

ad

$$v_0 = \frac{1}{\sqrt{2}} \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} \right) q_1 - \int_{-\infty}^{\infty} F^x \psi_1 dy \quad (70b)$$

(d) Lowest order solutions

Since for low frequency motions the forcing scales give $|\frac{\partial}{\partial t}, \frac{\partial}{\partial x}| \ll 1$, the eqns (66b),

(66b, 67b, 69), 70a,b
 q v in q, q

$$(66b) \Rightarrow q_{0t} + q_{0x} = - \int_{-\infty}^{\infty} F^x \psi_0 dy \quad (71a)$$

$$(67b) + (66b) \quad |q_1| \sim \left| \int_{-\infty}^{\infty} \psi_0 F^y dy \right| \sim |q_{0t} + q_{0x}| \sim O\left(\frac{q_0}{x_{scale}}\right) \ll 1 \quad (71b)$$

$$(69) \Rightarrow (2m+1) \frac{\partial}{\partial t} q_{m+1} - \frac{\partial}{\partial x} q_{m+1} = \int_{-\infty}^{\infty} F^x \left[m \psi_{n+1} - \sqrt{m(m+1)} \psi_{n-1} \right] dy \quad m=1, 2, \dots \quad (71c)$$

$$67a \quad r_{m-1} = \sqrt{\frac{m+1}{m}} q_{m+1} \quad m=1, 2, 3, \dots \quad (71d)$$

$$v_m = \sqrt{2(m+1)} \frac{\partial}{\partial t} q_{m+1} + \int_{-\infty}^{\infty} F^x \left[\sqrt{\frac{m}{2}} \psi_{m-1} - \sqrt{\frac{m+1}{2}} \psi_{m+1} \right] dy \quad m=1, 2, \dots \quad (71e)$$

$$(70b) \quad v_0 = -\frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} F^x \psi_1 dy \quad (71d)$$

(26)

note: to zeroth order, the zonal wind stress drives the motion (F_y does not appear in (70x))!

(c) The physical interpretation of (71).

Consider the unforced (low frequency) motion.

$$q_{tot} + q_{ox} = 0 \quad (72)$$

$$\text{and} \quad (2m+1) \frac{\partial}{\partial t} q_{m+1} - \frac{\partial}{\partial x} q_{m+1} = 0 \quad (73) \\ m=1, 2, \dots$$

Which has general solutions

$$q_0 = G_0(x-t) \quad (74)$$

$$q_{m+1} = G_{m+1}((2m+1)x-t) \quad (75)$$

Where G_m is an arbitrary wave form. The $m=0$ ($m \neq 0$) solution describes a wave form traveling eastward (westward) at speed 1 ($-\frac{1}{2m+1}$): the long Kelvin (Rossby) wave!

These waves are non-dispersive.

