GFD II: Balance Dynamics ATM S 542

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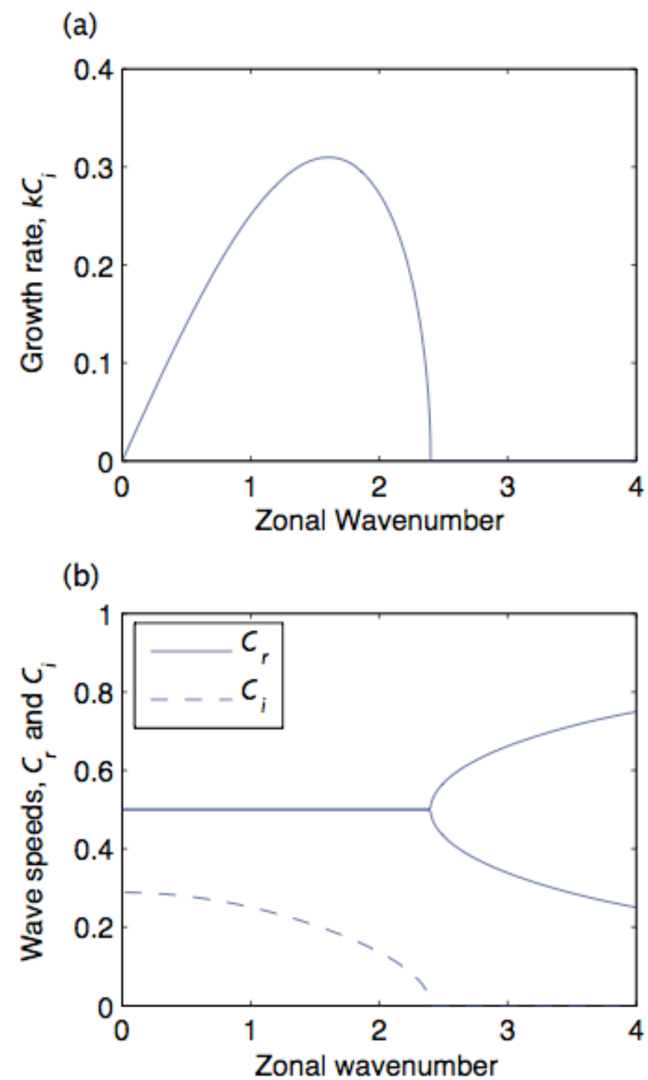
WEEK 8 SLIDES

Eady Model

Growth rates (imaginary part of frequency)

Wave speeds (real part of frequency divided by k)

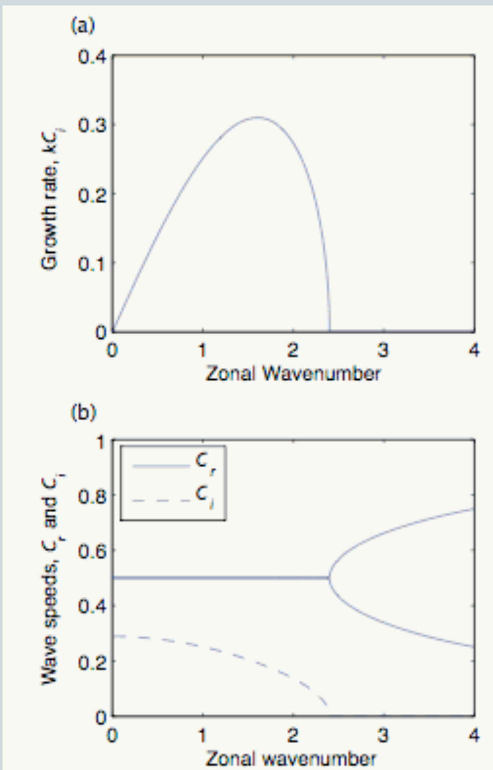


Stable for large wavenumbers, unstable for small wavenumbers

Phase speed for unstable modes = mean flow speed at midtroposphere.

Eady vs Barotropic Instability

Eady model



Barotropic instability

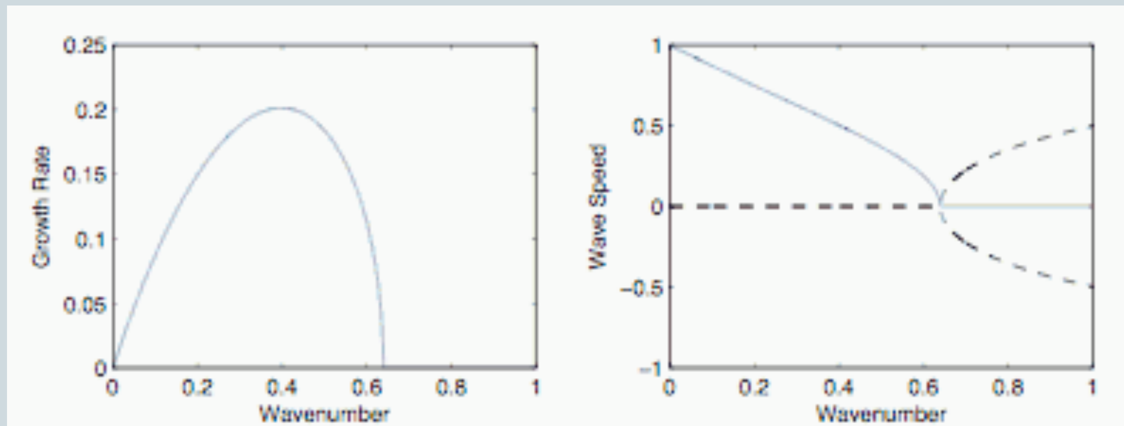


Fig. 6.5 Left: Growth rate ($\sigma = kc_i$) calculated from (6.42) with c non-dimensionalized by U_0 and k non-dimensionalized by $1/a$ (equivalent to setting $a = U_0 = 1$). Right: Real (c_r, dashed) and imaginary (c_i, solid) wave speeds. The flow is unstable for $k < 0.63$, with the maximum instability occurring at $k = 0.39$.

