Haurwitz Memorial Lecture

Scale Interactions and the Generation of Low-Frequency Variability in the Atmosphere

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Bernhard Haurwitz

• Born 14 August 1905, Glogau, Lower Silesia
• Studied at U's of Breslau, and Göttingen
• PhD under Ludwig Weickmann at Leipzig, 1927
  – Inferred the vertical structure of midlatitude storms from balloon measurements
• Joined Rossby at MIT in 1932
  – Studied the effect of cloud type on solar radiation from measurements (Manabe and Stickler, 1964)
  – Deduced from theory the compensation depth of hurricanes to be ~10km, and the eyewall to be funnel-shaped. Haurwitz(1936)
Bernhard Haurwitz

- Carnegie Fellow at U. of Toronto 1935-37, visiting lecturer until 1941 supported by Canadian Met. Service
  - Three classic papers on dynamic meteorology, 1937-1940 -> Rossby-Haurwitz Waves
  - Published *Dynamic Meteorology* in 1941
- Associate Professor MIT, July 1941
- Chair of Dept. of Meteorology at NYU 1947
- Moved to University of Colorado and HAO 1959
- Moved to NCAR in 1964
- Lectured at U. of Alaska Fairbanks
- Lectured at Colorado State University
Rossby-Haurwitz Wave Theory


\[
c = U \left(1 - \frac{L^2}{L_s^2}\right)
\]
\[
L_s = 2\pi \sqrt{\frac{U}{\beta}}
\]

  - some of his solutions on a sphere to the barotropic equations are exact, and people refer to that solution as the Rossby-Haurwitz Wave, which has been used to test dynamical codes (e.g. Bourke (1972) and Hoskins (1973)).
Rossby-Haurwitz Wave Theory

\[ c = U \left( 1 - \frac{L^2}{L_s^2} \right) \]
\[ L_s = 2\pi \sqrt{\frac{U}{\beta}} \]
\[ c = U - \frac{\beta}{4\pi^2} L^2 \]

- For fixed \( c \), \( L \) increases with \( U \)
- For fixed \( c \) and \( U \), \( L \) increases as \( \beta \) decreases
Outline

• Introduction to Eddy-Driven, Self-Maintaining Jets on Earth and in Earth-Like models
• Variability of Eddy-Driven Jets
• Barotropic models of eddy driven jet variability.
• Localized jets and Eddy Feedbacks
• Rossby Wave Scales and jet latitude
From the mass-averaged zonal-mean momentum balance, it seems the only way to induce midlatitude westerly jet is with meridional eddy momentum transport.

\[
\frac{\partial[u]}{\partial t} \approx - \frac{\partial}{\partial y} \left( u^* v^* \right) - K_D \left( u_{sfc} \right)
\]
Why Eddies move Momentum

• We now relate meridional eddy momentum flux to the generation, propagation and breaking of Rossby waves.
• Rossby waves are generated preferentially in the Extratropics, primarily by baroclinic instability.
• Rossby waves propagate away from their source regions, and flux momentum in the opposite direction toward the wave source region.
• Rossby waves propagate preferentially toward the Tropics and they break there as they approach critical lines deduced from linear theory.
• Acceleration of zonal momentum in the same direction as the rotation in regions of wave generation is required when a gradient in absolute vorticity is present.
Eddy, Mean-Flow Jet Stabilization Conceptual Model

- Wave Source in Jet
- Wave Breaking - Easterly Acceleration
- Wave propagation out of jet
- Westerly Acceleration
\[
\frac{\partial u}{\partial t} = -\frac{\partial}{\partial y} u'v' = v'\zeta'
\]
Eddy-Driven Jets

• Eddies must have source in jet – e.g. conversion from potential to kinetic energy as in baroclinic instability – thermal gradient is largest under jet, or in Tropics, convection.
• Eddies must be able to propagate out of jet, mostly meridionally, and break elsewhere in order to move momentum around “permanently”.

