Comments on "Gravity Wave Refraction by Three-Dimensionally Varying Winds and the Global Transport of Angular Momentum"

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1. Introduction

Hasha et al. (2008, hereafter HBS) recently examined the role of horizontal variations in the large-scale flow on the pseudomomentum flux in vertically propagating gravity waves, focusing on applications in climate dynamics. They conclude "… columnar parameterization schemes for topographic waves should produce results that are little affected by the neglect of three-dimensional effects, which is encouraging." While I do not dispute their conclusions about the net influence of horizontal atmospheric structure on monthly-averaged profiles of the pseudomomentum flux, it is important to note that results published previously by Chen et al. (2005, hereafter CDH) indicate that very different outcomes may be obtained if one considers the influence of horizontal inhomogeneity on topographically generated gravity wave momentum fluxes during specific synoptic-scale events. CDH show that horizontal variations in the large-scale flow over a mountain, together with the concomitant stretching or shrinking of horizontal wavenumbers along ray paths, can significantly modify the momentum flux associated with high-frequency topographic waves. In addition, CDH show that significant errors can also be produced if the temporal variations in the large-scale flow are neglected, as they are in the ray-tracing calculations in HBS and virtually all gravity-wave-drag (GWD) parameterizations.

2. Horizontal confluence and diffluence

CDH compared two time-dependent flows: a horizontally uniform flow and an eastward-translating barotropic jet with dynamically consistent regions of confluence upstream of the jet maximum and diffluence downstream. The temporal variation in the horizontally uniform zonal winds matched the temporal variation in the wind speed over the mountain top in the case with the propagating jet. The influence of horizontal variations in the large-scale flow is presented in Figs. 4c,d and 6 of CDH, which are reproduced here as Fig. 1. Figure 1a shows the domain-averaged mountain wave momentum flux as a function of height as diagnosed from a numerical simulation that explicitly resolves the gravity waves in the case with horizontally uniform flow; Fig. 1c gives the same information for the case with the barotropic jet. Figures 1b and 1d give the corresponding momentum flux profiles as diagnosed from ray tracing and the conservation of wave action. As a point of reference, if the fluxes calculated by a time-independent columnar GWD parameterization were displayed in the same format as in Fig. 1, all the contour lines would run vertically and have the same values at $z = 0$ as those in Fig. 1a.

The maximum momentum fluxes generated by gravity waves in the horizontally uniform flow are only 80% of those produced by waves in the jet. Larger discrepancies can be seen at specific times and vertical levels. Above 4 km, the momentum fluxes during the acceleration phase ($t < 25$ h) of the idealized jet case are generally weaker than those in the horizontally uniform flow. For example at $(z, t) = (16, 20)$, the ratio of the momentum flux in the jet case to that in the uniform case exceeds 1.5, as diagnosed from the simulated gravity waves, and exceeds 1.2 for the ray-tracing calculation. Conversely, the momentum fluxes aloft during the decelerating phase ($t > 25$ h) are generally weaker in the case with the idealized jet. The differences during the decelerating phase may be appreciated by comparing the values at $(z, t) = (8, 37.5)$, which is the point shown by the black dot in each panel. At this point the gravity wave momentum flux is almost 40% greater in the case with uniform large-scale forcing.