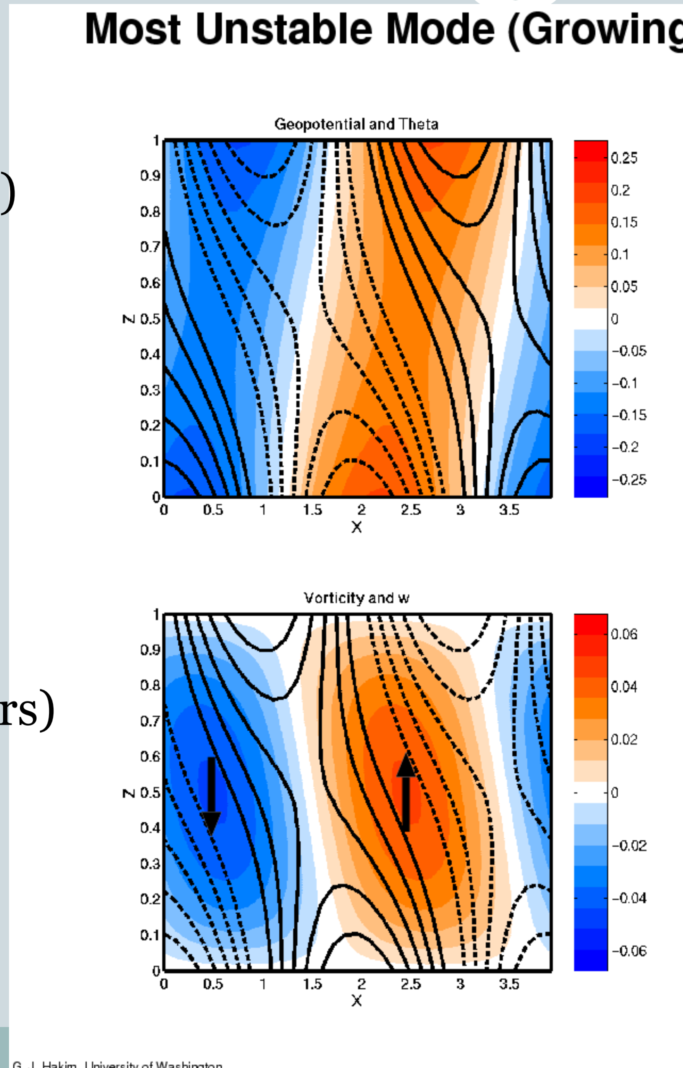
Eady vs. Barotropic Instability

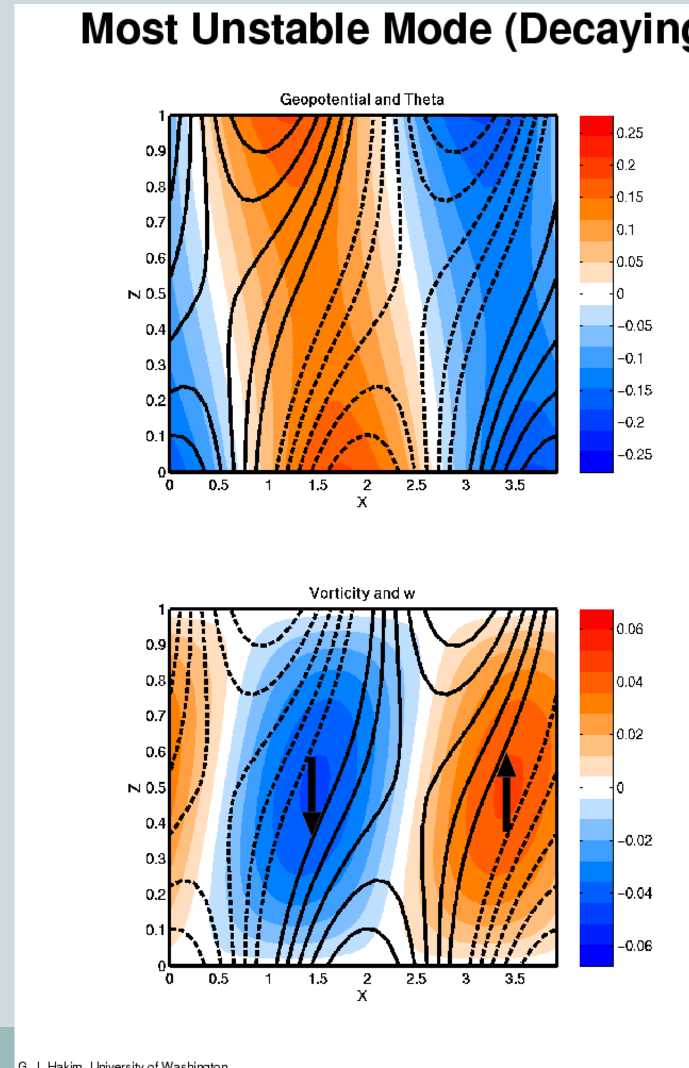
- Both require the following conditions on edge waves:
  - Phase speed matching
  - Amplitude requirement
    - Waves have to affect each other
    - Depends on their reach vs their separation
  - Phase tilt requirement
    - Must be so they reinforce each others perturbations
- Eady is 3-D though! Barotropic is only 2-D