Show George Figure 1

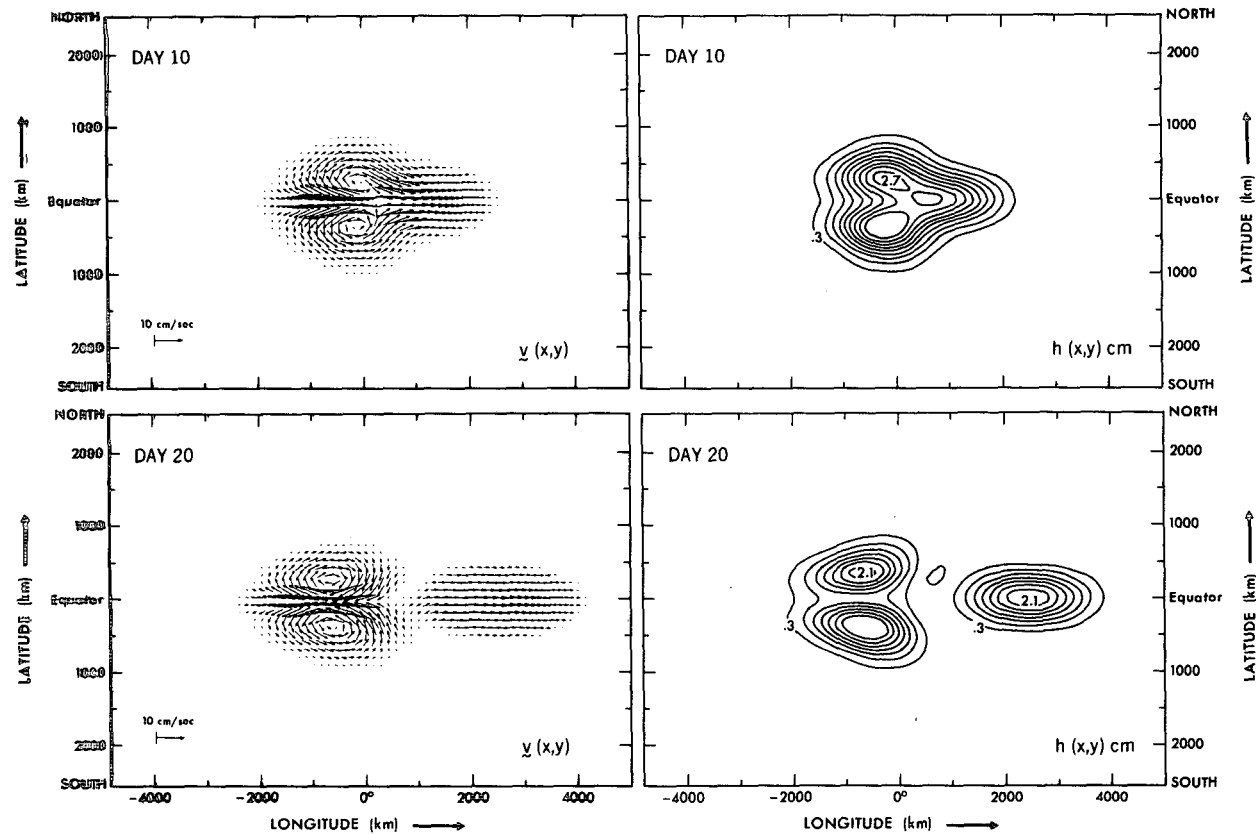


FIG. 3. Dispersion of perturbation in the depth of the thermocline.

Figure 3 shows the dispersion of an initial perturbation in the height field

$$h = 0.2 \exp[-(x^2 + y^2)/(500 \text{ km})^2] \text{ cm} \quad (4)$$

that is maintained for 10 days. In these calculations $a = 1/100$ days, $b = 1/100$ days and $C_0 = 140 \text{ cm s}^{-1}$ so that the equatorial radius of deformation is 250 km and the equatorial inertial time $(1/\beta c)^{1/2}$ is 1.5 days. The perturbation is seen to disperse into westward traveling Rossby waves and an eastward propagating Kelvin wave. (For details of the numerical method