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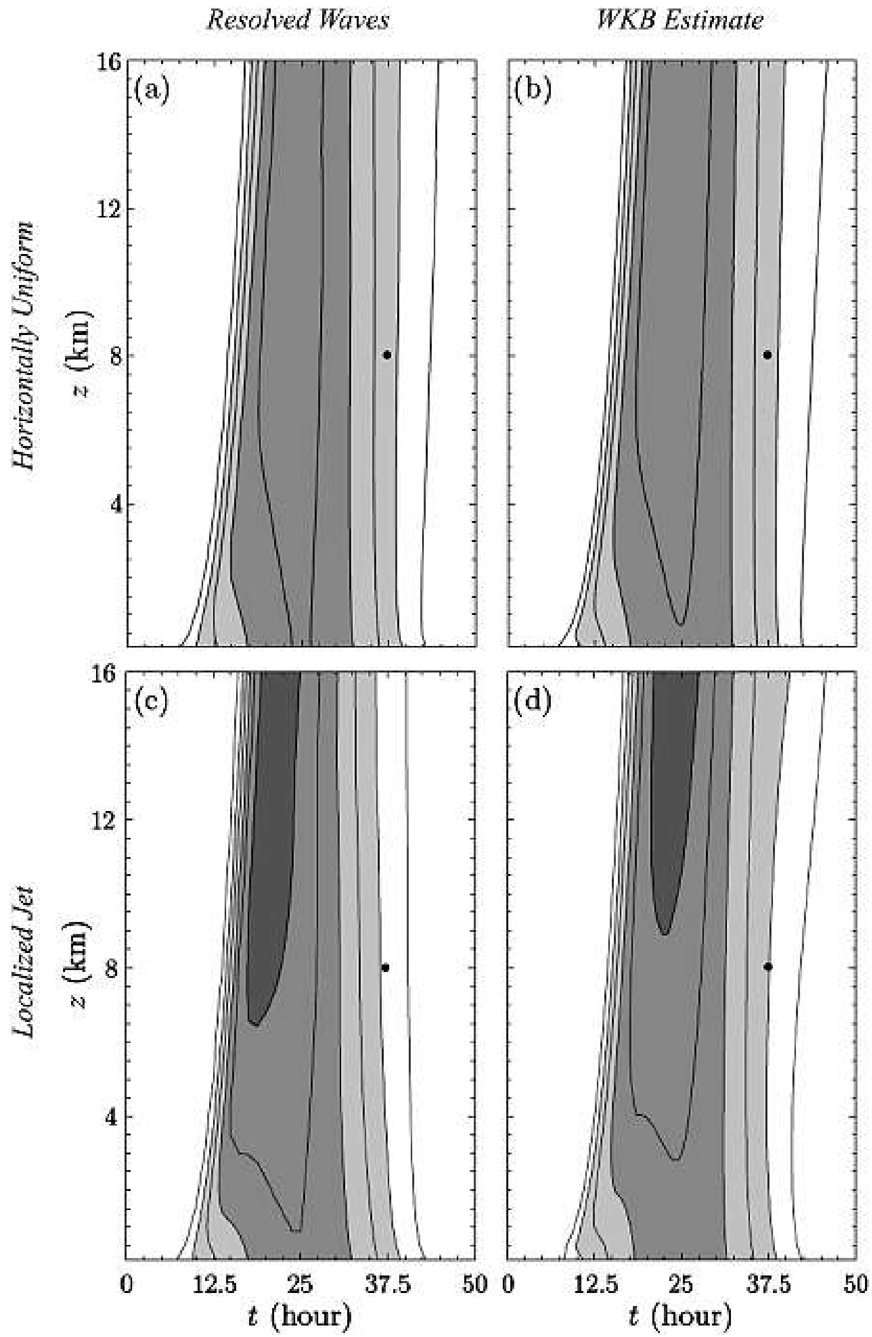

FIG. 1. Gravity wave momentum flux integrated over the horizontal domain as a function of t and z for (a),(b) the horizontally uniform flow and (c),(d) the barotropic jet. The direct evaluation of the fluxes due to gravity waves resolved in the numerical model is presented in (a) and (c); (b) and (d) show the fluxes computed using ray-tracing theory and conservation of wave action. The contour interval is 10^8 N; steps in the gray shading are at -2×10^8, -4×10^8, and -6×10^8 N.

Why is the sensitivity to horizontal variations in the large-scale flow documented in CDH so much larger than that found in HBS? It is hard to say for the case of the single ray shown in Fig. 1 of HBS, but a hypothesis is readily available for the results presented in their Fig. 3, which compares profiles of the January mean of the globally integrated pseudomomentum flux computed using columnar and fully three-dimensional ray tracing. HBS note "the relative difference between the two profiles averages 4% though most of the domain . . . ."

The primary factor responsible for the differences between the horizontally uniform and the jet cases in CDH is the change in cross-ridge horizontal wavenumber k associated with variations in the cross-ridge velocity U along an axis perpendicular to the ridge. Letting x denote the coordinate perpendicular to the ridge, the dominant terms in HBS's Eq. (4) are

$$\frac{dk}{dt} = -\frac{\partial U}{\partial x}k,$$

implying that k increases as waves propagate through regions of diffluence and decreases when the propagation is through confluent flow. In the barotropic case, wave action has a simple relation to the momentum flux [Eq. (17) of CDH], which implies that increases in k must be associated with increases in momentum flux to conserve wave action. Thus, as in Fig. 1, the momentum flux is enhanced in regions of flow diffluence and reduced where the flow is confluent.

If one were to sample the flow across some particular ridge below the midlatitude westerlies at a regular time intervals for an entire month, it is likely the sample would include several cases in which the cross-ridge flow was confluent and several in which it was diffluent. Although the horizontal variations in the large-scale flow might have a major influence on the momentum flux during individual confluent and diffluent flow events, these differences would tend to average out in the monthly mean. Similarly, the monthly averaged differences between the pseudomomentum fluxes estimated from columnar and three-dimensional schemes in HBS could be much smaller than the differences in individual cases.

3. Concluding remarks

Although here we have focused on cases in which horizontal variations in the large-scale flow produced nontrivial modifications to the momentum flux relative to that which would be estimated from a columnar parameterization, CDH also noted that wave packets launched in an accelerating cross-mountain flow have larger vertical group velocities than those launched earlier, and as they ascend they find themselves at the same elevation (but not the same three-dimensional spatial location) as their predecessors. The result is an accumulation of wave action aloft and, as visible in Fig. 1, a significant enhancement of the momentum fluxes during a period of mean flow acceleration (12.5 h $\leq t \leq$ 25 h). This suggests that in addition to problems arising from the neglect of three-dimensional effects, columnar schemes can also produce nontrivial errors if they neglect the time dependence of the large-scale flow and the concomitant variations in the vertical group velocities of individual wave packets.

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