A Simple Method for Estimating Barotropic Tidal Currents on Continental Margins with Specific Application to the M_2 Tide off the Atlantic and Pacific Coasts of the United States

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(Manuscript received 13 July 1981, in final form 23 October 1981)

ABSTRACT

Theory is developed to describe barotropic tidal currents on "smooth" continental shelves, that is, continental shelves with longshore scales much greater than the shelf width. Two models are considered, one in which sea level does not vary significantly across the shelf and the other in which it does. Both models include longshore gradients and friction (parameterized linearly in velocity). The models were tested by calculating the M_2 tidal currents off the Atlantic and Pacific coasts of the United States and then comparing the calculated currents to those measured. Results show that theory and observation are in very good agreement as far offshore as 300 km. Along the Atlantic coast, on account of the wide continental shelf, current velocities are typically O(0.10–0.15 m s^{-1}) north of Cape Hatteras and O(0.15–0.28 m s^{-1}) off Savannah, Georgia. Currents rotate anticyclonically and are highly elliptical (ellipticity $E \approx -0.4$), with the semi-major axis oriented normal to the coast. Friction is significant in the South Atlantic Bight and acts to rotate the zero-friction current ellipses in a clockwise direction. Off the Pacific coast, where the continental shelf is narrow, M_2 tidal currents are relatively weak (0.02–0.08 m s^{-1}) and strongly oriented in the longshore direction ($E \approx 0.1$). The currents rotate counterclockwise with negligible friction influence. The good agreement between calculated barotropic M_2 currents and the observed currents off the southwest Pacific coast and all along the Atlantic coast of the United States suggests that the semidiurnal tides along these coasts are largely barotropic.

1. Introduction

Since many coastal processes are dominated by the tides, a detailed knowledge of how tidal currents vary across the continental margins is important. This is especially true on wide continental shelves where the semidiurnal tides may be strongly amplified. However, obtaining measurements of tidal currents on continental shelves can be a difficult, time-consuming and costly endeavor. Therefore, a reliable analytical model of how tides vary across continental margin topography would be valuable.

Research on the effect of continental shelves on tidal currents has been carried out previously, most notably by Fleming (1938), Rattray (1957), Redfield (1958) and Clarke and Battisti (1981). Redfield used the canal theory of Sterneck (1915) to estimate semidiurnal tidal currents off the east coast of the United States and found that the strongest currents were located on the wide shelf off Savannah, Georgia. Fleming used a similar theory in describing currents in the Gulf of Panama. However, as the theory in this paper shows, both longshore gradients and rotation, which are omitted from canal theory, are, in general, important in describing tidal currents on continental shelves.

Studies by Rattray (1957) and Clarke and Battisti (1981) have taken into account longshore gradients and rotation. Rattray used a Maclaurin-series expansion for the sea surface height to determine frictionless tidal currents within a few kilometers of the coast. He found good agreement between calculated and observed currents off Washington and suggested that the poor agreement he found off Savannah, Georgia, might be due to the neglect of friction. Clarke and Battisti obtained simple analytic expressions for continental-margin tidal currents in the frictionless case.

The purpose of this paper is twofold:

1) To construct and verify observationally models that can be simply and inexpensively used to estimate barotropic tidal currents on continental margins.

2) To describe the M_2 tidal currents on the east- and west-coast continental margins of the United States.

The analytic models used in this paper are described in Sections 2 and 3. In Section 4 calculations of M_2 tidal currents along the east and west coasts of the United States are carried out and compared to those

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0022-3670/82/010008-09$06.25

measured. The conclusions are presented in Section 5.

2. Theory

Since only the tidal motions on continental margins will be considered, direct tidal generation by the large-scale astronomical forcing will be ignored. Therefore, the Laplace Tidal Equations with friction included are

$$u_t - fv = -g\eta_x - \frac{\tau_B^x}{\rho H}, \tag{2.1}$$

$$v_t + fu = -g\eta_y - \frac{\tau_B^y}{\rho H}, \tag{2.2}$$

$$\eta_t + (Hu)_x + (Hv)_y = 0. \tag{2.3}$$

In these equations u, v, t, f, g, η, τ_B^x, τ_B^y, ρ and H refer to the x and y velocities, time, the Coriolis parameter, the acceleration due to gravity, the deviation of the sea surface from some undisturbed level, bottom friction in the x and y directions, the water density and the water depth, respectively. In this work x is the Cartesian coordinate oriented perpendicular to and away from the coast, and y the alongshore coordinate. Subscripts denote differentiation.

As in Clarke and Battisti (1981, hereafter referred to as I), theory will be developed for "smooth" continental shelves, i.e., shelves where longshore variations in topography are on a much larger scale than offshore variations and where bends in the coastline occur on a scale much larger than the shelf width. If one defines

$$il = \frac{\eta_y}{\eta}, \tag{2.4}$$

then, since $l^{-1} \sim y$ scale, the condition that a shelf be smooth can be written

$$|l| \cdot (\text{shelf width}) \equiv \epsilon \ll 1. \tag{2.5}$$

For the semidiurnal tides $|l^{-1}|$ is O(2000 km). It is shown in I that l is independent of x and therefore can be estimated from easily obtained coastal data.