Eady Model of Baroclinic Instability

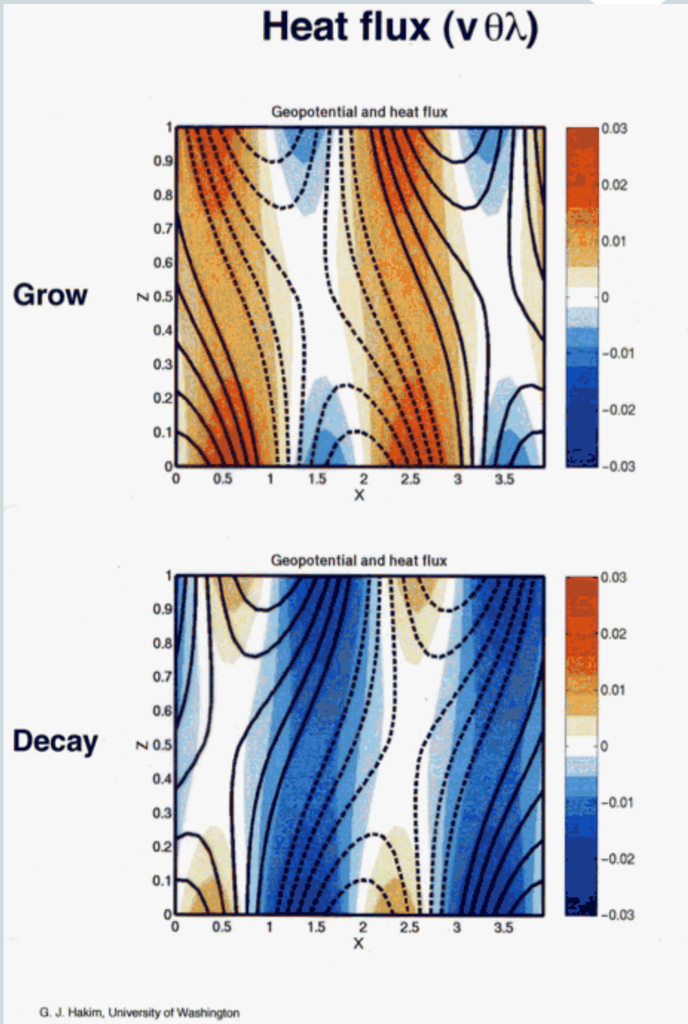
Height (contours)
& theta (colors)

Vorticity (contours)
& w (colors)





Heat fluxes



Growing mode has heat flux poleward

Decaying mode has equatorward heat flux (upgradient!)

Eady phase tilts (most unstable mode)