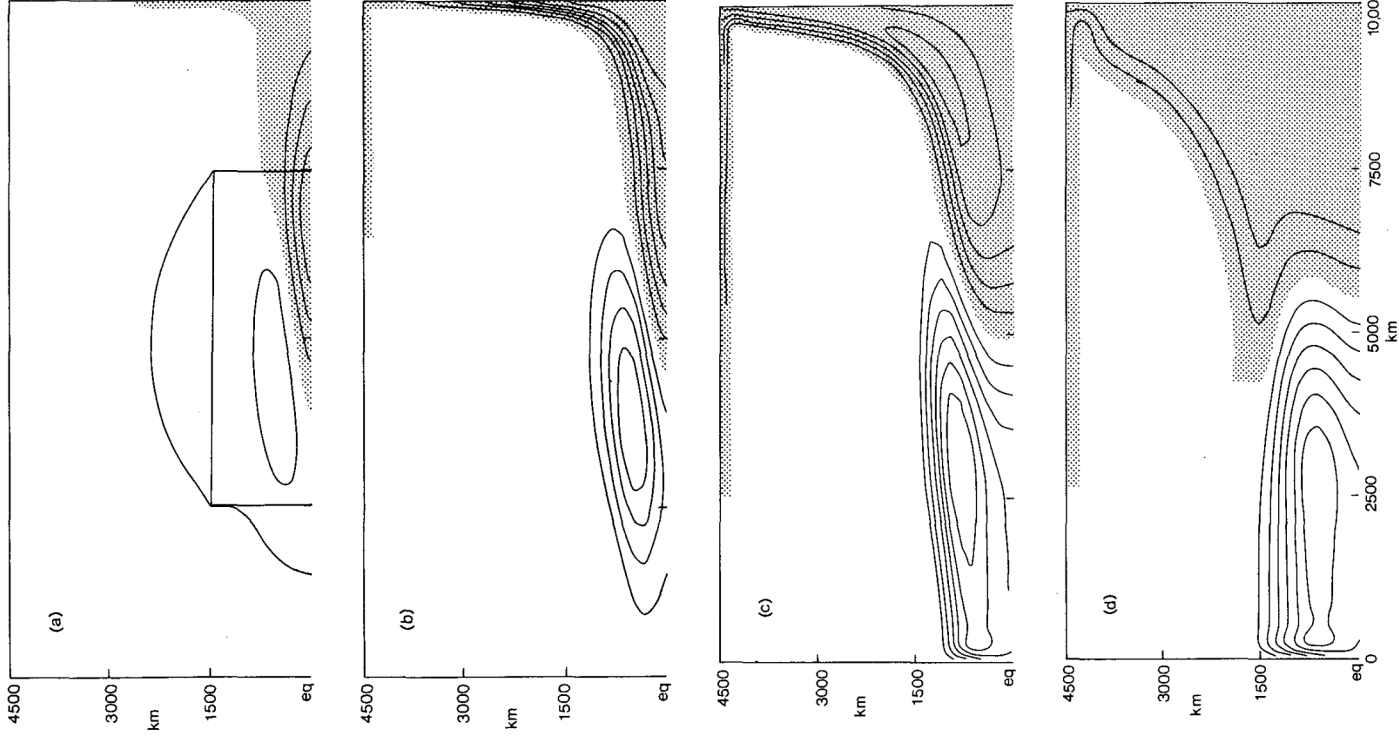


FIG. 2. The thickness of the model interface in response to τ_w alone at various times: (a) 1 month, (b) 3 months, (c) 13 months, and (d) 5 years. The location and structure of τ_w are indicated in (a), and are described precisely in Eqs. (5). The contour interval is 10 m, there is no 100 m contour, and the shaded region indicates thicknesses less than 100 m. To avoid displaying irrelevant, weak fluctuations about 100 m, the shaded region is not shown everywhere in the basin. The figure illustrates the propagation of Kelvin and Rossby waves about the ocean basin. The initial rise of h_e in Fig. 1 is caused by the arrival of equatorial Kelvin waves generated directly by the wind. The gradual relaxation of h_e toward h_w is caused by the arrival of a second set of equatorial Kelvin waves that are generated when Rossby waves reflect from the western ocean boundary.

(f) Reflection at an ^{mentioned} Eastern Boundary, $x = x_E$

In the low frequency, ~~limit~~ longwave limit, the only mode that can carry energy into the boundary from the west is the Kelvin mode. Upon reflection, it sends a series of long Rossby waves westward and some energy poleward along the coast (in the form of a coastal Kelvin wave).

The boundary condition is $u = 0$ on $x = x_E$.

Hence $q = r$ on $x = x_E$, so

$$q_m = r_m \text{ at } x = x_E \quad (76)$$

$$q_0 = r_0; \quad q_1 = 0 \quad (77)$$

$$r_{m+1} = q_{m+1} = q_m \left(\frac{m}{m+1} \right)^{1/2} \quad (78)$$

$m = 1, 2, \dots$

Hence $r_2 = q_2 = q_0 / \sqrt{2}$

$$q_4 = r_4 = q_0 \left(\frac{1}{2} \cdot \frac{3}{4} \right)^{1/2} \quad q_m = r_m = 0 \quad m = 1, 3, 5, \dots$$

$$\text{etc.} \quad \dots \quad q_{2m} = q_0 \left(\frac{1}{2} \cdot \frac{3}{4} \cdot \dots \frac{2m-1}{2m} \right)^{1/2}$$

In practice one starts with $r_0 = q_0$; the later is obtained by integrating the Kelvin solution from $x < x_E$ to the boundary. This sets the amplitudes of all q_m, r_m - the Rossby modes leaving the eastern boundary at speed $-(2m+1)^{-1}$.

