

1) Show that a fluid rotating in solid body rotation with angular velocity Ω has vorticity 2Ω .

2) Show that for an ideal gas in hydrostatic balance, the dry static energy ($s = c_p T + gz$) change with height can be related to the potential temperature change with height as $\frac{\partial s}{\partial z} = \frac{c_p T}{\theta} \frac{\partial \theta}{\partial z}$.

The quantity $\frac{c_p T}{\theta}$ is known as the Exner function and is denoted Π (the Exner function is sometimes alternatively defined as $\frac{T}{\theta}$). Dry static energy, and its moist counterpart the moist static energy ($m = c_p T + gz + Lq$) are frequently used as alternative measures of vertical stability in the atmosphere. Their linear, additive forms allow for easier computation in many cases.

3) Angular momentum. a) Show that the equation $\frac{Dm}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial \lambda}$ with $m = (u + \Omega r \cos \theta) r \cos \theta$, the axial angular momentum, is consistent with the zonal momentum equation in spherical coordinates.

b) For special cases of zonally symmetric flows (flows independent of longitude), the above equation reduces to $\frac{Dm}{Dt} = 0$. Consider a ring of fluid at rest at the equator. Displace this out to latitude θ . What are the winds at this latitude? Compute this “angular momentum conserving wind” value for $\theta = 10^\circ, 20^\circ, 30^\circ$. Compare with the observed upper tropospheric zonal winds at these latitudes.

c) What velocity will a ring of fluid initially at rest at 10° N have if displaced to the equator? How about a ring starting at 10° S?

4) Estimate the structure of the surface displacement for the large rotating tank in the lab at its fastest speed. How much higher is the water at the outside of the tank than in the center? If the inertial circles created at the fastest rotation speed had diameter 10 cm, what must the speed of the jet of dye be?