ψ

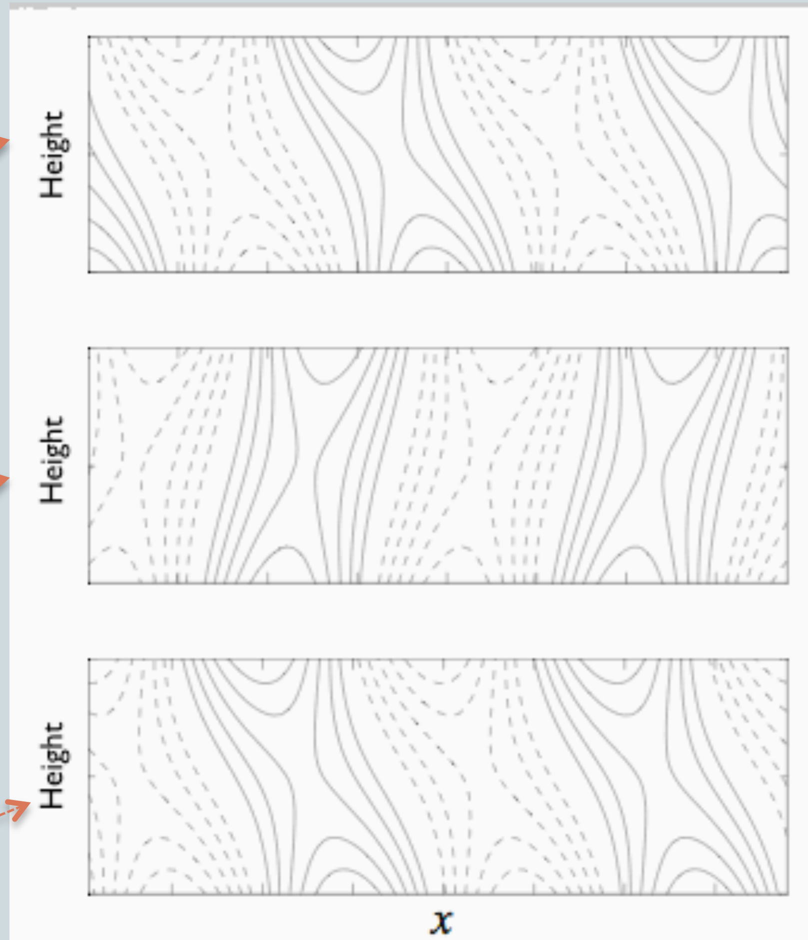
Streamfunction

Buoyancy

$$b = \frac{\partial \psi}{\partial z}$$

Meridional velocity

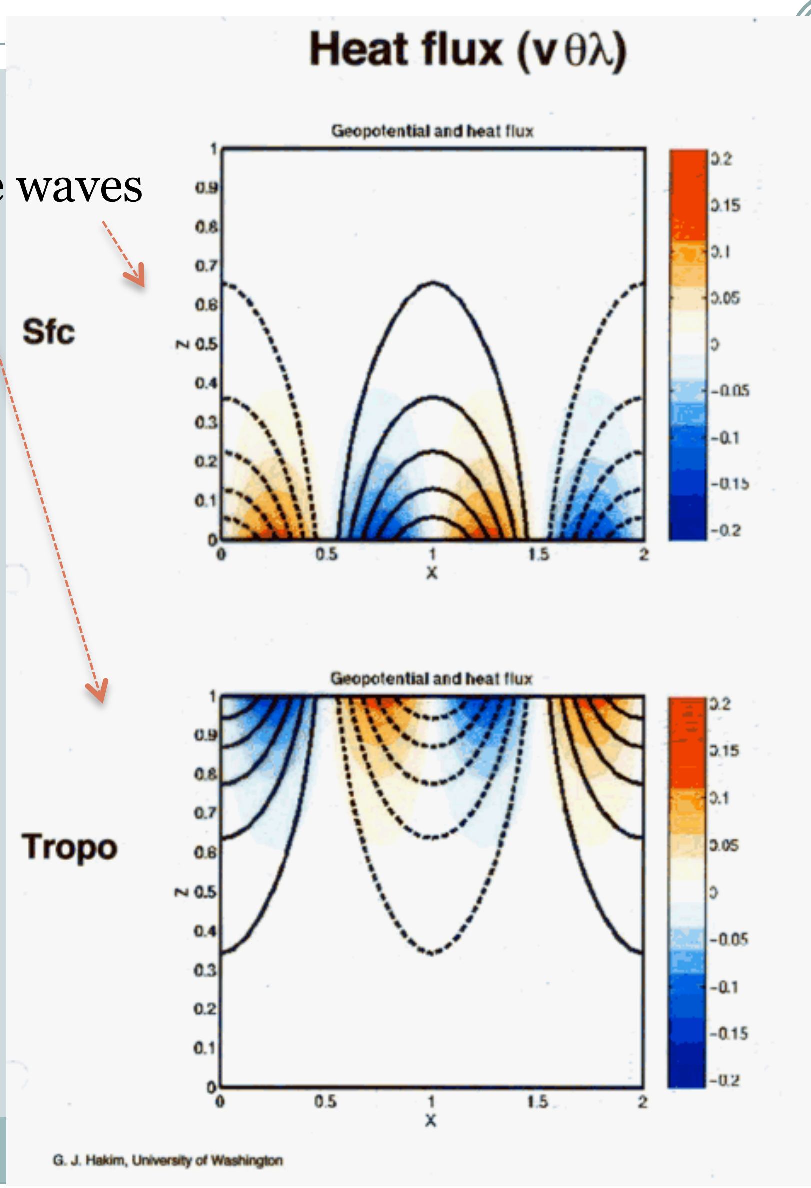
$$v = \frac{\partial \psi}{\partial x}$$

