JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 87, NO. C10, PAGES 7873–7878, SEPTEMBER 20, 1982

Estimation of Nearshore Tidal Currents on Nonsmooth Continental Shelves

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The theory of Battisti and Clarke (1982) to calculate analytically barotropic tidal currents across 'smooth' continental margin topography is shown to be valid near shore on 'nonsmooth' continental shelves. The theory includes longshore gradients and friction and produces formulae easily evaluated with coastal sea level data. The tidal currents across three such shelves (West Florida, south of Long Island, and Vancouver Island) are calculated and compared with those observed.

1. INTRODUCTION

In a previous paper (*Battisti and Clarke* [1982], hereinafter referred to as I), the authors presented analytic models for accurately calculating barotropic tidal currents across 'smooth' continental margin topography (i.e., where longshore variations in coastline and topography are on a much larger scale than the width of the continental shelf). In a test of the models, the M_2 tidal currents were calculated at locations off the east and west coasts of the United States. Where measurements were available for comparison, the calculated M_2 currents were in good agreement with those measured.

The key assumption in deriving the models is that the continental margin topography is 'smooth.' Mathematically, the smooth margin approximation can be stated as

$$|l|a \equiv \varepsilon \ll 1 \quad (1)$$

where a is the distance offshore to the shelf break and the complex function l is defined by

$$il \equiv \eta_y/\eta \quad (2)$$

In (2), $i^2 = -1$, η is the deviation of the sea surface from its undisturbed level, and the subscript y denotes differentiation alongshore. If x is defined as the distance from the coast, to order (lx), l is independent of x; l is consequently easily evaluated from coastal sea level data [see *Clarke and Battisti,* 1981]. On smooth continental margins, $|l|^{-1}$ is of the magnitude of the spatial scale for the deep-sea tide [*Clarke and Battisti,* 1981]. This spatial scale is 0(2000 km) for the semidiurnal tides and is even larger for the diurnal tides. Therefore, ε is small even on very wide smooth shelves. While many of the world's shelves are smooth, some are not, as strong longshore variations in the coastline or topography may generate significant longshore gradients (e.g., off of Southern Long Island due to nearby Long Island Sound and the New York Bight).

In I, calculations were made at locations where the smooth continental margin approximation was clearly valid ($|l|a \ll 1$). The purpose of this note is to show that the models can be used to calculate accurately the tidal currents on nonsmooth shelves provided calculations are done at distances x close enough to shore so that $|l|x \ll 1$. After presentation of the theory (section 2), the M_2 tidal currents on the west Florida shelf and off south Long Island, and the K_1 tidal current off Vancouver Island are calculated and compared with observed currents (section 3). Concluding remarks are presented in section 4.

Paper number 2C0750.
0148-0227/82/002C-0750$05.00

2. THE MODELS

In I, the analytic expressions for the barotropic tidal currents are obtained by solving the Laplace Tidal Equations, written here

$$u_t - fv = -g\eta_x - \frac{\tau_B{}^x}{\rho H} \quad (3)$$

$$v_t + fu = -g\eta_y - \frac{\tau_B{}^y}{\rho H} \quad (4)$$

$$\eta_t + (Hu)_x + (Hv)_y = 0 \quad (5)$$

where u, v, f, g, τ_B, ρ, and H are the offshore and longshore velocities, the Coriolis parameter, the acceleration due to gravity, the bottom stress, the water density, and the depth, respectively. The subscripts x, y, and t denote differentiation with respect to the spatial coordinates and time. The coordinate axis is chosen so the origin is centered at the coast.

2.1. *Model I*

Integrating the continuity equation (5) along the normal from the coast at $x = 0$ to x gives

$$-\int_0^x \eta_t \, dx = [Hu]_0{}^x + \int_0^x (Hv)_y \, dx \quad (6)$$

Because of the no flux boundary condition at the coast, $u \sim v$ and $x \leq$ shelf width, one has

$$u(x, y, t) = -\frac{1}{H}\int_0^x \eta_t \, dx \, [1 + 0(|l|x)] \quad (7)$$

Over most shelves the sea level does not vary significantly [*Clarke and Battisti,* 1981]. The change across the shelf is of order μx, where

$$\mu = \left\{\frac{\omega^2 - f^2}{g\alpha} + \frac{fl}{\omega}\right\} \quad (8)$$