Integrating the continuity equation (2.3) along the normal from the coast at $x = 0$ to x gives

$$-\int_0^x \eta_t dx = [Hu]_0^x + \int_0^x Hv_y dx. \tag{2.6}$$

Since Hu vanishes at the coast, $u \sim v$ and $x \leqslant$ shelf width, one has

$$u(x, y, t) = -\frac{1}{H}\int_0^x \eta_t dx[1 + O(\epsilon)]. \tag{2.7}$$

In I it was shown that over most shelves the sea level does not vary significantly. The change across the shelf and slope is of order (μx) where

$$\mu = \frac{\omega^2 - f^2}{g\alpha} + \frac{fl}{\omega}, \tag{2.8}$$

where α is the mean slope of the shelf and ω the tidal frequency. The result (2.8) was obtained in the case of no friction, but the same order of change also occurs in the frictional case. Thus to within an error $O(\epsilon, \mu x)$, Eq. (2.7) can be written

$$\boxed{u = -i\omega\eta(0)x/H}, \tag{2.9}$$

where $\partial/\partial t$ has been replaced by $i\omega$.

Assuming a frictional stress linear in velocity, τ_B can be written

$$\tau_B = \rho r \mathbf{u}, \quad r(x) = C_D|\mathbf{u}|, \tag{2.10}$$

where C_D is the drag coefficient and $|\mathbf{u}|$ will be taken, in practice, to be the root-mean-square (rms) velocity due to tides, wind, mean currents, etc. Use of (2.2), (2.4) and (2.10) gives

$$fu + \left(i\omega + \frac{r}{H}\right)v = -ilg\eta. \tag{2.11}$$

Thus, to within an error or order $(\epsilon, \mu x)$, Eqs. (2.9) and (2.11) yield the following expression for the longshore current v:

$$\boxed{v = \frac{\eta(0)}{[1 - (ir/H\omega)]} \cdot \left(\frac{xf}{H} - \frac{gl}{\omega}\right)}. \tag{2.12}$$

Eqs. (2.9) and (2.12), a generalization of the frictionless formulas in I, are the main formulas which will be used in Section 3 to estimate barotropic tidal currents off the east coasts of the United States. The simple analytic form of these formulas allows quick and inexpensive estimations of tidal currents on continental margins with general topography $H(x)$ given knowledge of just $|\mathbf{u}|$, $\eta(0)$ and l. The latter two are easily determined from coastal sea level measurements.[2] It should be noted that in Eq. (2.12) friction is a function of x and $r/H\omega$ can be O(1).

The form of the expressions (2.9) and (2.12) illustrates the importance of variable topography on tidal currents. If the topography changes rapidly over a small distance across the shelf, currents can change markedly. An example of this often occurs very near the shore where the bottom slope is frequently much steeper than the average slope over the shelf; nearshore currents predicted from (2.9) and (2.12) will be significantly smaller than those obtained using a shelf-averaged slope value.

By setting r, $l = 0$ in Eq. (2.12) one can obtain

[2] Since the coastal tide is of the form $\eta = Ae^{iG}$, l is given by $l = G_y - iA_y/A$. An accurate method for calculation of l from the coastal sea-level values is given in I. Usually $|l^{-1}| \sim 1000$ km.

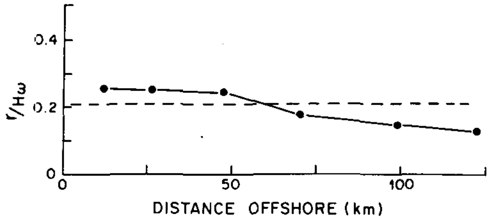

FIG. 1. The calculated frictional parameter $r/H\omega$ versus distance offshore of Brunswick, Georgia. Values of $|\mathbf{u}|$ are from M_2 tidal currents. The solid curve represents the actual calculated values of $r/(H\omega)$ and the dashed line the average value, (λ_0/ω). λ_0 is defined in Section 3.

an estimate for tidal currents on smooth continental-margin topography without friction or longshore gradients. When r, $l = 0$, $v/u = if/\omega$, indicating that the current ellipse rotates anticyclonically with principal axes parallel and perpendicular to the shore and in the ratio $|f/\omega|$. This is a generalization of a similar result that can be obtained for Poincaré waves on a step shelf. Since tidal currents on narrow shelves largely owe their existence to non-zero longshore gradients, v/u is only likely to be near if/ω on wide shelves where the topographic term xf/H dominates gl/ω in (2.12), and $l = 0$ is consequently a good approximation. Even in these cases frictional effects can significantly alter the value of v/u from if/ω. Despite this, $u/v = if/\omega$ is still a useful criterion for quickly checking whether frictional and/or longshore gradients are important to the dynamics.