Panetta (1993)

- Broad, Baroclinic, doubly periodic b-plane

- Flow organizes into strong westerly jets, separated by weak easterlies
- Temperature and Potential Vorticity gradients maximize in these jets, heat flux more uniform.
- Jets persist and slowly move around in latitude in presence of highly turbulent flow.

- In upper layer momentum balance is between eddy momentum flux convergence, driving jet, balanced by mean meridional circulation.
- In the lower layer (not shown here) the balance is between mean meridional circulation and drag.
Observed Zonal Flow in Southern Hemisphere

Eddy-Driven jet is characterized by Surface westerlies, Is clearly distinguished from the Hadley-driven subtropical jet, And is present in all seasons.

First EOF is robust and represents a meridional shift of the jet position.

Hartmann and Lo, 1998
Southern Annular Mode
Expressed as EOF of zonal mean wind

- The first EOF is a **shift** with latitude: larger variance and more persistent.
- The Second EOF is an **intensification** (**pulse**) of the climatological jet at 50S: less persistent.

Hartmann and Lo, 1998
Southern Hemisphere Eddy-Driven Jet.

First EOF represents N-S shift of eddy driven jet.

1.5 standard deviation of PC-1 corresponds to 10° latitude shift of surface westerlies.

Fig. 8. Zonal mean wind of high index composite, low index composite, and the high composite minus the low composite. Contour interval is 5 m s⁻¹, dotted line is zero contour, and dashed lines are negative.

Hartmann and Lo, 1998
Southern Hemisphere Eddy-Driven Jet.

Momentum Budget of Meridional Eddy-Jet Meandering

Transformed Eulerian Mean Formulation

\[
\frac{\partial}{\partial t} [u] \cos \varphi
\]

Residual Circ.  

\[
= f[v^*] \cos \varphi + \frac{1}{r \cos \varphi} \frac{\partial}{\partial \varphi} \left\{ -[u'v'] \cos^2 \varphi \right\}
\]

Barotropic  

\[
+ \frac{\partial}{\partial p} \left\{ f \left[ \frac{\partial [\theta']}{\partial [\theta]} \right] \cos \varphi \right\} + F_x \cos \varphi.
\]

‘Baroclinic’  
aka Form Drag

Drag determined as residual

Hartmann and Lo, 1998
Momentum Budget of Meridional Eddy-Jet Meandering

Barotropic

‘Baroclinic’ aka Form Drag

Total Eddy Forcing

Residual Circ.

Drag determined as residual

Hartmann and Lo, 1998

Fig. 9. Transformed Eulerian mean momentum balance for the high index composite (left column), low index composite (center), and high minus low difference (right). Terms are defined in text. Contour interval for composites is 1.5 m s$^{-1}$ day$^{-1}$ and for difference plots is 0.4 m s$^{-1}$ day$^{-1}$. Solid contours indicate a westerly acceleration and dashed lines an easterly acceleration. The zero contour is omitted.
Residual Circulation and Self-Maintenance of Anomalies

- Eddy momentum flux at top drives residual circulation that sustains surface zonal wind anomalies against drag.

- Adiabatic heating associated with residual circulation supports meridional temperature gradient against eddy heat flux convergence.

Hartmann and Lo, 1998
Self-Maintenance and Eddy Feedback

- Eddy-Driven jets are self-maintained, and
- Eddies can feed back on zonal flow anomalies to enhance them and enhance the persistence of anomalies
- **Counter argument**: The diagnostics are just showing dynamical consistency and its all random, zonal flow is more persistent because it is weakly damped.
Quantification of Eddy Feedback
Lorenz and Hartmann (2001)

• Demonstrate net positive eddy feedback for first EOF – meridional shift of jet. Feedback accounts for about half of the low-frequency variance of this mode of variability.

• Demonstrate no feedback for second EOF – pulsing of strength of jet.

