotential height $z_{0}$.

Let's do this to handle the $g$ is not a constant thing.
In lower atmosphere where $g = g_0$

$z \sim Z$

table

<table>
<thead>
<tr>
<th>$z$ (Km)</th>
<th>$Z$ (Km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1.00</td>
</tr>
<tr>
<td>10</td>
<td>9.986</td>
</tr>
<tr>
<td>20</td>
<td>9.941</td>
</tr>
</tbody>
</table>

Back to derivation of hyposometric
Using def of geopotential height $h = \frac{p_2}{g_0}$

$$\frac{dZ}{d\ln p} = \frac{1}{g_0} \int \frac{p_2}{Z} dz = \frac{R_0}{g_0} \int \frac{p_2}{Z} TV \, dp$$

or

$$Z_2 - Z_1 = \frac{R_0}{g_0} \int_{p_2}^{p_1} TV \, dp$$