Note that in (2.11) as $H \to 0$, $r/H \to \infty$, i.e., the frictional effect associated with the bottom stress $\to \infty$. Physically what happens is that as the coast is approached one eventually reaches a point where the boundary layer intersects the surface ($x = \delta$, say). In practice δ is very close to shore (usually <1 km). In this small region $0 \leqslant x \leqslant \delta$, the vertically averaged water velocity is not necessarily parallel to the bottom stress and Ekman-layer dynamics rather than (2.10) applies.

Finally, although diurnal currents are not discussed in detail in this work, it should be noted that Eqs. (2.9) and (2.12) can be used to estimate them. A difficulty arises in that l for the diurnal tides usually cannot be accurately estimated from historical data (see I). However, for smooth shelves the longshore gradient term gives rise to insignificant diurnal currents so that this difficulty is not very restrictive. [Typically $|l^{-1}|$ and η for smooth-shelf diurnal tides are O(10 000 km) and <0.1 m, so $\eta lg/\omega$ in Eq. (2.12) is O(10^{-3} m s^{-1})]. Diurnal currents are likely to be strongest on wide shelves where the topographic term xf/H is large compared to the longshore gradient term. The major diurnal currents on these shelves are usually smaller than the major semidiurnal currents because ω(diurnal)/ω(semidiurnal) < 1, and $|\eta$(diurnal)/η(semidiurnal)$|$ < 1 [see Eqs. (2.9) and (2.12)]. However, diurnal currents can be significant. Estimated diurnal currents on the wide shelf in the South Atlantic Bight show diurnal currents of O(0.03–0.04 m s^{-1}). Note that diurnal sea levels never change significantly across a shelf because $|\mu x| \ll 1$ always (see I). So for diurnal tidal currents on smooth continental margins, Eqs. (2.9) and (2.12) should always be adequate.

3. Theory for currents with large changes in η across the shelf

The expressions (2.9) and (2.12) for tidal currents are valid across many "smooth" continental margins. However, in a few cases the continental shelf is wide enough to induce a significant variation in the sea surface, η, across the shelf. Specifically, errors of order $|\mu x|$ will not be $\ll 1$. The shelf off Savannah, Georgia, is one such example ($\alpha \approx 5 \times 10^{-4}$ so $|\mu x| > 0.1$ for $x > 36$ km). For these cases, the expressions of Section 2 are applicable over the inner shelf, but are invalid on the outer shelf.

To derive an analytic solution for barotropic tidal currents for variable $\eta(x)$ consider the "smooth" continental margins discussed in Section 2 with the following approximations: 1) topography modeled as a linear profile in x ($H = x\alpha$) out to the shelf break at $x = a$; 2) friction modeled linearly in velocity [see Eq. (2.10)], with r/H being replaced by a shelf-averaged value

$$\lambda_0 = \frac{1}{a - \delta} \cdot \int_\delta^a \frac{r(x)}{H(x)}\, dx.$$

An example of how r/H does vary across the shelf is illustrated in Fig. 1 for Brunswick, Georgia, where the values of r/H reach a maximum for the east coast. Here the M_2 tidal contribution to r/H is slowly varying across the shelf, suggesting that setting r/H = constant is a reasonable approximation. Indeed the errors in neglecting other contributions to $|\mathbf{u}|$ (e.g., mean currents) may exceed that of setting r/H constant.

Under these approximations (2.1) and (2.2) can be written

$$(\partial/\partial t + \lambda_0)u - fv = -g\eta_x\,,$$

$$(\partial/\partial t + \lambda_0)v + fu = -g\eta_y\,.$$

From these equations one can obtain

$$u = \frac{-g}{(i\omega + \lambda_0)^2 + f^2}\,[\eta_x(i\omega + \lambda_0) + f\eta_y], \quad (3.1)$$

$$v = \frac{g}{(i\omega + \lambda_0)^2 + f^2}\,[f\eta_x - \eta_y(i\omega + \lambda_0)]. \quad (3.2)$$

Substitution of (3.1) and (3.2) into (2.3) and use of (2.4) gives the differential equation for the sea surface height. The equation is

$$\eta_{xx} + \frac{1}{x}\eta_x + \frac{\gamma}{x}\eta = 0, \qquad (3.3)$$

where γ is

$$\gamma = \frac{i\omega}{(i\omega + \lambda_0)} \cdot \left[\frac{lf}{\omega} - \frac{(i\omega + \lambda_0)^2 + f^2}{g\alpha}\right]. \qquad (3.4)$$