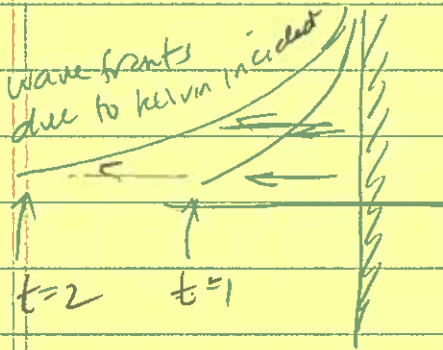
In terms of energy propagation, some does leak poleward.

A general result (not shown) is that poleward

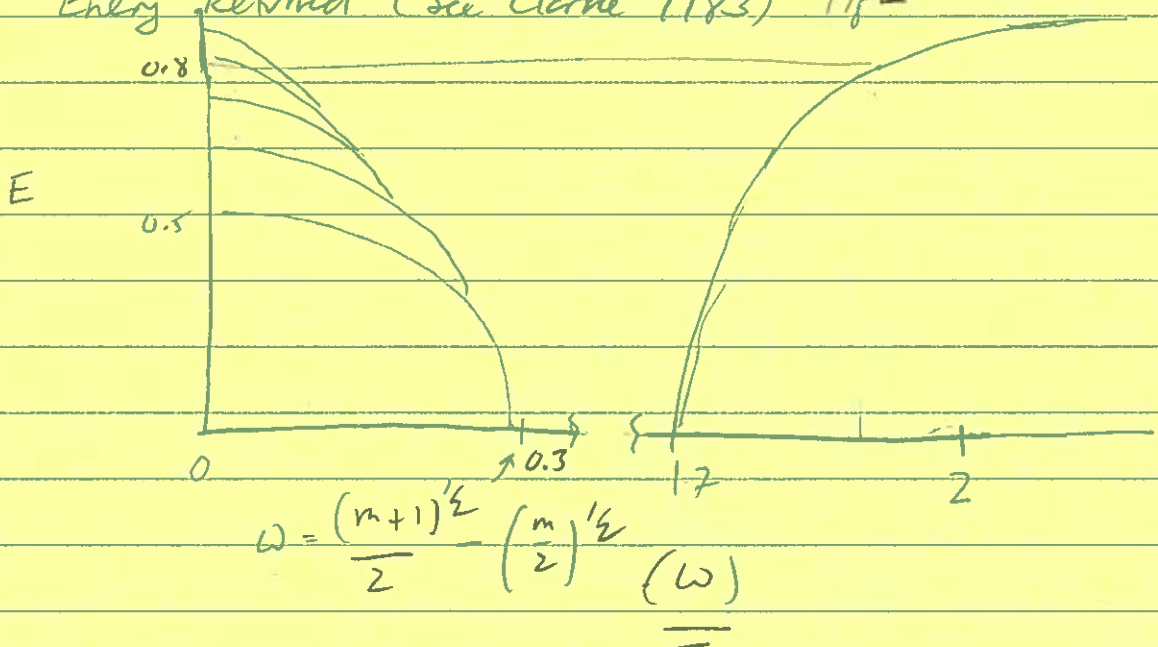
$y/|y| = y = (2\omega)^{-1}$ energy is trapped along the boundary