and where α is an offshore-averaged bottom slope and ω is the tidal frequency. Thus to within an error $0(\mu x, lx)$ (6) can be written

$$u = -i\eta(0)\omega x/H \tag{9}$$

where $\partial/\partial t$ has been replaced by $i\omega$ and $\eta(0)$ is the coastal sea level. Assuming a frictional stress linear in velocity, τ_B can be written

$$\tau_B = \rho r \mathbf{u} \qquad r = C_D|\mathbf{u}| \tag{10}$$

where C_D is the bottom drag coefficient and $|\mathbf{u}|$ is taken, in practice, to be the root mean squared velocity. Use of (2), (4), (9), and (10) yields the analytic solution for the longshore tidal current v:

$$v = \frac{\eta(0)}{\left(1 - \dfrac{ir}{H\omega}\right)} \cdot \left[\frac{xf}{H} - \frac{gl}{\omega}\right] \tag{11a}$$

In (9) and (11a) the depth H and r are general functions of x. Equations (9) and (11a) are valid where $|l|x$ and $|\mu x|$ are small.

Note that in deriving (9) and (11a) it was effectively assumed that $u \geq v$, this assumption enabling us to proceed from (6) to (7). However, such an assumption is not always valid. Consider the case when $v \gg u$. Since $f/\omega \sim 1$, the term fu in (4) is negligible and one has

$$v = \frac{\eta(0)}{\left(1 - \dfrac{ir}{H\omega}\right)} \cdot \frac{-gl}{\omega} \cdot (1 + 0(|l|x)) \tag{11b}$$

From (5) an estimate for u is $u < x\eta(0)\ \omega/H$. Therefore, $u/v \ll 1$ implies $\omega^2 x/(Hg|l|) \ll 1$ or, since $f/\omega \sim 1$, $(xf/H)/(g|l|/\omega) \ll 1$. Equations (11$a$) and (11$b$) are therefore effectively the same, and (11a) is consequently valid even when $v \gg u$. Note that the formula for u given by (9) may be invalid when $v \gg u$ ($\int_0^x (Hv)_y\, dx$ may not be negligible in (6)), but since $v \gg u$, the current vector has small error.

2.2. *Model II*

Across most continental margins $|\eta(x)/\eta(0)| \cong 1$ is valid, and (9) and (11a) can be used to calculate the currents. However, across wide shelves $|\mu x|$ may not be $\ll 1$ (e.g., in the outer regions of the continental shelf in the South Atlantic Bight). For these cases the following expressions should be used (see I):

$$u = \frac{\eta(0)g\,[\gamma(1 - \gamma x/2)(i\omega + \lambda_0) - il(1 - \gamma x)f]}{(i\omega + \lambda_0)^2 + f^2} \tag{12}$$

$$v = \frac{-\eta(0)g\,[\gamma(1 - \gamma x/2)f + il(1 - \gamma x)(i\omega + \lambda_0)]}{(i\omega + \lambda_0)^2 + f^2} \tag{13}$$

where

$$\gamma = \frac{i\omega}{(i\omega + \lambda_0)} \cdot \left[\frac{lf}{\omega} - \frac{(i\omega + \lambda_0)^2 + f^2}{g\alpha}\right]$$

and

$$\lambda_0 = \frac{1}{a - \delta} \cdot \int_\delta^a \frac{r(x)}{H(x)}\, dx$$

The error in (12) and (13) is $0(|l|x, (\gamma x)^2/2))$. Here bottom topography has been approximated as linear in x, and λ_0 is a shelf-averaged value for friction (see I for a discussion of λ_0). Equations (9), (11a), (12), and (13) are evaluated with the knowledge of just the bathymetry, $|\mathbf{u}|$, $\eta(0)$, and l. The latter two are easily determined from coastal sea level data. As previously stated, the first model has H and r as general functions of x, and so this model should be used whenever $|\mu|x \ll 1$.

To summarize, the two models combined can be used for calculating tides on any shelves with error $0(|l|x)$. Thus, results even for 'nonsmooth' shelves where longshore variations are comparable to the shelf width ($\varepsilon = |l|a \sim 1$) can be obtained provided calculations are done close enough to shore such that $|l|x \ll 1$. Specific examples are considered in section 3.