The solution of (3.3), which is finite at the coast, can be written in terms of zero-order Bessel functions, or more conveniently,

$$\frac{\eta(x)}{\eta(0)} = \sum_{n=0}^{\infty} \frac{(-\gamma x)^n}{(n!)^2}$$

$$= 1 - \gamma x + (\gamma x)^2/4 - (\gamma x)^3/36 + \cdots. \qquad (3.5)$$

In obtaining this result it has been assumed $\delta \ll x$. This is a very good approximation since δ is generally <1 km and this theory is meant to be used on the outer part of the shelf. For realistic continental shelves $|\gamma x| < 1$. Thus, since (3.5) converges very rapidly, one only needs to consider the first few terms of this expression. Using the expression (3.5) in (3.1) and (3.2) gives the following results for u and v:

$$u = \frac{\eta(0)g\left[\gamma\left(1 - \frac{\gamma x}{2}\right)(i\omega + \lambda_0) - ilf(1 - \gamma x)\right]}{(i\omega + \lambda_0)^2 + f^2}, \qquad (3.6)$$

$$v = \frac{-\eta(0)g\left[\gamma\left(1 - \frac{\gamma x}{2}\right)f + il(1 - \gamma x)(i\omega + \lambda_0)\right]}{(i\omega + \lambda_0)^2 + f^2}. \qquad (3.7)$$

Eqs. (3.6) and (3.7) are the expressions for barotropic tidal currents across a continental shelf of slope α with an offshore averaged friction λ_0. The error in these expressions is $O[(\gamma x)^2/2, \epsilon, \delta\gamma]$.

Note that since the theory of Section 2 is simpler than the theory discussed here and also allows for far more general topography, the formulas (2.9) and (2.12) should be used instead of (3.6) and (3.7) wherever possible. Thus, for example, even when η varies significantly across the shelf, (2.9) and (2.12) should still be used to describe the currents nearshore where x is small enough that $|\mu x| \ll 1$. In this way the inaccuracy mentioned earlier of using a shelf-averaged value for the nearshore slope is avoided.

Eqs. (3.6) and (3.7) do not apply over the continental-slope region where the bottom slope changes by at least an order of magnitude and $r/H\omega$ also changes rapidly. To determine the tidal currents in this region, consider an integration of the continuity equation similar to that carried out in Section 2. Such an integration is valid because over the continental slope η is very nearly constant [the error is $\leq$10%, see I, Eq. (3.2)]. Integrating (2.3) from $x = a$ to a general point x yields

$$u(x) = \frac{1}{H(x)}[u(a)H(a) - i\omega\eta(a)(x - a)]. \qquad (3.8)$$

Once off the continental shelf, H increases rapidly with increasing x so $r/H \rightarrow 0$. Substitution of (3.8) and (2.4) into (2.2) with $r = 0$ yields the expression for the longshore velocity

$$v(x) = \frac{-gl\eta(a)}{\omega} + \frac{if}{\omega H(x)} \times [u(a)H(a) - i\omega\eta(a)(x - a)]. \qquad (3.9)$$

In Eqs. (3.8) and (3.9) η and $u(a)$ are given by (3.5), (3.6) and (3.7) with $x = a$.

Using typical values for $H(a)$, $H(x)$ and a one finds that the terms proportional to $H(x)^{-1}$ in (3.8) and (3.9) quickly become small. Therefore, once off the shelf and into the deep sea the currents decrease rapidly, with $u \rightarrow 0$ and $v \rightarrow -gl\eta(a)/\omega$. This result is similar to that in the deep-sea region off of narrow shelves, described in Section 2.

4. Comparison of predicted and measured M_2 tidal currents off the east and west coasts of the United States

The simple analytic solutions for tidal currents across continental margins will be tested using the semidiurnal M_2 tide along the east and west coasts of the United States. The topography and form of the deep-sea tide off these coasts are very different; the west coast shelf is narrow [O(10–30 km)] compared with the wide [O(100 km)] east coast shelf. The M_2 species was chosen for comparison between observation and theory because the currents associated with the M_2 tide are the most pronounced—recall that currents go as the sea level at the coast $\eta(0)$ and $|\eta(0)_{M_2}/\eta(0)_{S_2,N_2} \cdots| > 1$.

Using readily available sea level data at the coast and bathymetric data from the United States Department of Commerce, Coast and Geodetic Survey nautical charts, M_2 current velocities given by Eqs. (2.9) and (2.12) were calculated along representative sections oriented perpendicular to the coast. The spacing of the sections along the coast was determined by the location of current measurements available for comparison with those calculated and by longshore variations in the bottom topography (see Figs. 2a, 2b). In the calculations l was estimated from the coastal sea level data as described by I. The first calculations used $r = 0$ in Eqs. (2.9) and (2.12). From the results $|\mathbf{u}|$ was estimated as the rms M_2 tidal velocity, and a nonzero value for r thus obtained. If friction was found to be important ($r/H\omega > 0.1$), an iteration using the estimated r and Eqs.