and propagates poleward. Equatorward of y_c ,

energy propagates westward at speed $-y^{-2}$



Energy Return (see Clarke 1983) Fig 2



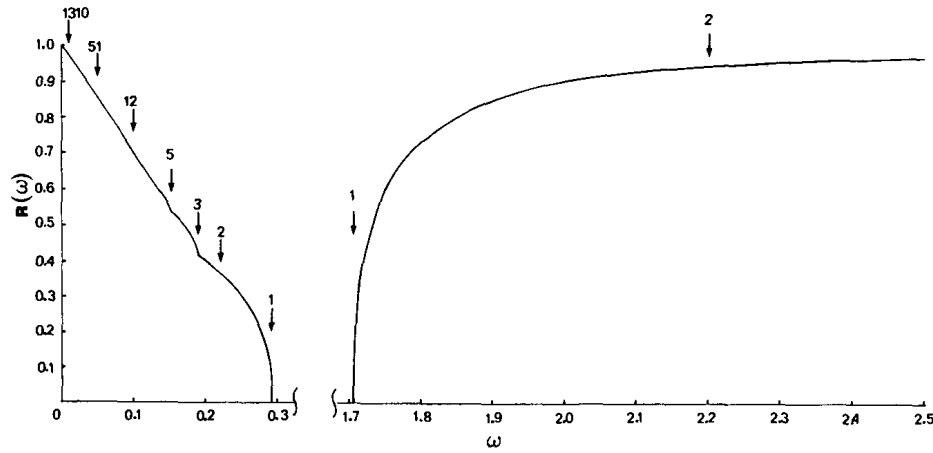


FIG. 2. The energy flux reflection coefficient of an equatorial Kelvin wave reflecting from an eastern meridional boundary. The graphs intersect the ω axis at $\omega = 1 - 2^{-1/2}$ and $\omega = 1 + 2^{-1/2}$, the frequencies corresponding to the highest frequency for equatorial planetary wave modes and the lowest frequency for inertia gravity wave modes. For $1 - 2^{-1/2} < \omega < 1 + 2^{-1/2}$ no waves with westward group velocity away from the boundary exist and $R(\omega) = 0$. For ease of plotting, the ω axis is broken between $\omega \approx 0.3$ and $\omega \approx 1.7$. The arrows and associated numbers point to the frequency when that number mode (planetary or inertia gravity) becomes available for reflection

$$\left[\omega = \left(\frac{m+1}{2} \right)^{1/2} - \left(\frac{m}{2} \right)^{1/2} \text{ or } \left(\frac{m+1}{2} \right)^{1/2} + \left(\frac{m}{2} \right)^{1/2}, \text{ respectively} \right].$$

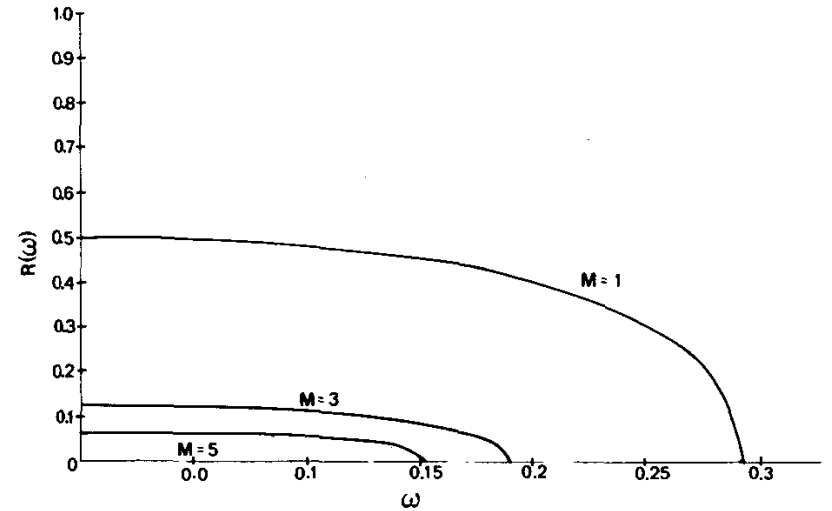
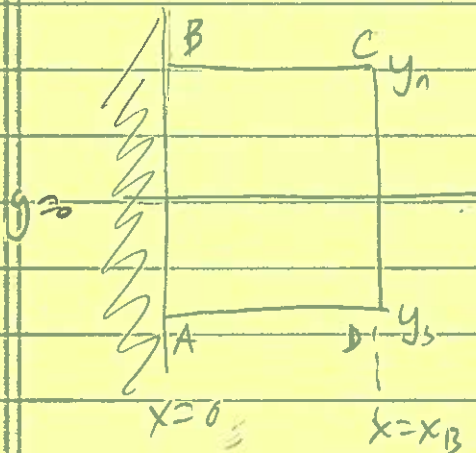


FIG. 4. The energy flux reflection coefficients for $M = 1, 3$ and 5 equatorial planetary wave modes reflecting from a western meridional boundary. The curves intersect the ω axis at $\omega = [(M+1)/2]^{1/2} - (M/2)^{1/2}$ ($M = 1, 3, 5$) and the $R(\omega)$ axis at $R(\omega) = 1/2, 1/8, 1/16$.

(g) Reflection at a meridional western boundary.

At the western boundary, the short Rossby waves contribute to the solution, but we have filtered them out in our solution (71).

These Rossby waves return mass and energy to the interior. The group velocity of these waves is very slow, so they do not go far into the domain before they are damped. In the end, these waves move mass ~~potentials~~ along the boundary.