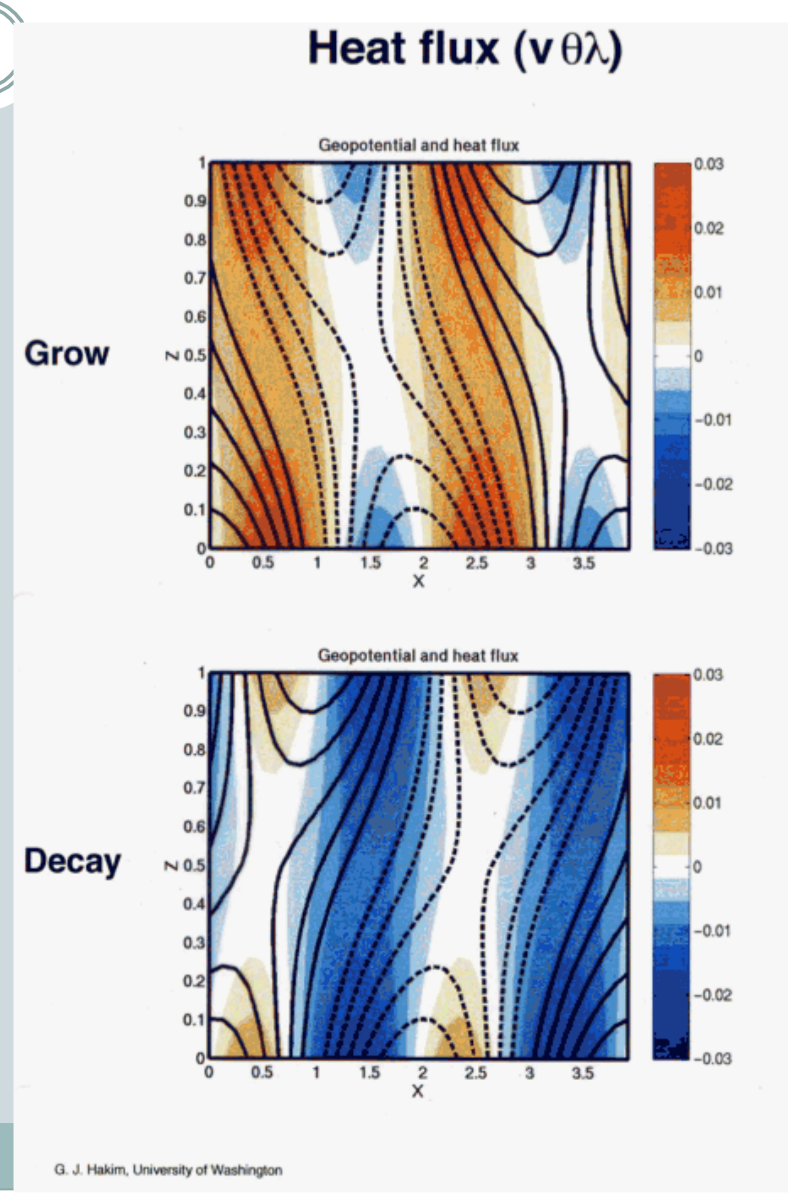


Note v and b tend to be correlated, i.e., there's a **poleward heat flux**

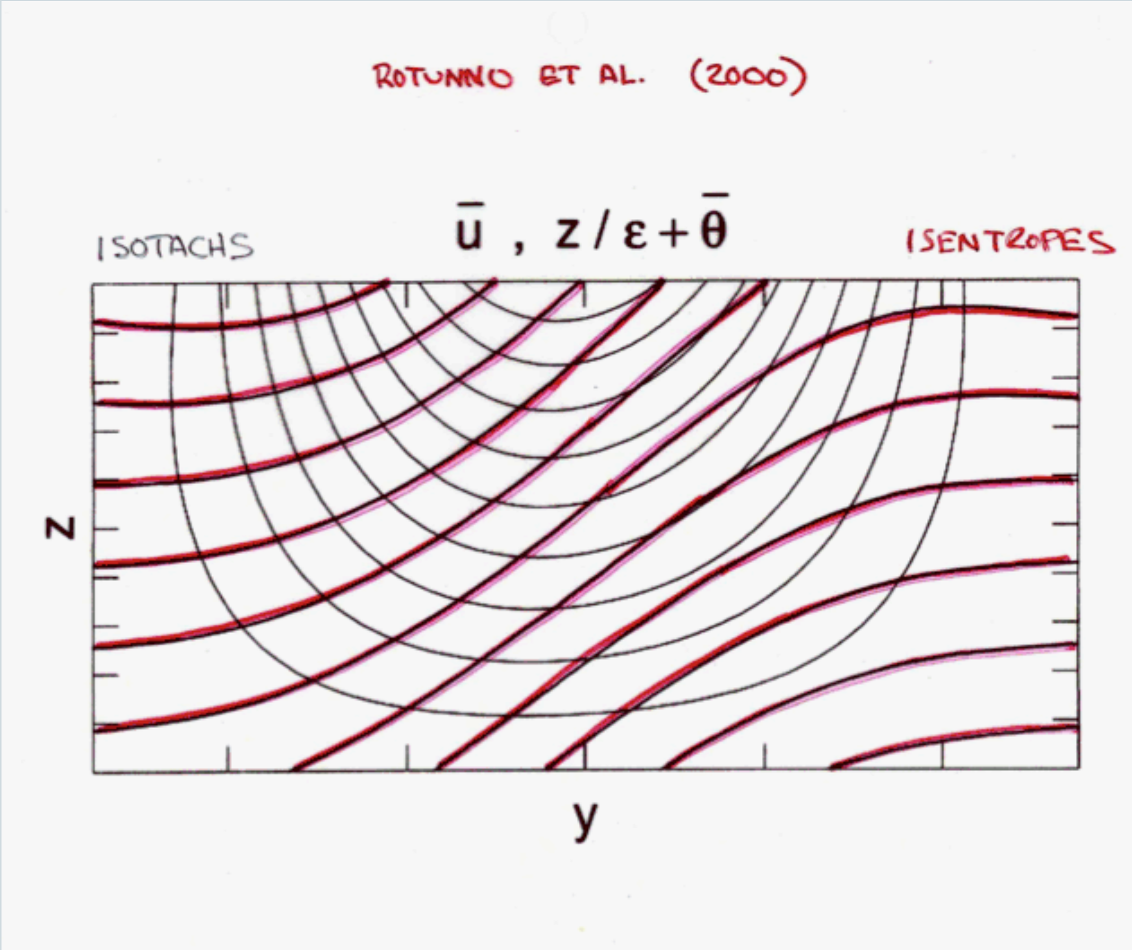
Heat fluxes

Edge waves