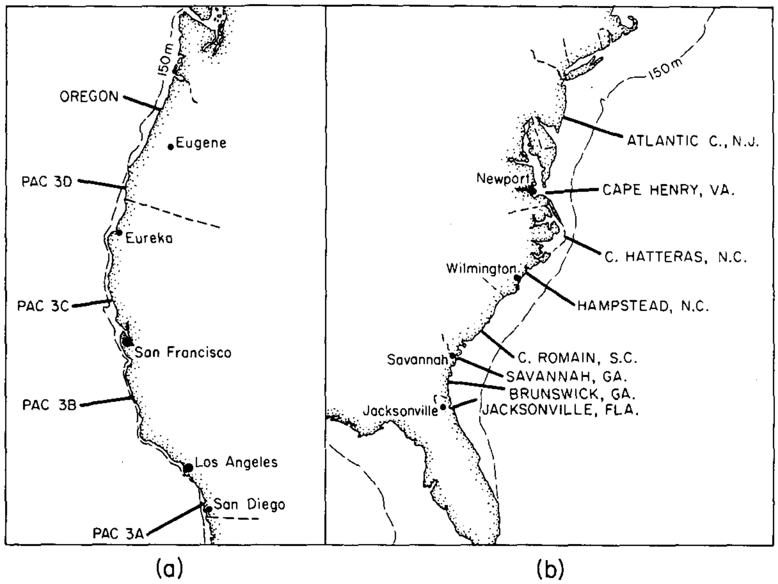

FIG. 2. (a) Location of Pacific Coast sections used in calculating M_2 tidal currents. (b) Location of Atlantic Coast sections used in calculating M_2 tidal currents.

(2.9) and (2.12) enabled more accurate results for u and v to be obtained. It is appropriate to note here that since the estimate of $|\mathbf{u}|$ did not include other tidal species or mean and wind generated currents, the value of r obtained is not very accurate. However, frictional effects are generally small, so errors in r are not crucial. Furthermore, without detailed knowledge of the nontidal shelf currents, $|\mathbf{u}|$ esti-

TABLE 1. The calculated barotropic M_2 tidal currents off the Pacific coast of the United States. The ellipticity E is the ratio: (semiminor current axis)/(semimajor current axis), with the sign designating the rotation sense; positive denoting cyclonic rotation. ϕ_{max} is the angle of the maximum current relative to the $+x$ axis. Maximum current occurs at a time ωt_{max} after high water at the coast. The values in parentheses are the observed currents at the location reported by the referenced source.

Location (lat, long)	x (km),	H (m)	Semimajor axis (m s^{-1})	Semiminor axis (m s^{-1})	E	ϕ_{max} (°)	ωt_{max} (°)	Observation source
Oregon	5	50	0.078	0.009	0.11	−97	−47	Torgrimson
(45°15′, 123°24′)	12	100	0.077 (0.049)	0.010 (0.009)	0.14 (−0.18)	−99 (20)	−48	and Hickey
	29	200	0.076	0.012	0.16	−101	−50	(1979)
	70	1100	0.080	0.006	0.07	−94	−44	
	200	2700	0.080	0.007	0.08	−95	−45	
	300	2700	0.076	0.010	0.12	−98	−47	
PAC 3D	3.6	55	0.047	0.006	0.12	−94	−33	
(42°15′, 124°24′)	14	185	0.046	0.007	0.15	−96	−34	
	35	900	0.049	0.003	0.07	−92	−31	
	50	1850	0.049	0.002	0.05	−92	−31	
	125	2600	0.048	0.004	0.09	−93	−32	
	160	2500	0.047	0.006	0.12	−94	−33	
	300	3250	0.046	0.008	0.17	−97	−35	
PAC 3C	6	90	0.030	0.004	0.14	−96	−37	
(38°35′, 123°33′)	63	900	0.031	0.002	0.08	−93	−35	
	80	3650	0.032	0.002	0.05	−92	−34	
	135	3950	0.032	0.002	0.07	−93	−34	
	230	4250	0.031	0.004	0.11	−95	−36	
	300	4450	0.030	0.004	0.14	−96	−37	
PAC 3B	7	110	0.039	0.005	0.12	−90	0	
(34°41′, 121°19′)	14	450	0.040	0.002	0.06	−90	0	
	36	900	0.040	0.003	0.07	−90	0	
	110	3900	0.040	0.002	0.05	−90	0	
	200	4100	0.040	0.003	0.08	−90	0	
	300	4250	0.039	0.005	0.13	−90	0	
PAC 3A	285	3640	0.021 (0.023)	0.006 (0.000)	0.026 (0.01)	−90 (−90)	0	Munk *et al.*
(32°55′, 117°16′)	350	3700	0.021 (0.022)	0.006 (0.006)	0.27 (0.24)	−90 (−91)	0	(1970)

TABLE 2. Calculated barotropic M_2 tidal currents off the Atlantic coast of the United States. The asterisk following the coordinate pair (x, H) indicates that calculations were made using the expressions cited in Section 3. In some instances the observed tidal currents were not decomposed into Fourier components. These stations are indicated by a dagger and plotted in Fig. 3.