The proper boundary condition for the longer waves (interior solution) is found by writing the closed zonal flow in terms of the boundary layer solution u_B and the

interior solution: $u = u_B + u_E$.

(low) frequency,

The short/ Rossby waves trapped near the Boundary are non divergent

$$\nabla \cdot \underline{u}_B = 0. \quad (79)$$

If the incoming energy is trapped near the equator, so too will be the poleward extent of the boundary layer solution. Hence we can integrate (79) using the divergence theorem to obtain

(30)

$$\int_A^B u dy + \int_B^C v dx + \int_C^D u (-dx) + \int_D^A v (-dy) = 0$$

$$\text{or } \int_A^B u dy = 0 \quad (80)$$

Since the western boundary layer condition

$$u = u_B + u_L = 0 \text{ at } x=0$$

$$\text{or } u_L = -u_B \text{ at } x=0$$

or integral of the boundary

$$\text{Hence } \int_A^B u_L dy = \int_{-\infty}^{\infty} u_L dy = 0 \quad (81)$$

th terms of u and r

$$\int_{-\infty}^{\infty} q dy = \int_{-\infty}^{\infty} r dy \quad (82)$$

$$\text{or } q_0(x=0, t) = r_0(x=0, t) + \sum_{m=1}^{\infty} (r_m - q_m) dm$$

$$\text{where } dm = \frac{\int_{-\infty}^{\infty} \psi_m dy}{\int_{-\infty}^{\infty} \psi_0 dy} = \left[\frac{m-1}{m} \cdot \frac{m-3}{m-2} \cdots \frac{1}{2} \right]^{1/2} \quad (83)$$

$m \geq 1$

where we have used (71d).

So we know the amplitude of the Kelvin mode q_0 issued from the western boundary in terms of the amplitude of each incoming Rossby mode $q_{m \geq 1}$ (r_0 is

known by 71a).

Assuming the wind stress is ~ symmetric about the equator, we can expand the ~~sea~~ kelvin amplitude into each Rossby contribution

$$q_0 = \frac{q_2}{\sqrt{2}} + \sum_{m=1}^{\infty} \frac{a_{2m}}{[2(m+1)(2m+1)]^{1/2}} q_{2m+2}$$

$$= 0.71 q_2 + 0.17 q_4 + 0.10 q_6 + 0.07 q_8 + \dots$$

That is, only the first few modes carry significant information.

(h) The irregular western Boundary

It is highly and far from meridional. Nonetheless, because the Rossby waves are zonally long and trapped near the equator, the collection of islands in the western Pacific act as an excellent wall for the incident q_2 and q_4 Rossby modes.

Cane & de Boyer (1983) and Clarke (83) ~~estimate~~ provide theoretical basis. Mantua & Bullock show observations are consistent with theory.

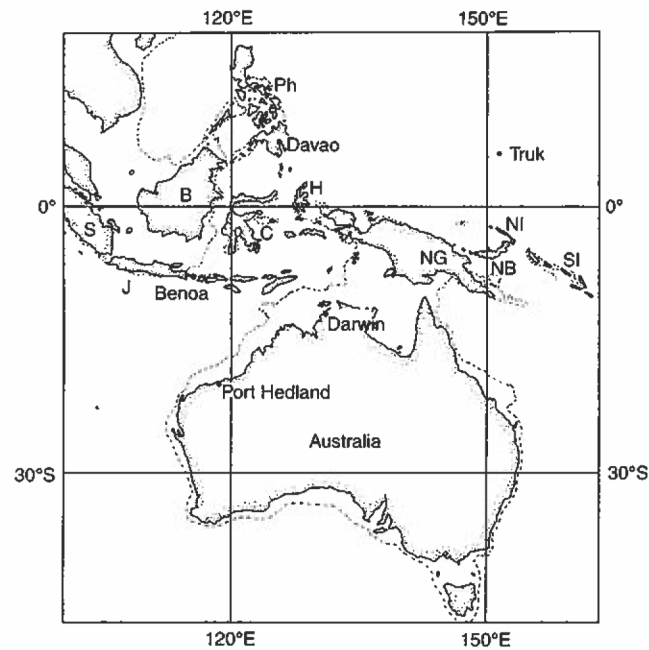


Fig. 4.2 The gappy, irregular western Pacific boundary region. The dashed line is the 200 m isobath. SI, Solomon Islands; NB, New Britain; NI, New Ireland; NG, New Guinea; H, Halmahera; C, Celebes; Ph, Philippines; B, Borneo; J, Java; S, Sumatra. (From Clarke 1991).