3. Application of Theory

To illustrate the use of the models in regions where the smooth continental margin approximation breaks down, tidal currents are calculated and compared with those measured in the following locations: (1) Off the West Coast of Florida. The Florida Keys mark the location of strong deviations from a smooth shelf approximation. The M_2 tide was calculated here due to the predominance of this constituent (largest signal to noise ratio). Here ε ranges from 0.16 to 0.77. (2) South Long Island. Here, strong longshore gradients in the M_2 tide are observed ($\varepsilon \cong 0.4$), due to the substantial deviations in the coastline in the form of Long Island Sound and the New York Bight. (3) Off West Vancouver Island. Here, the Straight of Juan de Fuca is a major deviation from a smooth coastline, and large longshore gradients for the diurnal tides are observed ($\varepsilon \cong 0.14$).

To calculate the tidal currents in these locations, sea level tidal constants and bathymetric data were obtained from the International Hydrographic Bureau and the U.S. Department of Commerce (National Ocean Survey Maps), respectively. Estimations of l were done by a least squares fitting of the coastal sea level data, as described in *Clarke and Battisti* [1981]. $|\mathbf{u}|$ was taken to be the root mean square velocity of the major tides (see I).

The tidal currents were calculated along transect sections oriented perpendicular to the coast, chosen to coincide with the locations of the measurements. The location of the transects and the locations of the current measurements are shown in Figure 1. As described in section 2, use of (9) and (11a) are restricted to regions where $\eta(x)$ does not substantially change from the coastal value, say $|\eta(x)/\eta(0) - 1| \leq 0.1$. Where $|\eta(x)/\eta(0) - 1|$ was not ≤ 0.1, (12) and (13) were used (see I for details).

With one exception off Florida, the distance offshore x to which calculations were made was limited to where $x \leq 0.15/|l|$ (see discussion). Calculated and measured currents are expressed in elliptical format and summarized in Table 1 and Figure 2. A discussion of the results follows.

3.1. *The West Florida Shelf*

The calculated and observed M_2 tidal currents on the West Florida Shelf are in very good agreement. The shelf can be divided into two distinct tidal regimes, from Naples to Tampa Bay and from Tampa Bay to Apalachee Bay. These regimes are discussed separately below.

From Naples to Tampa Bay. For the offshore region extending from Naples to St. Petersburg (Tampa Bay), the