• Strong high-frequency eddy feedback

• Slightly weaker negative feedback by other eddies – not as strongly sourced in jet and are refracted into jet.
Simple Model of Positive Eddy Feedback

Linear System
\[ \frac{dz}{dt} = m - \frac{z}{\tau} \]

Assume part of momentum forcing depends on zonal wind.
\[ m = \tilde{m} + b z \]

Choose \( b \) to explain long-term memory, then \( z \) without feedback can be computed.
\[ \frac{d\tilde{z}}{dt} = \tilde{m} - \frac{\tilde{z}}{\tau} \]

b. High-frequency eddies produce low-frequency forcing, because they respond to zonal flow.

Fig. 7. Observed statistics (thick line) and calculated statistics for no feedback (thin line): (a) cross-correlation between \( z \) and \( m \) and between \( \tilde{z} \) and \( \tilde{m} \); positive lag means \( z \) leads \( m \); (b) power spectra of...
Zonal Asymmetry

- Eddy feedbacks drive climatological zonal-mean wind jets and add persistence to north-south shifting anomalies.
- But atmosphere has zonal asymmetry.
- Repeat analysis, but shift to analysis of vorticity, rather than zonal-mean wind.
  - Simple scalar budget
  - Doesn't assume structure
Shading = Climatological zonal wind
Contours = SAM Structure

SAM is meridional shift of zonal wind, slightly downstream of eddy-driven midlatitude jet.

Barnes & Hartmann, 2010b JAS
Eddy Feedback Calculation

• Since we are not taking zonal averages, it is easier to use the vorticity budget, which is a scalar quantity, with a handy conservation equation.

\[
\frac{\partial \hat{\zeta}}{\partial t} = \left[ - (\zeta + f) \nabla \cdot \mathbf{u} - \hat{\zeta} \nabla \cdot \mathbf{u} \right]_{\text{stretching}} \\
+ \left[ - \nabla \cdot (\hat{\zeta} \mathbf{u}) \right]_{\text{eddy}} \\
+ \left[ - \mathbf{u} \cdot \nabla (\zeta + f) - \mathbf{u} \cdot \nabla \hat{\zeta} \right]_{\text{wave}} \\
+ \left\{ - \nabla \cdot [\mathbf{u} (\zeta + f)] \right\}_{\text{clim}} + \mathcal{F},
\]

(2)
Eddy Vorticity Flux Feedback in Southern Hemisphere

AAO/SAM Vorticity Structure at upper levels

\[ \frac{\zeta_{SAM}}{\nabla \cdot (V' \zeta')} \approx 2.5 \text{ days} \]

Synoptic eddy vorticity flux convergence associated with SAM

(all definitions based on EOF of sea level pressure)

Barnes & Hartmann, 2010b, JAS

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Does forcing pattern match response?

- Transient eddies provide a vorticity flux that reinforces vorticity signature of jets.
- Amplitude is sufficient to replace vorticity anomaly in less than a week.
- Is vorticity forcing that is in-phase with vorticity anomaly efficient for reinforcing the pattern? (e.g. would forcing in shape of mean pattern actually reinforce pattern or produce some other pattern, like a Rossby wave downstream?)
Barotropic response to Vorticity Dipole

\[
\frac{\partial \xi}{\partial t} + J(\psi, \xi + f) + \sigma \nabla^4 \xi + \frac{\xi}{\tau} = F_{\text{dipole}} + F'\\
\]

5-day linear drag
Steady Vorticity Dipole forcing = \(F_{\text{dipole}}\)
Random Red Noise added = \(F'\)

Vorticity Forcing  Vorticity Response  Zonal Wind Response

Haurwitz Lecture 2011  Zhu & Hartmann 2011
Eddy Feedback in 3-D

Eddies flux vorticity into vorticity maximum

Vorticity input at upper levels is balanced by a divergence term

Vertical motion to support upper level divergence
a. Cools center of vorticity anomaly.
b. Supports convergence at lower levels to balance drag.