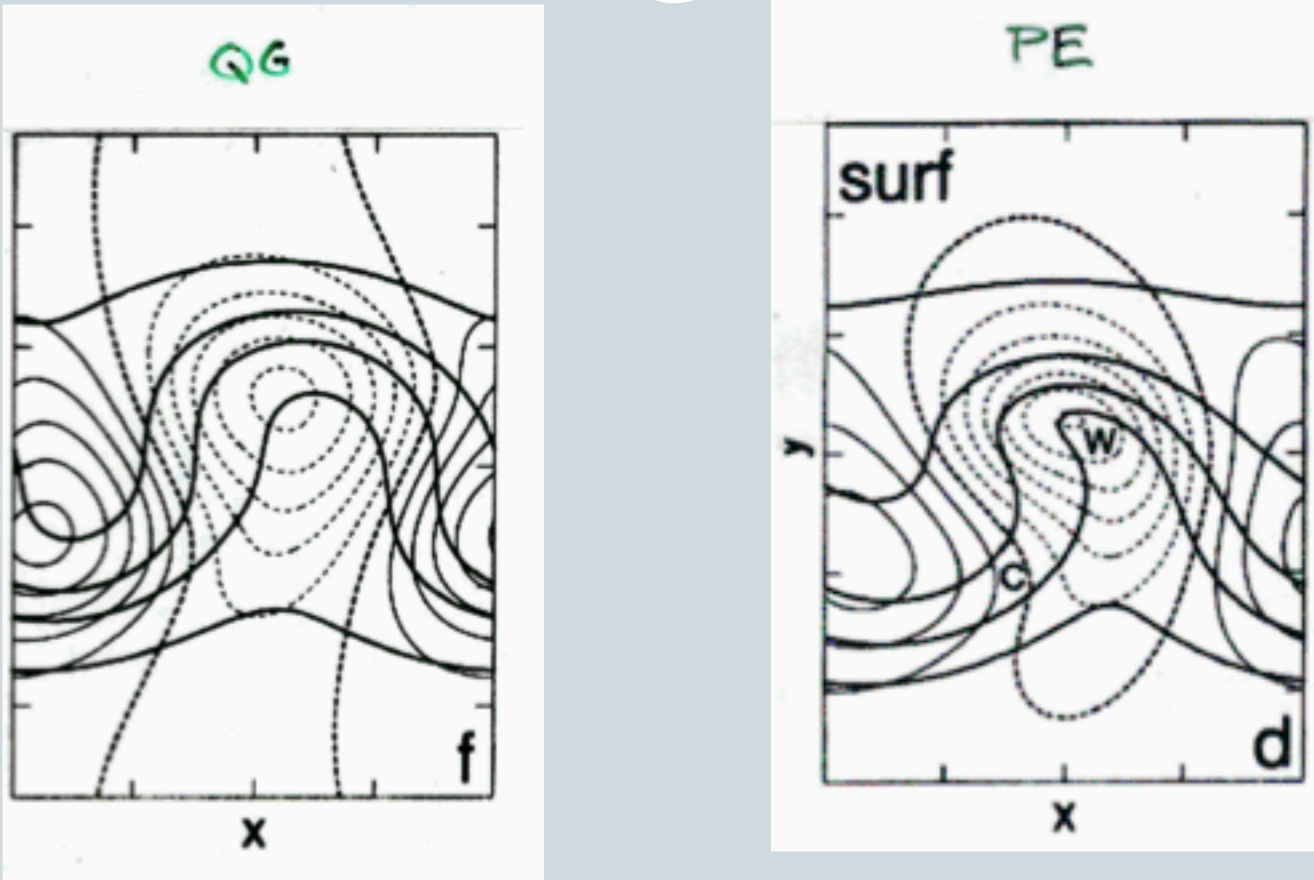




Idealized Cyclogenesis



Non-QG Effects



Nonlinear baroclinic instability simulations with a QG model and with a primitive equations model