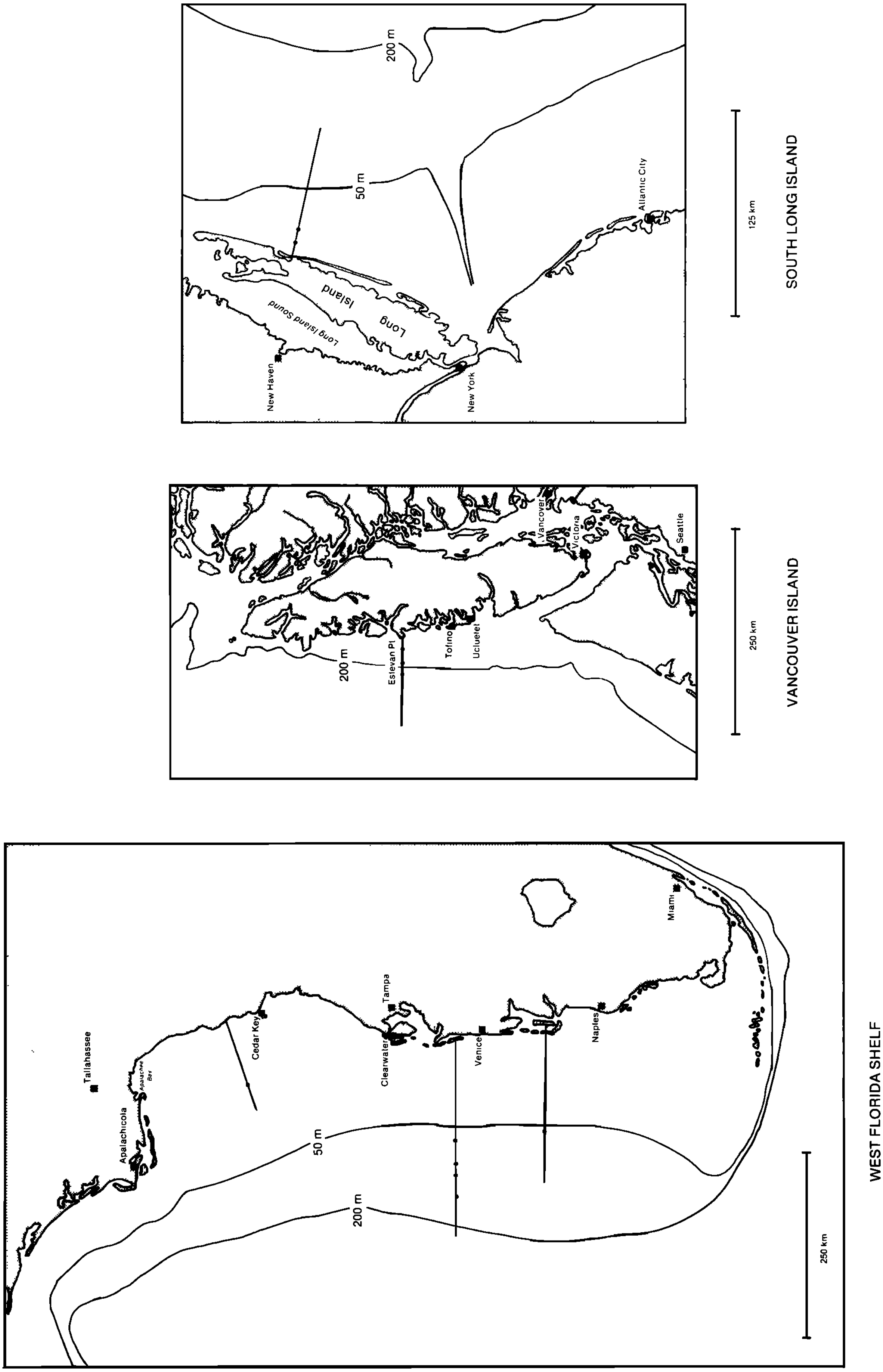

Fig. 1. Locations of the transect sections where calculations were made. The dots represent the locations of the current observations cited in the text. The locations of key isobaths (in meters) are included.

TABLE 1. Calculated and Observed Tidal Currents Across the Transects in Figure 1

		Current amp (m/s)					
x(km)	H(m)	Maximum	Minimum	E	ϕ(°)	ωt(°)	Observation Source
			Cedar Key, West Florida Shelf, M_2				
30	9.2	0.189	0.041	−0.22	1	87	*Mitchum and Sturges* [1982]
40	13	0.182	0.043	−0.24	1	86	
50	16	0.175	0.046	−0.26	0	85	
60	18	0.168	0.048	−0.29	0	84	
75	22	0.158	0.052	−0.33	0	81	
		(0.181)	(0.001)	(−0.00)	(4)		
			Venice, West Florida Shelf, M_2				
30†	22	0.041	0.016	−0.41	−21	99	*Koblinsky* [1981]
50	29	0.055	0.024	−0.44	−13	96	
70	40	0.053	0.023	−0.44	−13	95	
90	48	0.051	0.022	−0.44	−12	95	
110	57	0.049	0.021	−0.44	−12	95	
116	64	0.048	0.021	−0.44	−11	94	
		(0.050)	(0.017)	(−0.34)	(−15)		
140	90	0.046	0.020	−0.44	−11	94	
		(0.042)	(0.013)	(−0.30)	(−11)		
158*	120	0.037	0.017	−0.46	−15	93	
		(0.040)	(0.012)	(−0.30)	(−17)		
180*	158	0.027	0.012	−0.43	−17	96	
		(0.026)	(0.011)	(−0.42)	(−17)		
			Venice-Naples, West Florida Shelf, M_2				
30	19	0.076	0.030	−0.39	−20	98	*Koblinsky* [1981]
50	26	0.073	0.028	−0.39	−19	97	
70	35	0.069	0.026	−0.38	−18	97	
90	44	0.065	0.025	−0.38	−17	96	
110	53	0.062	0.023	−0.38	−16	95	
		(0.078)	(0.024)	(−030)	(−17)		
			Southern Long Island, M_2				
6†	27	0.106	0.006	−0.06	81	69	*May* [1979]
		(0.113)	(0.022)	(−0.19)	(76)		
9†	29	0.108	0.009	−0.09	78	67	
		(0.101)	(0.021)	(−0.21)	(77)		
12†	32	0.110	0.012	−0.11	76	67	
		(0.098)	(0.020)	(−0.20)	(82)		
20†	37	0.117	0.019	−0.16	71	65	
30†	46	0.122	0.024	−0.20	68	64	
40	50	0.129	0.031	−0.24	65	63	
50†	55	0.134	0.037	−0.27	62	62	
60	59	0.153	0.064	−0.42	51	64	
			Vancouver Island, K_1				
20	125	0.152	0.003	−0.02	−96	−79	*Crawford and Thomson* [1982]
		(0.15)	(0.02)	(−0.13)	(−98)		
35	170	0.153	0.004	−0.03	−98	−78	
		(0.11)	(0.04)	(−0.36)	(−93)		
55		0.154	0.006	−0.04	−99	−78	
		(0.05)	(0.03)	(−0.60)	(−106)		