Location (lat, long)	x (km),	H (m)	Semimajor axis (m s^{-1})	Semiminor axis (m s^{-1})	E	ϕ_{max} (°)	ωt_{max} (°)	Observation source
Atlantic City, NJ	40	23.5	0.132	0.071	−0.54	0	90	Moody and
(39°22′, 74°26′)	55	37	0.120	0.063	−0.53	0	90	Buttman (1980)
	90	55	0.133	0.072	−0.54	0	90	
	110	60*	0.149	0.082	−0.55	0	90	
			(0.155)	(0.087)	(−0.56)	(0)		
	125	90*	0.113	0.059	−0.52	0	90	
Cape Henry, VA	42	23.4	0.153	0.094	−0.62	−41	118	Haight (1942)
(36°57′, 75°59′)	70	36.5	0.152	0.107	−0.70	−39	119	
	85	73†	0.110	0.068	−0.62	128	−51	
	100	180	0.076	0.031	−0.40	112	−46	
	110	900	0.057	0.006	−0.10	97	−49	
	120	1530	0.055	0.004	−0.07	95	−50	
	175	2750	0.055	0.003	−0.05	94	−51	
Cape Hatteras, NC	24	55	0.031	0.013	−0.42	36	73	Haight (1942)
(35°15′, 75°30′)			(0.018)	(0.008)	(−0.39)	(88)		
	37	180	0.019	0.004	−0.23	54	73	
	57	1850	0.014	0.000	−0.01	83	86	
	100	3400	0.014	0.000	−0.01	83	86	
	145	3750	0.014	0.000	−0.02	81	85	
Hampstead, NC	24	22	0.085	0.031	−0.37	21	82	
(34°25′, 77°30′)	42	26	0.123	0.054	−0.44	15	83	
	60	31	0.144	0.066	−0.46	13	84	
	85	38	0.163	0.078	−0.48	12	84	
	110	90	0.094	0.037	−0.39	19	82	
	120	180	0.055	0.013	−0.24	29	82	
	160	550	0.033	0.001	0.02	51	91	
	190	975	0.030	0.002	0.07	61	98	
Cape Romain, SC	41	27*†	0.184	0.074	−0.40	21	81	Haight (1942)
(33°00′, 79°25′)	60	37*	0.178	0.071	−0.40	20	80	
	75	45*	0.173	0.069	−0.40	19	80	J. L. Pietrafesa
			(0.128)	(0.065)	(−0.51)	(10)		(pers. comm., 1981)
Savannah, GA	18.5	14.6	0.167	0.051	−0.31	10	87	Haight (1942)
(32°01′, 80°50′)			(0.18)	(0.05)	(−0.28)	(6)		
	40	18*	0.286	0.117	−0.41	3	88	
	60	26*	0.275	0.115	−0.42	3	87	
	80	37*	0.265	0.112	−0.42	2	86	
	100	46*	0.254	0.110	−0.42	2	84	
Brunswick, GA	26	15.2	0.234	0.110	−0.47	−22	101	Haight (1942)
(31°07′, 81°24′)		21*	(0.203)	(0.049)	(−0.24)	(−28)		
	48	30*	0.302	0.124	−0.41	−19	95	
	70	37*	0.287	0.117	−0.41	−19	93	
	96	43*	0.272	0.110	−0.40	−18	90	
	120		0.259	0.102	−0.40	−18	88	
Jacksonville, FL	7	17.4	0.100	0.028	−0.28	112	−54	Haight (1942)
(30°25′, 81°24′)			(0.099)	(0.005)	(−0.05)	(110)		

mated from tidal currents (or the strongly dominant tidal current as in this case) is as good as one can obtain.

As described in Section 2, use of Eqs. (2.9) and (2.12) is restricted to the case when η does not change greatly over the shelf ($|\mu a| \ll 1$). In the few cases in which this was not the case, the above iterative procedure taking into account frictional effects was carried out using expressions (3.6) and (3.7) of Section 3.

