On a Physical Mechanism for Rossby Wave Propagation

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ABSTRACT

A simple class of barotropic Rossby waves are shown to propagate westward as the geostrophically balanced meridional windfield periodically reverses in response to a small meridional pressure gradient arising from the latitudinal variation of the Coriolis parameter.

1. Introduction

The Rossby wave provides the simplest model for large scale disturbances in midlatitudes. It is therefore useful to have a thorough physical understanding of the mechanism by which these waves propagate. In the original discussion of the wave which now bears his name, Rossby (1939) draws on arguments proposed by Bjerknes (1937) to construct a physical interpretation of the wave propagation mechanism. Bjerknes and Rossby discussed the velocity changes experienced by an air parcel traveling through a region of equally-spaced sinusoidal pressure perturbations in a mean westerly flow. They noted that competing effects, due to the latitudinal variation of the Coriolis parameter and the tendency to maintain gradient wind balance, produce a pattern of convergence and divergence, causing the pressure perturbations to propagate westward relative to the mean flow. Subsequent authors (Platzman 1968; Hoskins et al. 1985) and many textbooks have explained the propagation mechanism as the response of the fluid to the constraint of potential vorticity conservation. "Potential vorticity reasoning" is a powerful tool that can provide important insight into a variety of complex meteorological problems, and the Rossby wave is a good example on which to illustrate its utility. However, the physical processes responsible for the propagation of a barotropic Rossby wave can be easily understood without recourse to arguments involving gradient wind balance or the circulations induced by potential vorticity anomalies. The purpose of this note is to provide an alternative physical explanation of the propagation mechanism.

2. Nondivergent Rossby waves on the midlatitude β-plane

Consider the horizontally nondivergent flow of an incompressible, homogeneous, inviscid fluid on the standard midlatitude β-plane. Let ψ be a streamfunction for the horizontal velocity field. As shown by Rossby (1939), ψ satisfies the vorticity equation

$$\frac{d\nabla_h^2\psi}{dt} + \beta\frac{\partial\psi}{\partial x} = 0. \tag{1}$$

A finite-amplitude solution to (1), representing a Rossby wave with uniform meridional structure, is

$$\psi = \psi_0\cos(kx - \omega t), \quad \omega = -\beta/k. \tag{2}$$

(Note that by convention $\omega > 0$, so $k < 0$.) The velocity perturbations in the wave are

$$u = -\frac{\partial\psi}{\partial y} = 0, \tag{3}$$

$$v = \frac{\partial\psi}{\partial x} = -\psi_0 k\sin(kx - \omega t). \tag{4}$$

Using the fact that $u = \partial v/\partial y = 0$, the horizontal momentum equations governing the disturbance reduce to

$$(f_0 + \beta y)v = \frac{1}{\rho_0}\frac{\partial p}{\partial x}, \tag{5}$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p}{\partial y}. \tag{6}$$

It follows that

$$p = (f_0 + \beta y)\rho_0\psi_0\cos(kx - \omega t). \tag{7}$$

These relationships may be used to construct a schematic (Fig. 1) of the pressure and velocity perturbations at a given moment in the wave. As shown by (5), the meridional wind is in geostrophic balance with the pressure field (and the zonal wind is zero). Equation (5) also shows that, because of the latitudinal variation in f, the magnitude of the pressure perturbation must increase with latitude along each line of constant phase in order to geostrophically balance v. As a consequence, $\partial p/\partial y > 0$ (and $\partial p/\partial y < 0$) in the shaded (unshaded) regions of Fig. 1. According to (6), these meridional pressure gradients produce local accelerations, $\partial v/\partial t$, that force the disturbance in the velocity field to propagate toward the west. In summary, *the geostrophically*

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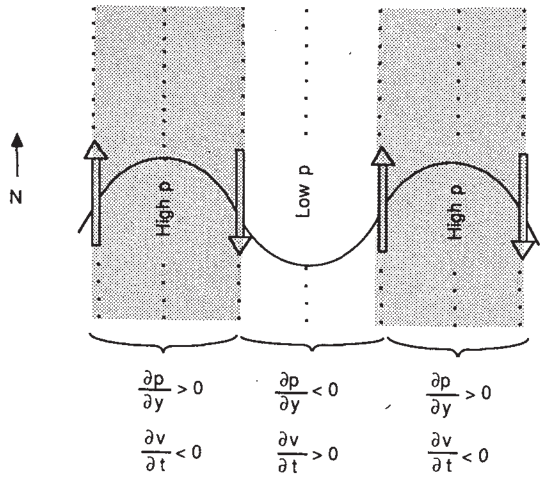

FIG. 1. The instantaneous distribution of velocity perturbations (heavy arrows) and pressure perturbations in a nondivergent, barotropic Rossby wave. The dotted lines are the lines of constant phase in the x–y plane. The solid curve shows the position of a string of fluid parcels (a material curve) which has been displaced by the wave motion. Those portions of the x–y plane where $\partial p/\partial y$ is positive are shaded.

balanced meridional windfield periodically reverses in response to a small meridional pressure gradient arising from the latitudinal variation of the Coriolis parameter.