The current magnitudes are in m/s, E is the ellipticity (defined in the text), ϕ is the angle of the maximum current relative to the $+x$ axis, and ωt is the time after high water of maximum current. The values in parenthesis are the observed currents reported by the referenced source. The dagger denotes the use of (9) and (11a) to calculate currents, otherwise (12) and (13) were used. On the smooth Venice shelf the asterisk denotes use of an extension of (12) and (13) to calculate the currents here (see (3.8) and (3.9) of I).

tidal currents are oriented normal to the shore and have anticyclonic rotation with ellipticity $E = -0.4$, close to the local value $-f/\omega \cong -0.5$. Here, ellipticity E is defined as the ratio of the minimum/maximum current, with the sign denoting the rotation sense (positive is cyclonic). As discussed in I, $E = -f/\omega$ is expected in a region where the topography dominates frictional and longshore gradient effects. Therefore, in this region the deviation in the coast in the form of the Keys does not effect the longshore gradient substantially ($|l|$ is $0(1.4 \times 10^{-6}\ m^{-1})$, the order of magnitude as one would expect for an open coastline).

From Tampa Bay to Apalachee Bay. The currents north of Tampa Bay to Apalachee Bay are much stronger ($0(0.16\ m\ s^{-1})$) than those calculated south of Tampa Bay. This is to be expected because the shelf is shallower in this region (i.e., the average bottom slope α is smaller). In fact, calculations of the M_2 sea level using the theory of *Clarke and Battisti* [1981] as well as the numerical calculations of R. O. Reid (personal communication, 1982) indicate that this part of the West Florida Shelf is resonant for the M_2 tide. When the coastal tide is amplified coastal tidal currents are also since u, v are proportional to $\eta(0)$.