Vorticity anomaly is more persistent than it would be in the absence of high-frequency eddies.

Barnes & Hartmann 2010a
Eddy Feedback Jet Self-Maintenance
Robinson (2000, 2006)

- Baroclinic eddies are generated
- They propagate away and dissipate elsewhere, leaving a barotropic westerly wind anomaly
- Surface drag generates baroclinicity (residual circulation is involved here)
- Baroclinic eddies are generated

Assumes continued forcing of broad baroclinic zone
Is a Movable Wave Source Necessary?

- Meridional eddy propagation seems critical, and this is determined by horizontal wind shear, not baroclinicity.
- Vertical eddy flux seems less critical to the variation than meridional eddy flux.
- A Barotropic model with fixed eddy stirring of vorticity explains most of the interesting features (Vallis et al 2004; Barnes et al. 2010)
Relative change of \( u'v' \) much bigger

Contour interval 1.5 m/s/day
Contour interval 0.4 m/s/day

Hartmann and Lo, 1998
Barotropic Model

Stirring of eddies vorticity in broad envelope in mid latitudes produces an eddy driven jet that has many of the characteristics of observed annular variability in data and baroclinic models (Vallis et al. 2004).

**Fig. 2.** The time- and zonally averaged zonal wind (solid line) from the zonally symmetric numerical model Z1 (see Table 1). The dashed line is the rms (i.e., eddy) velocity. The stochastic forcing is zonally uniform and is a Gaussian distribution in the meridional direction, centered at 45° with 12° half-width.

**Fig. 3.** The pseudomomentum stirring $F_{\xi\xi}$ and dissipation $D_{\xi\xi}$ and their sum [see Eq. (2.10)] for Z1. The distribution of dissipation is broader than the forcing, resulting in an eastward jet where the stirring is centered, with westward flow on the flanks.
**Observed Features that the Barotropic model produces**

- Shifting and Pulsing Modes (Vallis et al '04)
- Shifting mode has persistence similar to observations (Vallis et al 04)
- Shifting mode has positive eddy feedback, pulsing mode does not. (LH'01, Barnes et al '10)
- Negative skewness of the shifting mode –more persistent when jet is displaced toward the equator.
- Change from shift to pulse as the jet moves poleward
- Less eddy feedback and persistence as the jet moves poleward
- Weakened shifting mode enhanced pulsing mode as eddy-driven jet moves closer to subtropical jet (BH'11)
Barotropic Model – Latitude and Scale

Stir as in Vallis et al. (2004), but move eddy forcing across latitude from 30° to 40° to 50°. Scale of the resulting eddies is larger for higher latitudes.

Scale of Eddy Forcing

Scale of Eddy Response

Barnes & Hartmann, 2011
Barotropic Model – Latitude and Scale

Total wavenumber and zonal scales as a function of eddy-driven jet latitude, and quasi-linear theory.

Kidston et al. jet shift scale increase.

\[ K^* = \cos \theta \left( \frac{\beta^*}{u - c} \right)^{1/2} \]

Total Wavenumber Scaling

Zonal Length Scale

Barnes & Hartmann, 2011
Barotropic Model – Latitude and Scale

Quasi-Linear model: reduce wave amplitude by factor of 100 and increase momentum flux by corresponding factor -> linear waves, but comparable magnitude of eddy momentum flux to nonlinear integration.

Barnes & Hartmann, 2011
Barotropic Model – Latitude and Scale

Phase speeds and wind speeds are similar for the 30- and 50-degree jets, so the scale change seems mostly related to latitude.

Vorticity Power spectra as function of zonal wavenumber and phase speed
Barnes & Hartmann, 2011
Increase in eddy scale with more poleward eddy-driven jet

- Since zonal wind and phase speed don’t change much, it must be primarily latitude that causes the increase in scale, in these barotropic experiments.

\[ K^* = \cos \theta \left( \frac{\beta^*}{\bar{u} - c} \right)^{1/2} \]
Spatial Scales in ERA-40 Reanalysis

- Estimate scale through the one point correlation map of vorticity anomalies, after removing mean seasonal cycle from daily instantaneous fields.