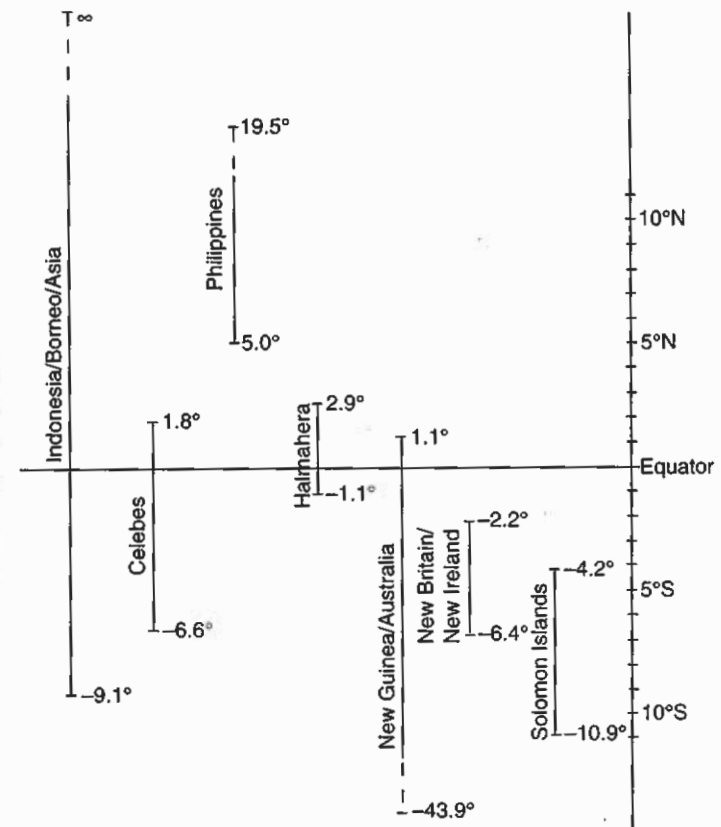


Fig. 4.3 Thin island approximations to the western Pacific boundary. The northern and southern latitudes were based on the northern and southern limits of the 200 m isobath for each island. (From Clarke 1991).

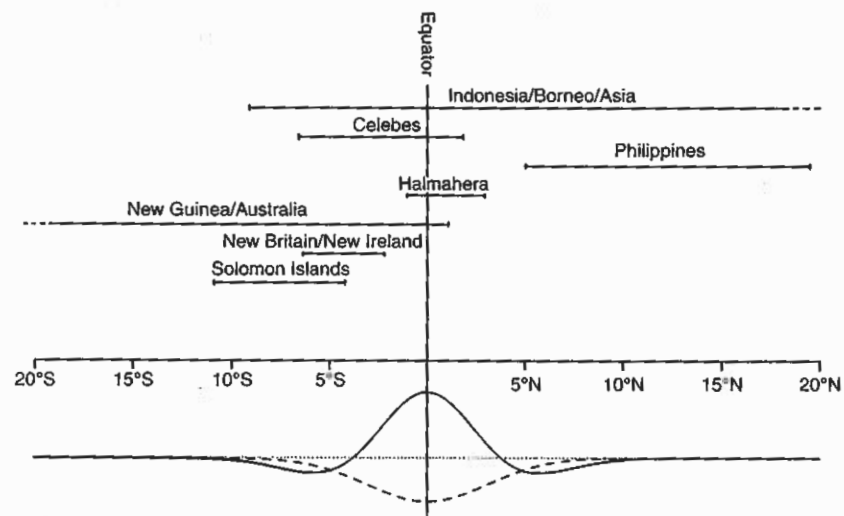


Fig. 4.4 First meridional mode first vertical mode ($c = 2.7 \text{ m s}^{-1}$) Rossby wave (solid curve) and Kelvin wave (dashed curve) non-dimensional u fields compared with the latitudinal extent of the islands. Only Indonesia/Borneo/Asia extends beyond the scale of the u fields. The dotted line is the zero amplitude line. (From Clarke 1991).