Observed Cyclogenesis

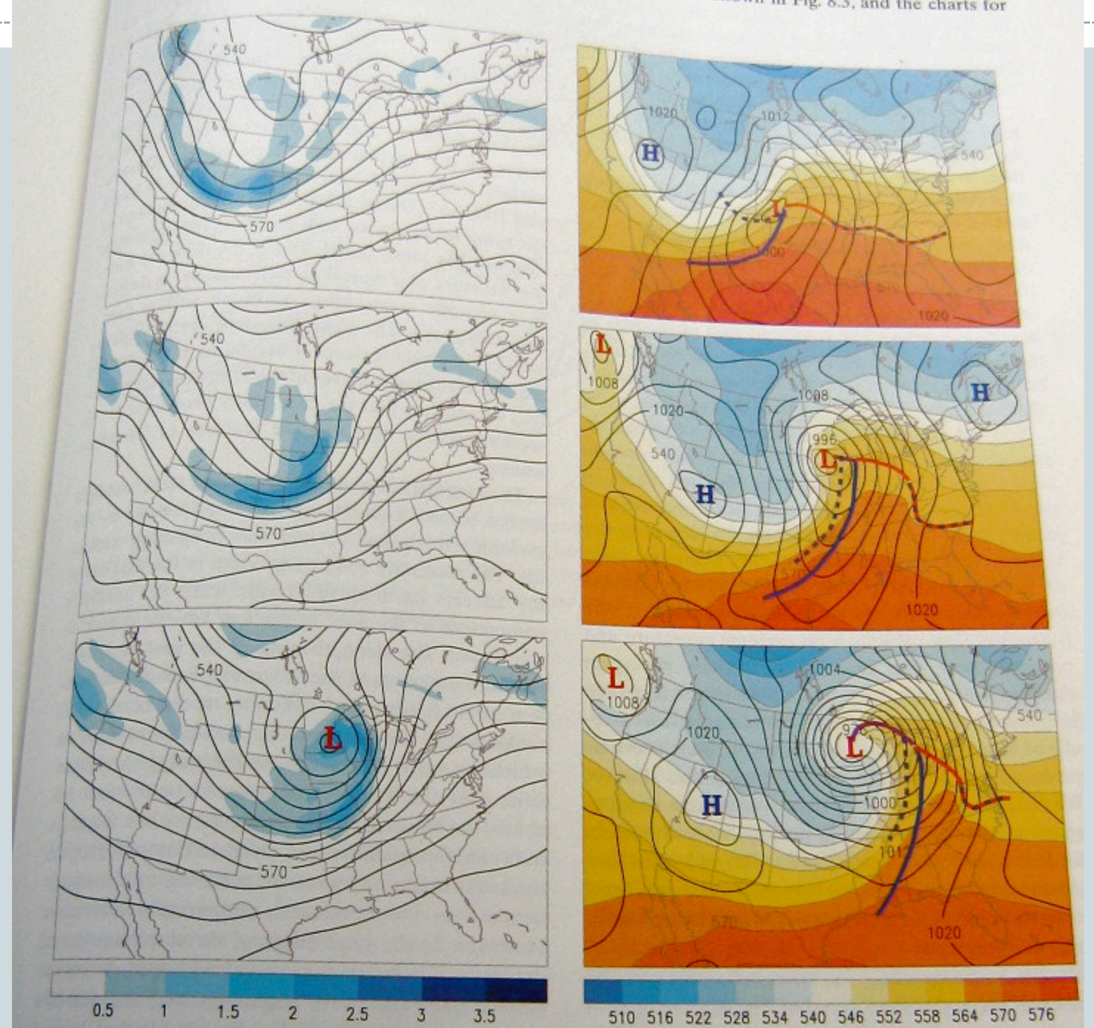


Fig. 8.3 Synoptic charts at 00, 09, and 18 UTC Nov. 10, 1998. (Left) The 500-hPa height (contours at 60-m intervals; lab in dkm) and relative vorticity (blue shading; scale on color bar in units of $10^{-4}\ s^{-1}$). (Right) Sea-level pressure (conto at 4-hPa intervals) and 1000- to 500-hPa thickness (colored shading: contour interval 60 m; labels in dkm). Surface fron positions, as defined by a skilled human analyst, are overlaid. [Courtesy of Jennifer Adams, COLA/IGES.]

Alternatives to Eady

- Other important models of baroclinic instability where linear analysis is useful:
  - Two-layer QG model ("Phillips model"): see Vallis Section 6.6
  - Charney model: see Vallis Section 6.9.1

Two layer model w/o beta

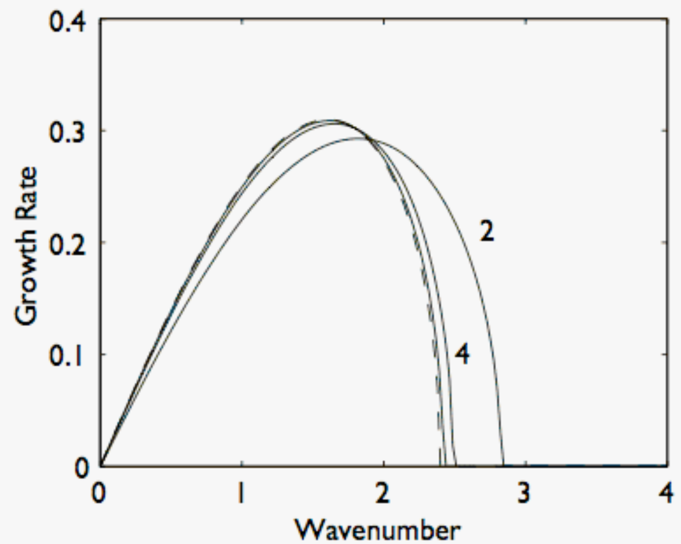


Fig. 6.13 Growth rates for models with varying numbers of vertical layers, all with $\beta = 0$ and the same uniform stratification and shear. The dashed line is the analytic solution to the continuous (Eady) problem, and the solid lines are results obtained using two, four and eight layers. The two- and four-layer results are labelled, and the eight-layer result is almost coincident with the dashed line.

2 layer problem closely resembles Eady qualitatively
Quantitative approach to Eady growth rates as layers are added