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Barnes & Hartmann, 2011
Midlatitude Zonal Length scales are about 20-30% greater in the Southern Hemisphere than the Northern Hemisphere
**Fig. 4.** 200 mb anomalous vorticity correlation e-folding lengths in the (left) zonal and (right) meridional directions. The climatological 200 mb zonal wind is contoured every 10 m/s starting at ±20 m/s, with negative contours dashed.
DJF, 300mb (-45°, 90°): lag = -10
DJF, 200mb (10º, 330º): \( \text{lag} = -10 \)
DJF, 200mb (2°, 215°): lag = -10
DJF, 300mb (48°, 335°): lag = -10
DJF, 200mb (-30°, 308°): \text{lag} = -10
Zonal and Meridional Vorticity Scales

DJF

JJA
Vorticity Scales at 50mb

Fig. 5. 50 mb anomalous vorticity correlation e-folding lengths in the (left) zonal and (right) meridional directions. The climatological 50 mb zonal wind is contoured every 10 m/s starting at ±10 m/s, with negative contours dashed.
JJA, 50mb (-2°, 260°): lag = -10
DJF, 50mb (5°, 210°): lag = 10
Eddy-Driven Jets and Change

• The troposphere is almost inviscid, but waves can move momentum around because they can propagate and break efficiently.
• Eddy feedback produces internal modes of low-frequency variability, notably meridional jet shifting.
• This gives large natural internal atmospheric variability that is unforced, but can respond to relatively modest external forcings (e.g. QBO, stratospheric ozone loss or greenhouse gas warming).

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Thank You

• You for your attention
• AOFD for this honor
• Grad Students (especially Libby Barnes and Brian Smoliak)
• Colleagues for ideas, encouragement and inspiration
Quasi-Biennial Oscillation

Downward propagating oscillation of tropical stratospheric zonal winds on the equator
Tropospheric Response to QBO

Garfinkel & Hartmann, 2010
Tropospheric Response to QBO

Dry GCM – No Eddies
Steady response to QBO momentum forcing

Garfinkel & Hartmann, 2010

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Eddy Interaction with QBO anomaly

Garfinkel & Hartmann, 2010

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Tropospheric Response to QBO

Dry GCM – With Eddies
Transient response to QBO momentum forcing
Eddies move easterly QBO anomalies downward by shifting jet

Garfinkel & Hartmann, 2010

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Observed response of zonal wind to QBO

Oct-Nov
AR-40 Pacific
WACCM Pacific

Dec-Jan

Feb-Mar
AR-40 Atlantic
Garfinkel & Hartmann, 2010

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Eddy-Driven Jets and Feedbacks

- A very interesting problem in large-scale dynamic meteorology
- Very important for understanding the intraseasonal and interannual variability of the General Circulation
- Very important for understanding the response of the general circulation to natural and human influences.
- Thank You!
Wave Propagation Theory

• If the jet-eddy feedback explains extratropical jets and their variability, we should be able to use wave propagation theory to explain some of their observed behavior.

\[ K^* = \left( \frac{\bar{q}_y \cos^2 \theta}{\bar{u} - c} \right)^{1/2} \]

\( k < K^* \) propagate, \( K^* \to 0 \) turning latitude for all waves, waves are refracted toward larger \( K^* \), poles in \( K^* \) indicate critical lines and wave breaking.