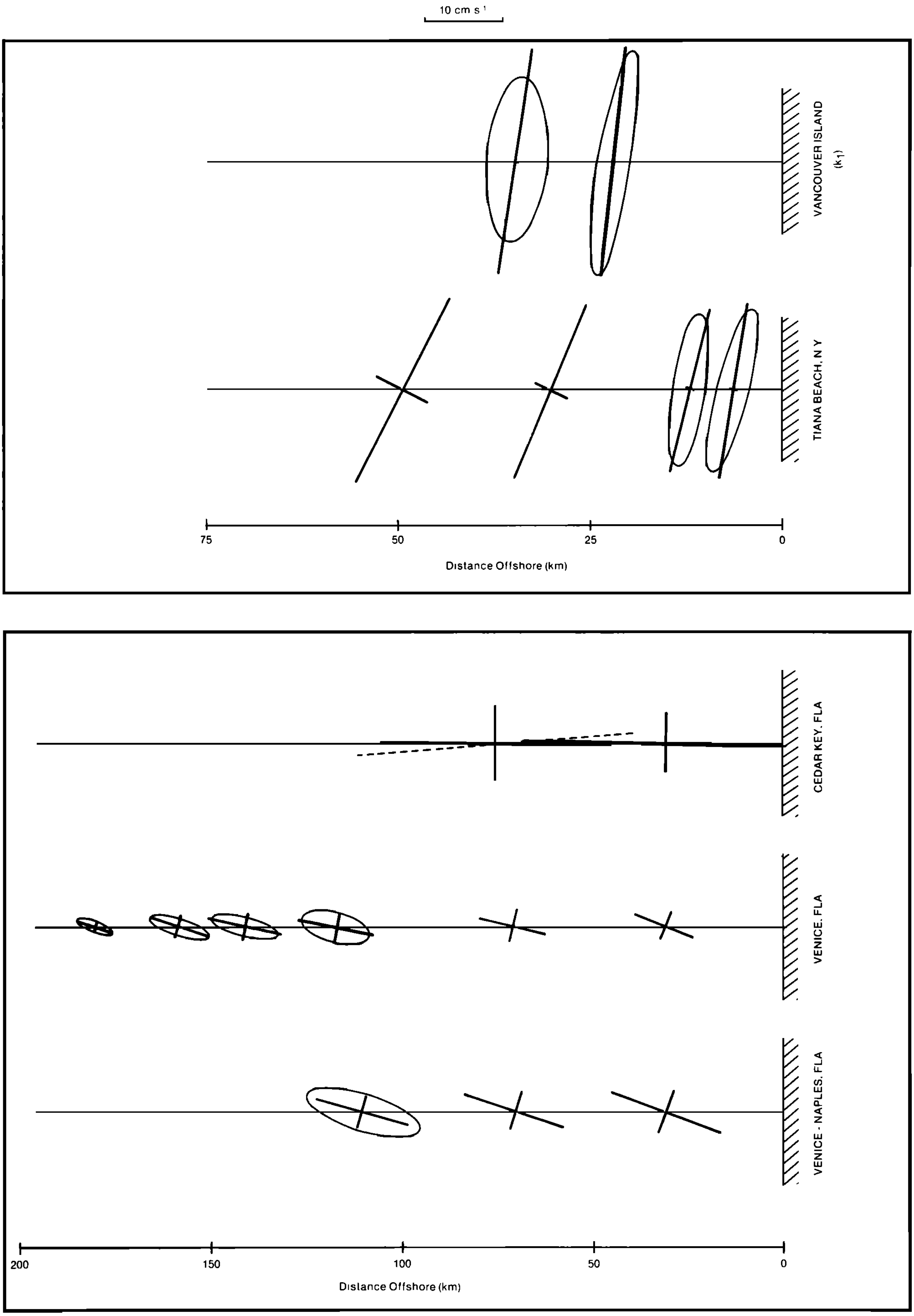

Fig. 2. Calculated versus observed currents. The solid ellipse axes denote the calculated currents. The ellipses are the observed currents. The oblique dashed line off Cedar Key is an observed current ($E = 0$). Note that the Florida offshore scale is different from that of the other cases.

There is a strong longshore gradient ($|l|$ is $0(2.9 \times 10^{-6}$ $m^{-1}))$ in Apalachee Bay due to the complex nature of the coastline here. This longshore gradient gives a significant error $|l|x$ of 20% at the location 75 km offshore. However, the agreement between observation and calculation is qualitatively correct and demonstrates the two very different M_2 tidal regimes on the west Florida shelf.

The strong longshore gradient off Cedar Key reduces v and thus the magnitude of the ellipticity from a purely topographical value of $|-f/\omega|$ to 0.28. This gradient may be due to the dent-like feature in the coast from Apalachee to Clearwater. If this feature were to act like a channel, then v would be zero, and, since v is proportional to $f\eta_x - \eta_{yt}$, there would be a strong longshore gradient. This dent-like feature is clearly not a channel. However, it does extend across half the shelf and has a scale comparable to $|\mu|^{-1}$, so it may partially act like a channel in reducing v and causing a strong longshore gradient.

3.2. *South Long Island*

Agreement between theory and observation off south Long Island is very good. Near the coast, the M_2 currents are anticyclonic and strongly oriented in the longshore direction ($E \cong -0.1$). This is because close to the coast the longshore gradient contribution to the current is large compared with the topographic contribution ($fx/H \ll |lg/\omega|$ in (7)).