Two layer (Phillips) model with beta

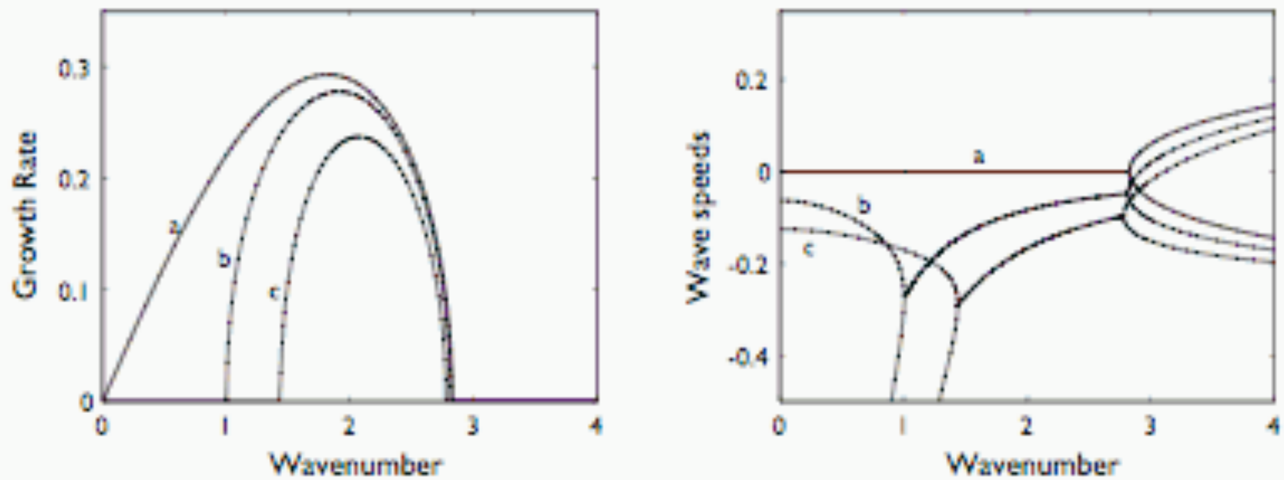


Fig. 6.14 Growth rates and wave speeds for the two-layer baroclinic instability problem, from (6.115), with three (non-dimensional) values of β: a, $\gamma = 0$ ($\hat{k}_\beta = 0$); b, $\gamma = 0.5$ ($\hat{k}_\beta = \sqrt{2}$); c, $\gamma = 1$ ($\hat{k}_\beta = 2$). As β increases, so does the low-wavenumber cut-off to instability, but the high-wavenumber cut-off is little changed. (The solutions are obtained from (6.115), with $\hat{k}_d = \sqrt{8}$ and $U_1 = -U_2 = 1/4$.)

Two layer model with beta

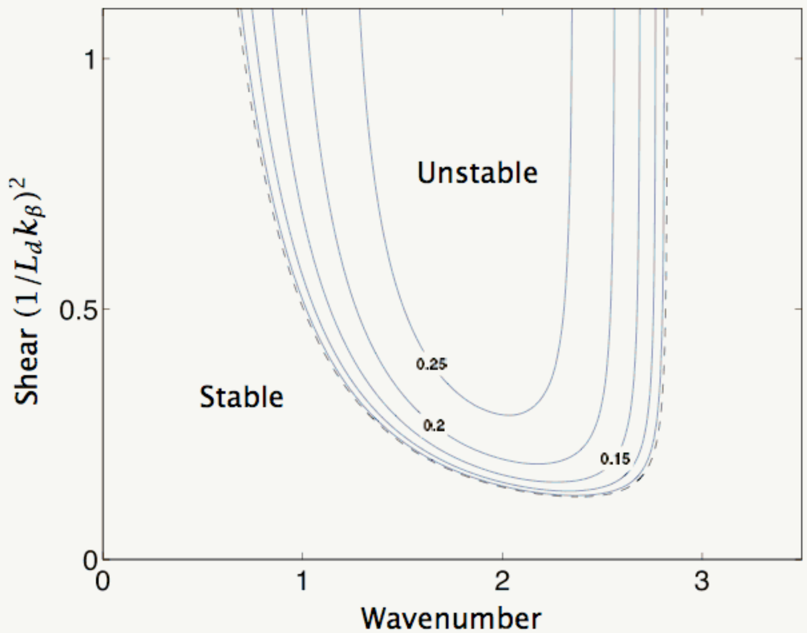


Fig. 6.15 Contours of growth rate in the two-layer baroclinic instability problem. The dashed line is the neutral stability curve obtained from (6.121), and the other curves are contours of growth rates obtained from (6.115). Outside of the dashed line, the flow is stable. The wavenumber is scaled by $1/L_d$ (i.e., by $k_d/\sqrt{8}$) and growth rates are scaled by the inverse of the Eady time scale (i.e., by U/L_d). Thus, for $L_d = 1000\,\text{km}$ and $U = 10\,\text{m s}^{-1}$, a non-dimensional growth rate of 0.25 corresponds to a dimensional growth rate of $0.25 \times 10^{-5}\,\text{s}^{-1} = 0.216\,\text{day}^{-1}$.

Phillips model summary

- Two-layer baroclinic instability:
  - Still have interacting waves leading to instability
  - Can qualitatively reproduce Eady results with beta = 0
  - With beta, have:
    - Critical shear for instability
    - Longwave cutoff
    - Shortwave cutoff

Charney model

- Setup:
  - Unbounded in vertical, density decreasing with height
  - Constant shear
  - Beta effect included
- Mechanism:
  - Edge wave at surface interacts with a Rossby wave in the troposphere

Summary: Extensions to Eady

- Beta:
  - How to add:
    - 2-layer model allows for easy analysis of this effect.
    - Charney model: beta is key in this model
    - Can also run Eady model numerically with beta.
  - Results:
    - In 2-layer model, stabilizes everywhere, causing weaker instability and stabilizing growth rates. This happens preferentially at larger scales.
    - In Charney model, larger beta pushes shallow modes downward.
    - In Eady model plus beta, pushes modes downward.

Summary: Extensions to Eady

- No upper boundary:
  - How to add:
    - Eady model without an upper boundary cannot produce baroclinic instability
    - With beta, this is the Charney model
  - Results:
    - Charney model has baroclinic instability (provided there's beta)
      - Deep modes have depth set by density scale height, and are similar to Eady model
      - Shallow modes also exist though: Rossby waves in interior locking to surface edge waves
      - Presence of shallow modes imply no short wave cutoff in Charney model

Summary: Extensions to Eady

- Non-uniformity in y
  - How to add:
    - Make a localized jet. Then can test linear stability of this, run nonlinear simulations, etc.