ATM S 509/OCEAN 512
Frierson
Midterm Exam: Due 2-15-16

Work completely independently – no discussions at all are allowed.
Open book, open notes.

1) Indicate (providing reasoning) whether the following flows are
    i) Hydrostatic/non-hydrostatic
    ii) Suitable for modeling on the sphere/f-plane/beta-plane/rotation is not important (also calculate the Rossby numbers, assuming $f_0 = 10^{-4} s^{-1}$ where appropriate)
    iii) Suitable for modeling with the Boussinesq equations/anelastic equations/neither

    A) The atmospheric jet stream (L = 2000 km, H = 10 km, U = 50 m/s)
    B) The Gulf Stream (L = 1000 km, H = 1 km, U = 1 m/s)
    C) Sea breeze convection (L = 5 km, U = 10 m/s)
    D) Sound waves propagating in a class room
    E) Flow in a rotating tank filled with water (Ω= 30 rpm, L = 1 m, H = 50 cm, U = 0.1 m/s)
    F) Tidal flow through Deception Pass (U = 6 knots = 3 m/s, L = 300 m, H = 30 m)
    G) Global overturning circulation on Titan (U = 5 m/s, Rotational period = 16 days, L = 1000 km, H = 40 km)

2) Planet Cylinder. A cylindrical planet, with radius R and very long length (assume infinite), attracts a thin atmosphere due to its gravity. It rotates about its axis at rate Ω.
    i) Derive the momentum equations for this planet, as seen in the rotating frame of the surface. You may omit metric terms (or include them for extra credit).
    ii) Summarize the action of the Coriolis and centrifugal forces on this planet. Is motion on the surface of the cylinder analogous to horizontal motion at any particular location on Earth?
    iii) Describe the deflection due to the Coriolis force on convection forced by a surface temperature gradient varying only in the angular direction.

3) Pressure and geopotential for hydrostatic, constant lapse rate atmospheres. Let $\frac{dT}{dz} = -\Gamma$, and the surface pressure and temperature be p_0 and T_0, respectively, for the following exercises.
    i) Show that the geopotential height for a constant lapse rate atmosphere (i.e., Γ= constant) satisfies

$$z = \frac{T_0}{\Gamma}\left(1 - \left(\frac{p}{p_0}\right)^{\frac{R\Gamma}{g}}\right).$$

ii) Invert the expression from part i to solve for the pressure as a function of geopotential, and calculate the height at which the pressure vanishes. Calculate this height for an atmosphere with constant potential temperature. Do you think this poses a problem for the usefulness of the anelastic equations, which assume a constant potential temperature reference state?

4) Ageostrophic winds and vertical velocities.

i) Show that if friction can be neglected, the horizontal equations of motion can be written as $\frac{Du}{Dt} + f \times u_a = 0$ with $u_a = u - u_g$ = the ageostrophic velocity.

ii) Show that the ageostrophic velocities are thus O(Ro) smaller than the full velocity.

iii) Use the Boussinesq equations on an f-plane to show the geostrophic winds are nondivergent, and the vertical velocity scales as Ro U H/L. Estimate this vertical velocity for the Gulf Stream using the magnitudes in Part 1.