Calculated and measured results are expressed in elliptical format for the four Pacific and eight Atlantic sections shown in Fig. 2. The results, summarized in Tables 1 and 2 and Figs. 3 and 4, are discussed below.

a. *Pacific coast*

Comparison of the calculated M_2 currents off the Pacific coast with measured currents reported by

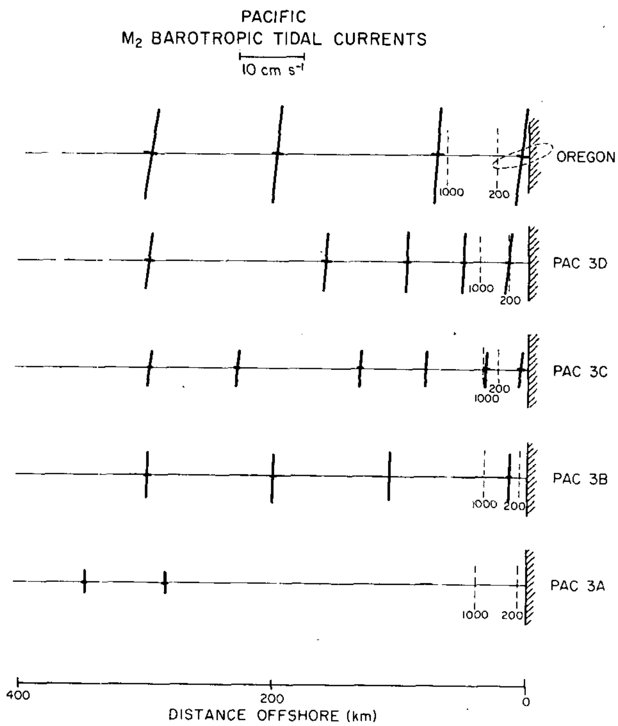

FIG. 3. Calculated versus observed currents along sections off the Pacific coast of the United States. The solid ellipse axes denote the calculated currents, the dotted ellipses the currents observed. The dashed lines are bathymetric contours (in meters). The observed currents along PAC 3A are plotted, but are not distinguishable from those currents calculated.

Munk, *et al.* (1970) shows excellent agreement. Along the Pacific coast of the United States the calculated M_2 tidal currents are consistent with those associated with a northward propagating Kelvin wave. Currents are weak [O(2–8 cm s^{-1})] due to the nature of the bathymetry and are strongly oriented in the alongshore direction with ellipticity greater than zero and ≤ 0.1 [Here ellipticity is defined as the ratio: (minor ellipse axis)/(major ellipse axis), with positive (negative) values denoting clockwise (anticlockwise) rotation]. The effect of friction is negligible throughout this region. Since the topographic contribution to the currents is very small (mathematically $fx/H < gl/\omega$), currents off this coast owe their character to the longshore gradient.

The poor agreement between theory and observation off Oregon is troublesome. However, bathymetry in the measurement area suggests that longshore gradients in topography are not small compared to gradients perpendicular to the coast. In such circumstances the "smooth" continental margin theory breaks down and one cannot expect the theory to be even qualitatively accurate.

b. Atlantic coast

Agreement between calculated and observed currents off the east coast of the United States is good. M_2 tidal currents off this coast are stronger than those off the Pacific coast on account of the wider continental shelves found there. Mathematically, the wider shelf implies that the topographic terms proportional to x/H in (2.9) and (2.12) are larger. In some areas currents are found to be in excess of 20 cm s^{-1}, rotating anticyclonically with the semi-major axis polarized perpendicular to the coast. Although having the correct orientation and rotation sense, current ellipses often do not satisfy the relation $u/v = if/\omega$ mentioned in Section 2; $|u/v|$ is generally somewhat less than the local ratio f/ω because of the influence of longshore gradients and friction on the currents.

Since the depth of the shelf break is roughly constant along the coast, the largest currents are found on the widest shelf near Brunswick, Georgia. The importance of shelf width is clearly illustrated by comparing the currents found on the shelf off Savannah with those off Cape Hatteras where the shelf

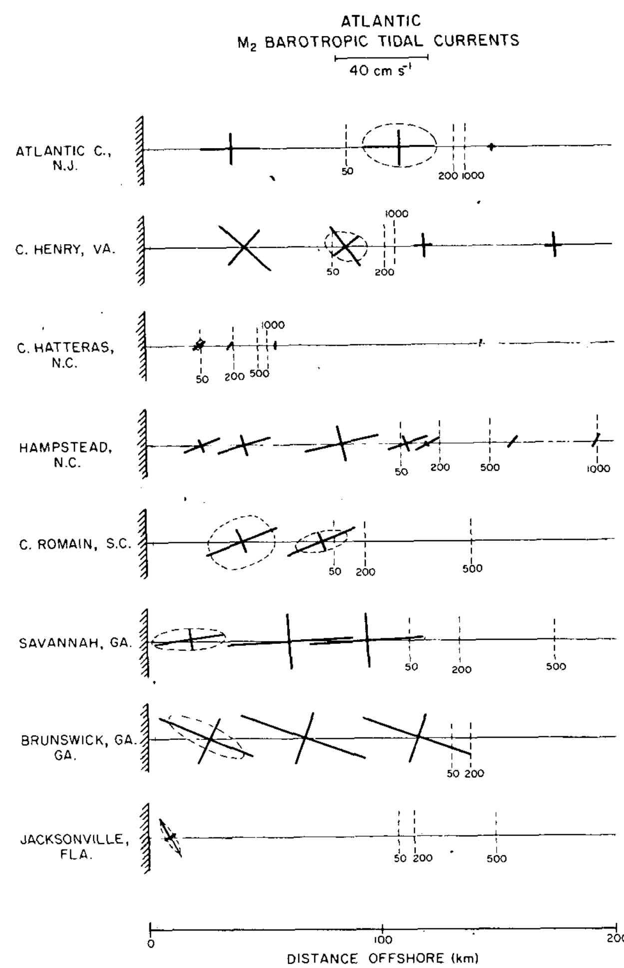

FIG. 4. Calculated versus observed currents along sections off the Atlantic coast of the United States. The solid ellipse axes denote the calculated currents, the dotted ellipses are the currents observed. The dashed lines are bathymetric contours.