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Bernhard Haurwitz - Sources

- BAMS, 1985, Meteorology in the 20th Century – A Participants View, Bernhard Haurwitz – 4 parts
  - Contains complete publication list
  - Interesting papers in which P.D. Thompson and J. Tribbia reflect on the contributions of Rossby and Haurwitz to the theory of Rossby-Haurwitz waves

Would someone like to consult these references and write a Wikipedia page for Bernhard Haurwitz?
Fig. 1. Leading EOF of DJFM monthly-mean sea level pressure over the Atlantic region 25°–90°N, 90°W–30°E. Contours are drawn every 2.6 mb with negative contours dashed and zero contour line omitted.
NAO Vorticity Anomalies and High-Frequency, Synoptic, Eddy Forcing.

High-frequency eddy vorticity flux convergence projects very well onto low level vorticity anomaly.
Quantification of Eddy Feedback using Southern Hemisphere
Lorenz and Hartmann (2001)

Focus on vertical average momentum balance
and meridional wave propagation.

\[
\frac{\partial \langle[u]\rangle}{\partial t} = - \frac{1}{\cos^2 \phi} \frac{\partial (\langle[u'v']\rangle \cos^2 \phi)}{a \partial \phi} - F,
\]

where \( \langle u \rangle \) is the vertical average of \( u \), \( [u] \) is the zonal

Then reduce to first EOF of \( U \), call it \( Z \), and corresponding structure in
eddy momentum forcing, call it \( M \),
then consider the system,

\[
\frac{\partial Z}{\partial t} = M - \frac{Z}{\tau}
\]

Lorenz & Hartmann, 2001

Fig. 2. (a) EOF1 of the zonal-mean zonal wind; (b) EOF1 of the
vertical and zonal-mean zonal wind. The percent variance explained
is given at top-right corner of plot.
Linear model of Annular Mode

\[
\frac{\partial Z}{\partial t} = M - \frac{Z}{\tau}
\]

\[Z = \text{amplitude of first EOF of } \hat{[u]}\]

\[M = \text{corresponding } - \frac{\partial}{\partial y} [u^* v^*]\]

Fourier Cross-Spectra of Z and M behave like linear equation is a good model.

Lorenz & Hartmann, 2001
Positive Eddy Feedback

\[ Z = u \]
\[ M = -\frac{d}{dy}(u'v') \]

**Clues**

a. M remembers Z

b. High-frequency eddies produce low-frequency forcing.

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**Fig. 5.** Cross correlation between z and m.

**Fig. 6.** Power spectra of the zonal index forcing by the time-filtered eddies: (a) synoptic-eddy forcing and (b) residual-eddy forcing. Vertical axis is power spectral density in m² s⁻² day⁻².

Lorenz & Hartmann, 2001
Eddy Fluxes and Zonal Momentum

Consider a non-divergent, barotropic fluid

\[
\frac{\partial [u]}{\partial t} = - \frac{\partial}{\partial y} [u^* v^*] = [v^* \zeta^*]
\]

\[
\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

Enstrophy Equation

\[
\frac{\partial}{\partial t} \left[ \frac{1}{2} \zeta^{*2} \right] + \beta_{\text{eff}} [v^* \zeta^*] = [F^* \zeta^*]
\]

\[
\beta_{\text{eff}} [v^* \zeta^*] = [F^* \zeta^*]
\]

Up-gradient eddy vorticity flux

Source of eddy enstrophy
Stirring Eddies to Generate Zonal Flow

Steady Enstrophy Equation

$$\beta_{\text{eff}} [v \star \zeta^*] = [F \star \zeta^*]$$

If vorticity source $F^*$ adds enstrophy, eddy vorticity flux must be up-gradient (normally northward) to maintain steady state.

Zonal Wind Equation

$$\frac{\partial[u]}{\partial t} + K_D[u] = -\frac{\partial}{\partial y}[u \star v^*] = [v \star \zeta^*] = \beta_{\text{eff}}^{-1}[F \star \zeta^*]$$

So that a source of wave enstrophy can support a westerly zonal mean wind against friction in the presence of a positive gradient of mean vorticity.

Also: Meridional Wave Propagation

$$c_{gy} \propto -[u \star v^*]$$