The large longshore gradient in this region seems to be generated by the New York Bight/Hudson Canyon feature and the entrance to Long Island Sound. There is a steady increase in sea surface height along the coast from eastern Long Island ($\cong$0.40 m) to the New York Bight ($\cong$0.65 m). Additionally, Long Island Sound acts to distort the phase of the M_2 tide, creating a strong longshore gradient along the southeastern edge of the island. If the longshore gradient were insignificant here, one would expect the ratio v/u to be $-f/\omega$ ($\cong -0.7$) with maximum current normal to the coast, and maximum current (offshore) 3 hours after high water at the coast (friction is not important since $r/\omega H \cong 0.07$). Instead, what is observed (and calculated) in the nearshore region is maximum currents to the east about 3 hours after high water.

As the shelf break is approached the topographic term in (7) increases so that fx/H is $0(|l|g/\omega)$, and the ellipse axes in the mid-shelf region rotate in a clockwise direction from those observed near shore. The magnitude of the ellipticity $|E|$ increases to $\cong$0.4.

3.3. *Vancouver Island*

Calculations of the K_1 currents off Vancouver Island are in good agreement with the observed currents very close to shore. The currents are nearly alongshore, highly elliptical, rotate clockwise, and are relatively fast ($0(0.15$ m s^{-1})). Maximum current is to the north at approximately 78° before high water at the coast, in contrast to the Kelvin wave which has η and $-v$ in phase. The current is an order of magnitude larger than that expected if l were given by the deep-sea value ($0(10^{-7}$ m^{-1})), due to the strong longshore gradients here ($|l| \sim 5 \times 10^{-6}$ m^{-1})). *Crawford and Thomson* [1982] attribute these strong currents (large l) to a continental shelf wave. They show numerically that a trapped barotropic shelf wave of frequency K_1 does produce the coastal velocity magnitude and structure observed, with a wavelength ~330 km. This wave length is clearly recognizable in coastal sea level data (a 5 cm oscillation is observed over the 'background' Kelvin wave signal of 40 cm).

Calculations of currents away from the shore quickly give rise to much larger errors than one would expect from the estimate $|l|x$. For example, in the extreme case of the shelf edge, calculated and observed currents (not shown) are very different, yet $|l|x$ is only 0.17. However, it is true that $|l|x$ is only an order of magnitude estimate and that stratification effects have been ignored, so that perhaps this result is not surprising. Whatever the reason, at the coast where l is known unambiguously, calculated and observed currents are in good agreement.

4. CONCLUDING REMARKS

The analytic models developed in *Battisti and Clarke* [1982] for calculating barotropic tidal currents across 'smooth' continental margin topography have been shown to be valid (to $0(|l|x)$) on nonsmooth continental shelves (i.e., where ε is not $\ll 1$). Using these models, the tidal currents were calculated across three such shelves: the West Florida Shelf, off Southern Long Island (both for the M_2 tide), and off Vancouver Island (the K_1 tide). The agreement between the calculated currents and those observed is very good.

In conclusion, it should be noted that particular care is required in calculating l for semidiurnal and diurnal tides above the critical latitudes where shelf waves may exist. These shelf waves generally have a longshore scale of 0(100 km) and a relatively small sea surface signal. Therefore, to estimate accurately l coastal tidal stations must be close enough together so that noise can be eliminated from the sea level signal and aliasing problems can be avoided.

Acknowledgment. This work was supported by NSF grant OCE-8118051.

REFERENCES

Battisti, D. S., and A. J. Clarke, A simple method for estimating barotropic tidal currents on continental margins with specific application to the M_2 tide off the Atlantic and Pacific coasts of the United States, *J. Phys. Oceanogr.*, *12*, 8–16, 1982.

Clarke, A. J., and D. S. Battisti, The effect of continental shelves on tides, *Deep Sea Res.*, *28A*, 665–682, 1981.

Crawford, W. R., and R. E. Thomson, Continental shelf waves of diurnal period along Vancouver Island, submitted to *J. Geophys. Res.*, 1982.

Koblinsky, C. J., The M_2 tide on the west Florida shelf, *Deep Sea Res.*, *29A*, 1517–1532, 1981.

May, P. M., Analysis and Interpretation of tidal currents in the coastal boundary layer, Doctoral thesis, Woods Hole Oceanographic Institution/Massachusetts Institute of Technology, 1979.

Mitchum, G. T., and W. Sturges, Wind-driven currents on the west Florida shelf, submitted to *J. Phys. Oceanogr.*, 1982.

(Received March 4, 1982;
revised May 12, 1982;
accepted May 13, 1982.)