TABLE 3. Calculated currents off Brunswick, Georgia. Notation is the same as in Tables 1 and 2. The data are presented in the format (non-frictional/frictional). Meaning of asterisks as in Table 2.

x (km),	H (m)	Semimajor axis (m s^{-1})	Semiminor axis (m s^{-1})	E	ϕ_{max} (°)
26	15.2	0.227/0.234	0.128/0.110	−0.56/−0.47	−16/−22
48	21*	0.291/0.302	0.161/0.124	−0.55/−0.41	−10/−19
70	30*	0.277/0.287	0.153/0.117	−0.55/−0.41	−9/−19
96	37*	0.263/0.272	0.145/0.110	−0.55/−0.40	−9/−18
120	43*	0.249/0.259	0.136/0.102	−0.55/−0.40	−8/−18

is very narrow; current strength varies by more than an order of magnitude.

The effect of varying offshore topography on the orientation of the tidal ellipse is seen off Hampstead, North Carolina. Once off the shelf edge, x/H decreases rapidly, and the ellipse orientation is governed by the longshore gradient, as the topographic contribution to the current decreases.

The inclusion of friction in the calculation of the currents causes the current ellipse axes to be rotated slightly in a clockwise direction relative to those calculated without friction. This is illustrated in Table 3 for Brunswick, Georgia, where friction effects are strongest on the east coast.

At a location 85 km off Cape Henry, Virginia, observed and calculated currents did not agree well. In this instance the observed current was a net tidal current. That is, the current record at these locations was not reduced to obtain currents associated with the M_2 tidal constituent. Hence, the disagreement between observation and theory is not surprising.

Finally, there have been several tidal measurements made north of Atlantic City, New Jersey, but because the smooth-shelf approximation breaks down at several places in this region (mathematically $\epsilon \sim 1$) no comparison of theory with observation was attempted. The smooth-shelf approximation breaks down because of coastal features which induce short longshore scales, e.g., Hudson Canyon, Long Island Sound.

5. Conclusion

Simple and accurate models for calculating barotropic tidal currents on smooth continental topography have been developed. The first model includes linearized bottom friction on variable topography $H(x)$. The tidal currents [Eqs. (2.9) and (2.12)] can be evaluated with easily obtainable knowledge of coastal sea levels and the offshore bottom topography. The expressions are valid for friction of O(1) wherever the smooth-continental-topography approximation holds and where the variation in sea surface does not substantially change across the shelf ($|\mu x| \ll 1$). The model shows that currents should be proportional to the coastal sea surface height and a linear function of the ratio: (distance offshore)/(water depth). In the few cases where the sea level changes substantially across the shelf ($|\mu x|$ not $\ll 1$), Eqs. (2.9) and (2.12) are no longer accurate and the more complicated expressions (3.6) and (3.7) of the second model apply.

Calculated and measured M_2 tidal currents along both the east and west coasts of the continental United States at distances offshore O(100–300 km) are found to be in good agreement with the models. Currents along the Pacific coast are weak [O(0.02–0.08 m s^{-1})], highly elliptical ($E \approx 0.1$) and oriented in the longshore direction. Unlike the west-coast M_2 tidal currents, which are mainly due to longshore gradients, the currents along the Atlantic coast of the United States are much stronger [O(0.10–0.25 m s^{-1})] on account of the presence of the wide continental shelf. The semi-major axis of the current ellipse is oriented perpendicular to the coast, with ellipticity −0.3 to −0.6. The currents decrease in magnitude quickly upon leaving the continental shelf.

Results do show that friction is important in describing the M_2 tidal currents on the wider sections of the east-coast continental shelf, rotating the current ellipse axes in a clockwise sense from those of the inviscid calculations. The good results for calculated M_2 tidal currents off both the southeastern Pacific coast and all along the Atlantic coast of the United States indicate that the M_2 tide off these coasts is largely barotropic.

Acknowledgments. We would like to thank Dr. H. O. Mofjeld for his comments and suggestions and Dr. L. J. Pietrafesa for supplying current measurements off Cape Romain, South Carolina. This work was supported by National Science Foundation Grant OCE79-07042 and is Contribution No. 1245 from the Department of Oceanography, University of Washington, Seattle.

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