3. The quasi-geostrophic shallow-water system

The same fundamental mechanism governs the propagation of quasi-geostrophic Rossby waves in the shallow-water system, in which the free-surface displacement (η) satisfies the potential vorticity equation

$$\frac{D_0}{Dt}\left(\nabla_h{}^2\eta_0 - \frac{f_0{}^2}{gH}\eta_0\right) + \beta\frac{\partial\eta_0}{\partial x} = 0, \qquad (8)$$

where

$$\frac{D_0}{Dt} \equiv \frac{\partial}{\partial t} + u_0\frac{\partial}{\partial x} + v_0\frac{\partial}{\partial y}$$

and H is the depth of the undisturbed fluid. In the above, (η_0, u_0, v_0) are the geostrophically balanced free-surface displacement and velocity fields. The order Rossby number (Ro) corrections to these fields will be denoted by (η_1, u_1, v_1). A finite-amplitude solution to (8), representing a quasi-geostrophic Rossby wave with uniform meridional structure is

$$\eta_0 = \tilde{\eta}\cos(kx - \omega t), \quad \omega = \frac{-\beta k}{k^2 + f_0{}^2/gH}. \qquad (9)$$

The geostrophically balanced velocities associated with this wave are $u_0 = 0$ and

$$v_0 = -\frac{gk\tilde{\eta}}{f_0}\sin(kx - \omega t). \qquad (10)$$

The O(Ro) corrections are not unique; the simplest behavior is obtained by setting $u_1 = 0$. Then the O(Ro) balance in the y-momentum and continuity equations reduces to

$$\frac{\partial v_0}{\partial t} = -g\frac{\partial\eta_1}{\partial y}, \qquad (11)$$

$$\frac{\partial\eta_0}{\partial t} = -H\frac{\partial v_1}{\partial y} \qquad (12)$$

and, therefore

$$v_1 = -\frac{\omega y\tilde{\eta}}{H}\sin(kx - \omega t), \qquad (13)$$

$$\eta_1 = -\frac{\omega ky\tilde{\eta}}{f_0}\cos(kx - \omega t). \qquad (14)$$

Except for the addition of the meridional ageostrophic velocities v_1, the schematic illustrating this shallow-water Rossby wave (Fig. 2) is identical to the previous example. As in the nondivergent case, westward propagation of the geostrophically balanced meridional velocity field is produced by the north–south pressure gradient $g\partial\eta_1/\partial y$ arising from the latitudinal variations in the Coriolis parameter. The patterns of convergence and divergence associated with $\partial v_1/\partial y$ raise the free surface west of the ridgeline and account for the westward propagation of the pressure pattern.

Since the choice $u_1 = 0$ is arbitrary, the schematic

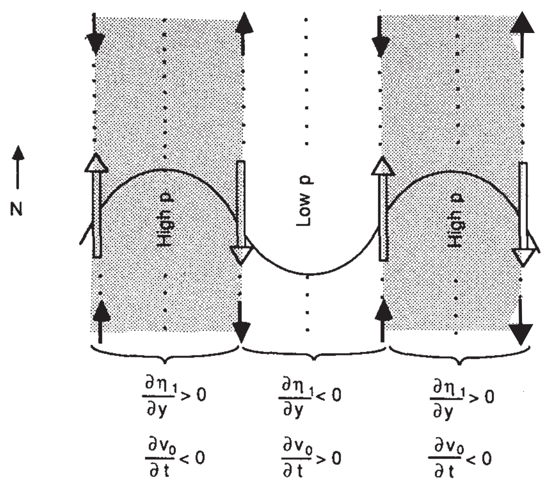

FIG. 2. As in Fig. 1 for a shallow-water Rossby wave. The small arrows show the O(Ro) correction to the meridional velocity responsible for the westward propagation of the perturbations in the free surface displacement. (v_1 has been plotted according to (13) as if $y = 0$ in the center of the figure.)

in Fig. 2 should be understood to apply to this particular special case. A more common practice (though still arbitrary) is to choose $\eta_1 = 0$ and to divide the velocity into a geostrophically balanced part (u_0, v_0) and an ageostrophic part, which in the framework of quasi-geostrophic theory, becomes (u_1, v_1). Still other possibilities can be obtained by setting different linear combinations of u_1, v_1 and η_1 to zero. Each choice produces a somewhat different O(Ro) circulation. Those differences should not be allowed to obscure the essential point, which is that any time there is a wave-like pattern of geostrophically balanced north–south flow, small meridional gradients must be present (in v and/or p) due to the variation of the Coriolis parameter with latitude. The perturbations associated with these meridional gradients act to reverse the direction of the north–south flow and shift the wavelike pattern to the west.

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